Frequency Estimation Using SAMP-SVD Based on Nyquist Folding Receiver

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Abstract. The Nyquist Folding Receiver (NYFR) is an efficient analogy-to-information (A2I) architecture that folds the broadband RF inputs by subsampling so that can sample with a low-speed ADC. The compressive sensing (CS) model of the NYFR can be built in the sparse environment to recover by traditional CS algorithms. This paper presents an improved algorithm SAMP-SVD based on sparsity adaptive matching pursuit (SAMP) and singular value decomposition (SVD) which is intended to deal with the problem caused by SAMP recovering—the inversion of matrix, avoiding the problem of whether matrix is singular and reducing the time of computation. A comparative analysis between the presented method and sparse reconstruction by separable approximation (SpaRSA) is discussed; simulation results show the algorithm can reconstruct the signal effectively and estimate the frequencies accurately.

1 Introduction

The bottleneck in reconnaissance receiver performance is in the analog-to-digital conversion (ADC) process, where sampling rate and accuracy in limitation [1], cannot matching to radar coverage. Given this discord, we need to improve the receiving system on the present device level, breaking the limitation of ADC.

There are currently several sub-sampling architectures: the random demodulator (RD) [2] and the modulated wideband converter (MWC) [3] as a practical implementation of compressive sensing (CS) [4], where the signals is sampled completely randomly for suppression of aliasing and guaranteed reconstruction. Another efficient analogy-to-information (A2I) architecture is called the NYFR [5] in 2006 presented, which folds the broadband radio frequency (RF) input by sub-sampling with a stream of short pulses. The modulation sampling induces a bandwidth widening that varies with the Nyquist zone (NZ, i.e. folding index number). It is a periodic non-uniform sampling, which is a tradeoff between folding and aliasing. The undersampled signals then digitized by a narrowband ADC, which the same as CS techniques by using only a few non-zero coefficients in some suitable sparse representation, allowing recovery techniques in addition to traditional CS recovery methods such as reconstruction via $l_1$-minimization [6].

There are a number of classical approaches to recover information from signals, which are obtained as the solution to the constrained minimization problem in a CS setting, including $l_1$-minimization, Sparse Reconstruction by separable approximation (SpaRSA) [7], iterative greedy unfolding (IGU) [8] and matching pursuit (MP) [9], etc. $l_1$-minimization and SpaRSA are by means of optimization theory, the former tend to be slow for extremely large sampled data sets and, ill-suited for non-positive definite measurement matrix, if speed is not an issue, SpaRSA obtains higher resolution accuracy. MP and orthogonal matching pursuit (OMP) [10] techniques are the basis methods on pursuit algorithms, which enjoy highly efficient, low computational complexity and a restricted isometry property (RIP) [11]. stagewise orthogonal matching pursuit (StOMP) [12] reduces the complexity of the algorithm, but its accuracy not as good as OMP. regularized orthogonal matching pursuit (ROMP) [13] and compressive sampling matching pursuit (CoSaMP) [14] require the sparsity knowledge of signals, subspace pursuit (SP) [15] algorithm performance is similar to CoSaMP, and combination with OMP to get a powerful methods named SAMP [16].

This paper focuses on SAMP because of its efficiency and ease of implementation, and applies SVD to solve the least squares (LS) problem, for a high-accuracy frequency estimation under the condition of a low signal-to-noise ratio (SNR).
2 NYFR Sensing Matrix Model

CS builds upon the fundamental fact that signals can be represented by using only a few non-zero coefficients in a suitable basis or dictionary, projecting a high-dimensional signal onto a low-dimensional signal model, recovered such signals from very few measurements by optimization algorithm, to break through the traditional limits of sampling theory.

Assumptions about the analog RF input signals $x_s(t)$ of the NYFR architecture in a usual CS setting [8]; namely:

1) the input $x_s(t)$ can be sampled according to the Nyquist theorem, denoted as $x[n]$.

2) the discrete Fourier transform (DFT) $X[k]$ of $x[n]$ is sparse or highly compressible.

Note that the relationship between the NYFR architecture in a usual CS setting [8]; namely: $X_k$ is given by

$$y[n] = \phi x[n] \quad \text{or} \quad y[n] = \Phi X_k [k]$$

where $\phi \in \mathbb{C}^{n \times N}$ is measurement matrix and, $\Phi \in \mathbb{C}^{n \times n}$ is sensing matrix. Note that $N = K \cdot Z$, where $Z$ denotes index number (or folding number).

