Modeling and Simulation for Nonlinear Pressurized Water Reactor Cores Using Gap Metric and Fuzzy Model with Transfer Function

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Abstract. This investigation is to deal with the modeling issue for nonlinear pressurized water reactor cores. Nonlinearity measure with gap metric and T-S fuzzy modeling are exploited to build the fuzzy model of a nonlinear core which approximates this nonlinear core model. The gap metric of the core is proposed to quantify the core nonlinearity. The curve in the whole range of core power level that is solved adopting the gap metric is the core nonlinearity measure. In terms of the measure, six linearized core models at six power levels are selected as local models of the nonlinear core model. Based on the local models and the introduction of the triangle membership function, the core fuzzy model is achieved. The core fuzzy model and nonlinear core model are simulated. Simulation results show that the core fuzzy model can approximate and substitute the nonlinear core model.

1 Introduction

Nuclear energy as a kind of clean energy provides an incentive to sustainable development of nuclear power plants (NPPs) for electricity generation. Meanwhile, of research fields for NNPs, the modeling study on plants of NPPs such as reactor cores is always popular. The pressurized water reactor (PWR) cores in NPPs possess nonlinear characteristics, and parameters of them are varying at different operating conditions. Hence, cores are essentially time-varying complex systems so that modeling their models is difficult. It is necessary and meaningful to research the modeling issue of nonlinear cores.

Many researchers have utilized the point reactor core modeling method to construct PWR core models at the full power level. Kerlin et al.[1] modelled the PWR core of the H. B. Robinson NPP at the full power level which was also verified to be correct. PWR cores at the full power level were modeled by Edwards et al.[2-5]to design core power control systems.

However, such modeling for cores based on one power level cannot represent the dynamics of nonlinear core within the global ranges of power level. Stimulated by the considerations, the work in this paper is to model a nonlinear PWR core within the global ranges of power level by the use of gap metric to quantify the core nonlinearity measure and the T-S fuzzy principle. Finally, the T-S fuzzy model and nonlinear model of the core are simulated, and conclusions are drawn.

2 PWR core model

The point reactor core modeling as a traditional modelling method is adopted to model a PWR core, which is a lumped parameter method. For this modeling, a core is regarded as one point without any space profile, and parameters of the core only vary with time and have nothing to do with space positions.

Table 1. Main model parameters.

Parameter	Name
P_r	core power level
c_{ri}	ith group normalized precursor concentration
c_r	normalized precursor concentration
T_f	fuel average temperature
T_i	coolant inlet temperature
T_e	coolant outlet temperature
δrod	position variation of the control rod (fraction of core length)
δ	deviation of a parameter from initial steady-state value

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According to the point reactor core modeling [1-3], the nonlinear PWR core is modelled adopting the point kinetics equations with six groups of delayed neutrons and reactivity feedbacks due to control rod movement and variations in fuel temperature and coolant temperature. Main model parameters are given in Table 1. The nonlinear core model is showed as Eqs.(1)-(6).

$$\frac{dP_r}{dt} = \frac{\rho - \beta}{\Lambda} P_r + \sum_{i=1}^{g} \frac{\beta_i c_{ri}}{\Lambda} \tag{1}$$

$$\frac{dc_{ri}}{dt} = \lambda_i n_r - \lambda_i c_{ri}, i = 1, 2, \ldots, g \tag{2}$$

$$\frac{dT_f}{dt} = \frac{f_f P_0}{\mu_f} P_r - \frac{\Omega}{\mu_f} T_f + \frac{\Omega}{2\mu_f} T_i + \frac{\Omega}{2\mu_f} T_e \tag{3}$$

$$\frac{dT_e}{dt} = \frac{(1-f_f)P_0}{\mu_c} n_r + \frac{\Omega}{\mu_c} T_f + \frac{2M-\Omega}{2\mu_c} T_i - \frac{2M+\Omega}{2\mu_c} T_e \tag{4}$$

$$\rho = \rho_{rod} + \alpha_f (T_f - T_{f0}) + \frac{\alpha_c}{2}(T_i - T_{i0}) + \frac{\alpha_c}{2}(T_e - T_{e0}) \tag{5}$$

$$\delta\rho_{rod} = \alpha_{rod} \delta rod \tag{6}$$

The small perturbation linearization methodology is utilized to linearize this nonlinear core model, and the linearized core model is expressed by Eqs.(7)-(11).

$$\frac{d\delta P_r}{dt} = -\frac{\beta}{\Lambda}\delta P_r + \sum_{i=1}^{g} \frac{\beta_i}{\Lambda} \delta c_{r_i} + \frac{P_{r0}}{\Lambda}\delta\rho \tag{7}$$

$$\frac{d\delta c_{ri}}{dt} = \lambda_i \delta n_r - \lambda_i \delta c_{ri}, i = 1, \ldots, g \tag{8}$$

$$\frac{d\delta T_f}{dt} = \frac{f_f P_0}{\mu_f}\delta n_r - \frac{\Omega}{\mu_f}\delta T_f + \frac{\Omega}{2\mu_f}\delta T_i + \frac{\Omega}{2\mu_f}\delta T_e \tag{9}$$

$$\frac{d\delta T_e}{dt} = \frac{(1-f_f)P_0}{\mu_c}\delta n_r + \frac{\Omega}{\mu_c}\delta T_f + \frac{2M-\Omega}{2\mu_c}\delta T_i - \frac{2M+\Omega}{2\mu_c}\delta T_e \tag{10}$$

$$\delta\rho = \alpha_{rod}\delta rod + \alpha_f \delta T_f + \frac{\alpha_c}{2}\delta T_i + \frac{\alpha_c}{2}\delta T_e \tag{11}$$

One group delayed neutron model is utilized and the coolant inlet temperature is treated as a constant [2-4]. The transfer function and the state equation of the core are separately calculated and expressed by

$$G = \frac{\delta P_r}{\delta rod} = \frac{\sum_{i=0}^{3} a_i s^i}{\sum_{i=0}^{4} b_i s^i} \tag{12}$$

$$\begin{cases} \dot{\mathbf{x}} = A\mathbf{x} + Bu \\ y = C\mathbf{x} + Du \end{cases} \tag{13}$$

where $u=\delta rod$-the input; $y=\delta P_r$-the output; a_i (i=0,1,2,3)-numerator coefficients; b_i (i=0,1,2,3,4)-denominator coefficients; s-laplace operator; $\mathbf{x}=[x_1, x_2, x_3, x_4]^T=[\delta P_r, \delta c_r, \delta T_f, \delta T_e]^T$-the state matrix; A-the $R^{4\times4}$ system matrix; B-the $R^{4\times1}$ input matrix; C-the $R^{1\times4}$ output matrix; D-the zero matrix.

A transfer function of the core at a power level 10*i% is denoted by $G_{10*i\%}$ (i=1, . . . ,10). These transfer functions are calculated by using parameters from Ref. [6], in which the total primary heat output is 2200 MW, the primary coolant inlet temperature is 285°C, the primary coolant outlet temperature is 317°C, the primary coolant average pressure is 15.5 MPa, and the primary coolant mass flow is 12861.1 kg/s.

3 Nonlinearity measure with gap metric for core

The nonlinearity measure on the basis of the gap metric for the core is proposed to quantify the core nonlinearity. One group of linearized core models can be selected to approximate the nonlinear core in terms of distributing situations of the core nonlinearity measure in the while range of power level, and each linear model is regarded as a local model of the core.

According to the work [7, 8], the gap metric of the core is developed as follows to solve the core nonlinearity.

Let O_i (i=1,2) be two closed linear operators in a Hilbert space H_s. $H_s \times H_s$ is also a Hilbert space with inner product derived from H_s. The graph of O_i is denoted by $g_r(O_i)$. As O_i is linear, $g_r(O_i)$ is a subspace of the product Hilbert space $H_s \times H_s$.

Definition 3.1. The gap between two closed linear operators O_1 and O_2 is defined by the gap between $g_r(O_1)$ and $g_r(O_2)$:

$$\Delta(O_1, O_2) = \Delta(g_r(O_1), g_r(O_2)) \tag{14}$$

where $\Delta(\cdot)$is a metric.