Motivated by RD model, the direct measurement matrix (D-measurement matrix) is given by

$$\phi = \text{diag} (P_1, P_2, \ldots, P_K)$$

where $P_i = \left[ p_{i \cdot (i-1)Z}, p_{i \cdot (i-1)Z}, \ldots, p_{i \cdot (N-1)Z} \right]$, $i = 1,2,\cdots,K$, and $p_j = \exp(-j(2\pi kf_j t + k_n \theta(t)))$, $j = 1,2,\cdots,N$, namely that the modulation sampling function is sampled by the Nyquist rate of the input $x_s(t)$.

A next consideration is signal reconstruction from $y[n]$ and $\phi$ by CS techniques. Actually, the NYFR signal reconstruction is dependent of $k_n$ (namely, folding index number) estimated correctly and the synchronization of measurement matrix sampled with wideband-input signals under Nyquist theorem, from the mathematical point-of-view. Thus the sensitivity of folding index number estimation and the synchronization of sampling with inputs are important to estimate accurately, which show the deficiencies.

3 Information Recovery

The OMP and the SP are the typical greedy algorithm of the CS algorithm, which can search for answers in the several regions of a solution space and jump out of local optimization at a greater probability, thus the global optimization can be found. The SAMP is the combination of the OMP look-ahead searching and the SP backtracking searching, modifying the support set (SS) with a fixed step length in iterative process, eliminating the poor atoms via new and the backtracking searching until residuals no longer shrinking.

The SAMP brings solution for blind signal recovery which does not require information of sparsity $s$ of input signals as a prior, following greedy iteration principles.
through stage by stage to estimates the sparsity. The OMP and the SP are viewed as its special cases, namely that when \( \hat{s} = 1 \), SAMP can be roughly regarded as OMP, which takes a few more iterations on more accuracy. In addition, when \( \hat{s} = s \), SAMP becomes exactly SP. Thus, the stage is generally chosen one from \( \hat{s} = 1 \sim s \). The improved SAMP algorithms which applied SVD on the basis of previous study for high-resolution reconstruction, called SAMP-SVD.

The following Table I shows the steps of SAMP algorithm [16].

**Table 1. SAMP Steps**

| Input:  | Sampling matrix \( \Phi \), Sampling vector \( y \), step size \( d \) |
| Output: | A \( s \)-sparse approximation \( \hat{x} \) of the input signal |

**Initialization:**
- \( \hat{x} = 0 \) (Trivial initialization)
- \( r = y \) (Trivial initialization)
- \( F_k = \emptyset \) (Empty finalist)
- \( l = d \) (Size of the finalist in the first stage)
- \( k = 1 \) (Iteration index)
- \( j = 1 \) (Stage index)

**Repeat:**
- \( S_k = \text{Max}\{\|\Phi \|, 1\} \) (Preliminary Test)
- \( F_{k+1} = F_k \cup S_k \) (Make Candidate List)
- \( \hat{x}_k = \text{arg min}_{x} \| y - \Phi \hat{x}_k x \| \) (Final test)
- \( r = y - \Phi \hat{x}_k \) (Compute Residue)

**If** halting condition true
- **then** quit the iteration;
**else if** \( \| r \| \geq \| r \| \) (Preliminary Test)
- **then** [stage switching]
  - \( j = j + 1 \) (Update the stage index)
  - \( l = j \times d \) (Update the size of finalist)
**else**
- \( F_{k+1} = F_{k+1} \cup S_k \) (Update the finalist)
- \( \hat{x}_k = r \) (Update the Residue)
- \( k = k + 1 \)

**End**

**Until** halting condition true;

**Output:** \( \hat{x} = \text{arg min}_{x} \| y - \Phi \hat{x} \| \) (Prediction of non-coefficients)

An optimum solution is found by the minimum of residual to \( \hat{x}_k = \text{arg min}_{x} \| y - \Phi \hat{x} \| \), which is equivalent to a least squares problem as \( \Phi \hat{x}_k = \Phi \hat{x}_k y \), hence, namely \( \hat{x}_k = (\Phi \hat{x}_k \Phi \hat{x}_k)^{-1} \Phi \hat{x}_k y \). However, in solving the normal equation we are interested in the singularity of \( \Phi \hat{x}_k \Phi \hat{x}_k \), unfortunately, the inverse of a nonsingular matrix costs a large computation and is difficult to implement in computer arithmetic. The SVD definitely point out the generalized inverse numerical solution of \( \Phi \hat{x}_k \Phi \hat{x}_k \) which is avoided in the problem of singularity.

The solution of LS regular system by using SVD algorithm

\[
\hat{x}_k = \sum_{i=1}^{l} u_i (u_i^T \Phi \hat{x}_k y / \sigma_i^2)
\]

where \( u_i \) is the right singular vector, \( \sigma_i \) represents nonzero singular value and, the number of singular values is \( l \) which depends on \( \Phi \hat{x}_k \Phi \hat{x}_k \).