Linear systems such as G of the core are viewed as mappings in a Hilbert space. Hence, G is treated as an operator.

Lemma 3.1. Frequency response systems are closed [7].

According to Lemma 3.1, G as a frequency response system is a closed linear operator in H_s. The gap metric in Definition 3.1 can be used to qualify the gap between two linearized systems of the core. So Eq.(15) is obtained by

$$\Delta(G_{10*i\%}, G_{10*j\%}) = \Delta(g_r(G_{10*i\%}), g_r(G_{10*j\%})) \tag{15}$$

where $\Delta(\cdot) \in [0\ 1]$; $i \neq j$; i,j=1,…,10.

Let H_∞ and H_2 be the standard Hardy spaces of functions, and let RH_∞ be the subspace of real rational functions in H_∞. $G_{10*i\%}$ is a 1×1 rational transfer matrix and has the normalized right coprime factorization as follows.

$$G_{10*i\%} = N_1 M_1^{-1},\ G_{10*j\%} = N_2 M_2^{-1} \tag{16}$$

with $M_1^* M_1 + N_1^* N_1 = I$ and $M_2^* M_2 + N_2^* N_2 = I$.
where for i=1 or 2, M_i and N_i belong to RH_∞, $M_i(s)^* = M_i(-s)^T$, $N_i(s)^* = N_i(-s)^T$.

$g_r(G_{10*i\%})$ or $g_r(G_{10*j\%})$ is a closed subspace of $H_2 \times H_2$ that consists of all pairs (δrod, δP_r) such that $\delta P_r = G_{10*i\%} * \delta rod$ or $\delta P_r = G_{10*j\%} * \delta rod$. They are presented by

$$g_r(G_{10*i\%}) = \begin{bmatrix} M_1 \\ N_1 \end{bmatrix} H_2,\ g_r(G_{10*j\%}) = \begin{bmatrix} M_2 \\ N_2 \end{bmatrix} H_2 \tag{17}$$

And

$$\Delta(g_r(G_{10*i\%}), g_r(G_{10*j\%})) = \left\| \Pi_{g_r(G_{10*i\%})} - \Pi_{g_r(G_{10*j\%})} \right\| \tag{18}$$

where $\prod_{(\,)}$ is the orthogonal projection onto $(\cdot)$.

Lemma 3.2. The gap between $G_{10*i\%}$ and $G_{10*j\%}$ is calculated by

$$\begin{aligned} &\Delta(G_{10*i\%}, G_{10*j\%}) \\ &= \left\| \Pi_{g_r(G_{10*i\%})} - \Pi_{g_r(G_{10*j\%})} \right\| \\ &= \max\{\vec{\Delta}(G_{10*i\%}, G_{10*j\%}), \vec{\Delta}(G_{10*j\%}, G_{10*i\%})\} \\ &= \max\{\inf_{q \in H_\infty} \left\| \begin{bmatrix} M_1 \\ N_1 \end{bmatrix} - \begin{bmatrix} M_2 \\ N_2 \end{bmatrix} q \right\|_\infty, \inf_{q \in H_\infty} \left\| \begin{bmatrix} M_2 \\ N_2 \end{bmatrix} - \begin{bmatrix} M_1 \\ N_1 \end{bmatrix} q \right\|_\infty \} \end{aligned} \tag{19}$$

where $\vec{\Delta}(\cdot)$ denotes a directed gap.

4 Fuzzy modeling

The T-S fuzzy method [9] can be used to handle modeling issues of nonlinear plants such as the core at varying operating conditions. According to this modeling, the power level operating range of the nonlinear core is partitioned into separate subregions, local dynamics in each region are represented by one linear model as a local model of this core; fuzzy rules are proposed adopting membership functions that are utilized to develop fuzzy sets of which the intersection of two ones is nonempty, the T-S fuzzy model that is the overall model to approximate the nonlinear core is then achieved via "blending" of all linear local models based on fuzzy rules.