And the following Table II shows the steps of SVD decomposition.

**Table 2. SVD Decomposition Steps**

| Step 1: the singularly valuable decomposition of \( \Phi \hat{x}_k \) |
| all the nonzero singular value is \( \sigma_1, \sigma_2, \ldots, \sigma_l \) |
| the corresponding right singular vector \( u_1, u_2, \ldots, u_l \) |

**Step 2:** output the numerical solution \( \hat{x}_k = \sum_{i=1}^{l} u_i (u_i^T \Phi \hat{x}_k y / \sigma_i^2) \)

**Fig. 1** The Flow Chart of SAMP-SVD Algorithm

where the output gets the numerical solutions from SVD algorithm to avoid the complexities of matrix inversion. SAMP-SVD obtains high-resolution reconstruction by combining the merits of SAMP and SVD, not requiring sparsity and singularity as prior. The flow chart of the SAMP-SVD algorithm is presented in Fig.1.

4 Results and Discussion

In this section, the performance of SAMP-SVD algorithm derived above is evaluated through simulation and compared with the SpaRSA that is one of traditional CS convex recovery algorithm, and we briefly discuss
their strengths and weaknesses. The inputs can be written as fellows shown in (7)

\[ x(n) = \sum_{m=1}^{N} A_m \cos(2\pi f_m n + \phi_m) + w(n) \]  

(7)

where the \( A_m \) is the amplitude of the \( m \)th signal, \( f_m \) and \( \phi_m \) represent the frequency and phase of input signal, and \( w(n) \) is a vector of Gaussian white noise.

**Example 1:** At the first experiment, we will show the information recovery performance of the two matrices mentioned above. We consider the parameters of input signals are \( f = 5.36 \text{GHz}, A = 1, \phi = \pi/6 \), and assume the environment is without noise. For measurement matrix \( \Psi \), we choose average sampling rate \( f_s \) of 2GHz, and the modulated function \( \theta(n) \) is written as \( \sin(2\pi f_s n) \). As discussed earlier, in order to reconstruct more highly accuracy, we should give a true index zones. The NYFR sensing matrix \( \Phi \) has the same modulated function as \( \Psi \). Fig.2 shows the SAMP-SVD reconstruction spectrum obtained from both of two. Notice that there is an extra frequency at 6.65GHz in the first picture which use D-measurement matrix, but it is pure in the spectrum of second. As a consequence, we will use the NYFR sensing matrix in the following experiments.

![Fig. 2 Spectrum of reconstruction signal](image)

**Example 2:** In this experiment, we assume that the input of NYFR consisting of monopulse (MP), linear frequency modulation (LFM) and binary phase shift keying (BPSK) signals at 2.1, 5.56, and 12.94GHz, and consider the amplitude of each signals are 1, the phase are \( \pi/6 \), \( \pi/3 \) and \( \pi/2 \), respectively. LFM signal is given a fixed-bandwidth of 5MHz, for the BPSK signal case, we choose random codes. Besides, the relative SNR is 10dB. Fig.3 shows the SAMP-SVD reconstruction magnitude spectrum of three input signals, in which algorithm uses the NYFR sensing matrix \( \Phi \). Notice that the modulated function \( \theta(n) \) is written as \( \sin(2\pi f_\theta n) \), where \( f_\theta \) is 10MHz.

![Fig. 3 Spectral of original and SAMP-SVD reconstruction signal](image)

**Example 3:** Here we consider the situation almost the same with example 1, and we define normalized root mean square error (NRMSE) to evaluate the performance of the frequency estimation by using SpaRSA and SAMP-SVD, the results are based on 500 Monte Carlo simulations and shown in Fig.4, respectively.

![Fig. 4 NRMSE of frequency estimation (SAMP-SVD & SpaRSA)](image)

The simulation shows the ability of the proposed algorithm on information reconstruction of inputs more faster than SpaRSA, because of SAMP-SVD without the process of matrix inversion, if time is a factor should be considered, the SAMP-SVD is a better choice. Fig.4 shows the NRMSE tends to a certain value when the SNR is greater than 8dB, and the performance of two algorithm is close to each other.

5 Conclusions

SAMP-SVD exhibits a tradeoff in computational complexity versus reconstruction performance if the RIP condition of measurement matrix is satisfied, contributes that not requiring sparsity of the receiving signals and singularity of sensing matrix as prior. And simulation
shows that the algorithm achieves accurate frequency estimation than the existing SpaRSA in the low SNR environment. Future work will concentrate on wideband signals recovery and the effect of RIP on signal recovery based on the NYFR with information recovery methods of CS framework, promising for selecting the parameters and improving the structure of the NYFR.

References