4.1 Core local models based on nonlinearity measure

$G_{10\%}$ is chosen as a referenced model and a gap metric between $G_{10\%}$ and G_{Pr} for $P_r \in [0.1\ 1]$ is calculated by means of Section 3. This calculated gap metric is the core nonlinearity measure between 10% and P_r.

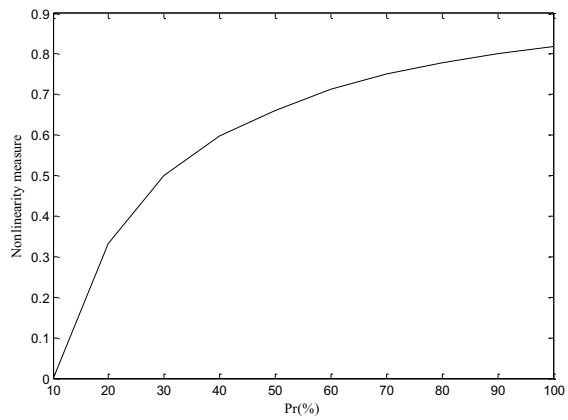

Figure 1. Nonlinearity measure of the core.

The nonlinearity measure curves of the core are obtained and shown in Fig.1, where the curve approximates a parabola. Comparing with two level areas [0.1 0.6] and (0.6 1.0], the curve change relatively rapidly in [0.1 0.6], and vary relatively gently in (0.6 1.0] in the form of an approximated straight line. These indicate that the core has the strong nonlinearity in [0.1 0.6] and the weak nonlinearity in (0.6 1.0]. Therefore, choose three linear models at 10%, 20%, 40%and 60% to substitute the nonlinear core in [0.1 0.6] with the strong nonlinearity, select two linear models at 80% and 100% to substitute the nonlinear core in (0.6 1.0] with the weak nonlinearity. Finally, linearized core models at five levels are selected as local models of the core to substitute the nonlinear core in (0 100%].

Let $G_1 = G_{10\%}$, $G_2 = G_{20\%}$, $G_3 = G_{40\%}$, $G_4 = G_{60\%}$, $G_5 = G_{80\%}$, $G_6 = G_{100\%}$. The state equation of G_i is represented by (A_i B_i C_i D_i) (i=1,…,6). Parameters of G_5 are enumerated: [a_3, a_2, a_1, a_0]=[5088 7021 1853 137.2], [b_4, b_3, b_2, b_1, b_0]=[1 401.4 692.2 389.9 43.2]. G_1, G_2, G_3, G_4 and G_5 are used to respectively denote dynamics of the nonlinear core located in the five power level ranges (0, 10%], [10%,

20%], [20%, 40%], [40%, 60%], [60%, 80%] and [80%, 100%] in turn.

4.2 Core fuzzy model

The triangle membership function is utilized to set up fuzzy rules. The membership function of the core fuzzy model at whole power levels is shown in Fig.2.

The fuzzy logic system (FLC) is of the following form.

Rule i: if P_r is M_i, then $Y_i=G_i$ (i=1,…,6)

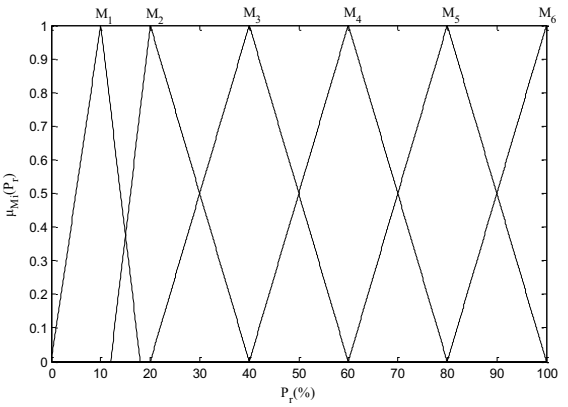

Figure 2. Membership function of core fuzzy model at whole power levels

where Rule i denotes the ith fuzzy rule; M_1, M_2, M_3, M_4, M_5 and M_6 respectively represent the fuzzy sets corresponding to the power levels 10%, 20%, 40%, 60%, 80% and 100%; $\mu_{Mi}(P_r)$ represents the membership when a power P_r belongs to the fuzzy set M_i, which is calculated in the light of a relationship between the membership and the core power level.

In the light of Fig.2, weight values q_i for the "blending" of G_i (i=1,…,6) is calculated as Eq. (20).

$$q_i = \frac{\mu_{M_i}(P_r)}{\sum_{i=1}^{6} \mu_{M_i}(P_r)}, i = 1,\ldots,6 \tag{20}$$

where $P_r \in (0\ 1]$, $q_i \in [0\ 1]$.

One value q_i indicates how the ith local model at a power level belongs to a core fuzzy model at this power level. Finally, the core fuzzy model is expressed by Eq.(21) or (22).

$$G_{FM} = \sum_{i=1}^{6} q_i G_i \tag{21}$$

$$\begin{cases} \dot{\mathbf{x}} = \sum_{i=1}^{6} q_i [A_i \mathbf{x} + B_i u] \\ y = \sum_{i=1}^{6} q_i C_i \mathbf{x} \end{cases} \tag{22}$$

5 Simulation

To verify effectiveness and correctness of this fuzzy modeling for the nonlinear core, the core fuzzy model (21) and the core nonlinear model are compared via simulations.

When the input δrod is taken as a 0.01 step, the fuzzy modeland the nonlinear model at ten typical power levels 10×i% (i=1,…,10) are simulated, and output responses of the fuzzy modeland nonlinear model at these levels are shown in Fig.3, which is namely responses of δP_r. In fig.3, the curve FMi (i=1,…,10) represents the output response of the fuzzy model at 10×i%, and the curve NLi (i=1,…,10) is the output response of the nonlinear model at 10×i%. From FMi and NLi, it can be observed that FMi approaches NLi. Similarly, the similar simulation results are able to be achieved for other power levels.

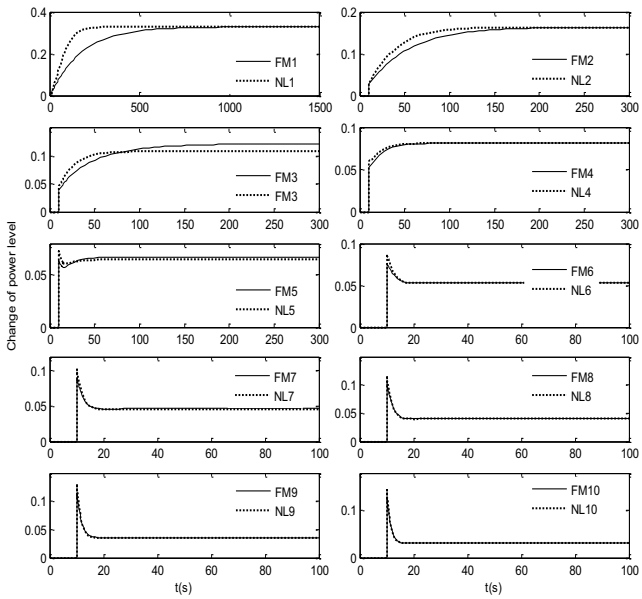

Figure 3. Output responses of fuzzy model and nonlinear model for the core at ten power levels

Hence, the core fuzzy model can approximate the core nonlinear model according to simulation results.

6 Conclusions

The work handles the modeling problem of nonlinear PWR cores. On the basis of modeling a nonlinear core and calculating the core nonlinear measure with the introduction of the gap metric, one group linear local models of the core is defined. In terms of local models and the triangle membership function, the T-S fuzzy model for the core is developed. From numerical simulations, it is obtained that the core fuzzy model can substitute the core nonlinear model. The proposed modeling principle as a reference method can be applied to model other reactor cores and nonlinear plants, and also applicable for stability analysis and control system design of nonlinear systems.

Acknowledgments

The authors would like to thank anonymous reviewers for their valuable comments. The work is funded by National High Technology Research and Development Program of

China (863 Program) (No.2015AAXX46611) and China Postdoctoral Science Foundation (No.20159200078).

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