

Jammer Suppression in DS-CDMA Communications using Parafac-based blind separation

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Abstract. In this paper we propose to apply parafac-based source separation techniques for jammer suppression in direct spread spectrum communication systems. The jammer excision is formulated as an optimization problem and a new algorithm is presented which is based on the parafac tri-iterative least square algorithm. By jointly diagonalizing the time delay correlation matrix of the observed signals and using the new proposed method, a better solution is achieved. The proposed algorithm can successfully separate communication signals and jamming signals. Simulation results reveal that our proposed algorithm has the better blind signal separation performance than joint diagonalization method. Our proposed algorithm doesn't require whitening processing. Moreover our proposed algorithm works well in the underdetermined condition, where the number of sources exceeds than the number of antennas.

1 Introduction

Jammer suppression techniques in spread spectrum (SS) communications have been under active research due to the inevitable gains in the overall system performance and capacity [1]. Although bandwidth expansion gives an inherent temporal mitigation capability for SS communications, usually called a processing gain, additional techniques are needed. This is because bandwidth expansion results in bandwidth dependent mitigation capability. Interference mitigation techniques, however, alleviate the need for wider spectrum for reliable communications.

In commercial cellular SS and DS-CDMA systems, many types of interference can appear, starting from multiuser interference inside each sector in a cell to inter-operator interference. Unintentional jamming can also be present due to co-existing systems at the same band, whereas intentional jamming arises mainly in military applications. Jamming can be mitigated by the use of multiple antenna sensors utilizing spatial diversity. However, when using conventional array receivers, directions of arrivals of signals must be estimated first. This in turn requires exact prior knowledge of the positions of the receiving antenna sensors. By using blind source separation (BSS) techniques as proposed in [2-3], DOA estimation can be relaxed, making it possible to achieve performance gains when applied to uncalibrated arrays in which the positions are known only roughly or not all. Belouchrain and Amin [2] were the first to use the blind source separation (BSS) techniques to aid conventional detection for jammer mitigation. They have used BSS based on second-order statistics to separate

desired signal and jamming signals. However, their work suffers from some weakness. In general, there are two types of BSS methods for separating signal, which are namely second order statistic-based BSS [4-5] and high order statistic-based BSS [6]. Second-order statistic-based BSS methods require assumption about the second-order statistics such as nonstationarity or nonwhiteness. Only one source with gaussian characteristic is required. Joint diagonalization of a set of covariance matrices in [4] can be used for blind signal separation, but this algorithm requires whitening processing. The process will influence the separate performance, as the statistical error of this stage can not be modified in the separation stage. However, conventional blind signal separation methods including joint diagonalization method cannot work in the underdetermined condition, where the number of source exceeds the number of sensors. On the other hand, high-order statistic methods require high computation.

This paper introduces a new source separation technique exploiting the time coherence of the source signals. The proposed approach is based on parafac blind separation. Parafac separation has been first introduced as a data analysis tool in psychometrics. Parafac decomposition is thus naturally related to linear algebra for multi-way arrays [7, 8]. Parafac decomposition was used widely in blind estimation of multi-input multi-output system, polarization sensitive array signal processing, array parameter estimation and so on [9-10].

Our work links the blind signal separation problem to the parafac model and derives a novel blind signal separation algorithm whose performance is better than joint diagonalization method [4]. Our proposed algorithm doesn't need whitening processing. Instead, our proposed

algorithm relies on a fundamental result of Kruskal [8] regarding the uniqueness of low-rank three-way array decomposition. Our proposed algorithm works well in the underdetermined condition, where the number of sources exceeds than the number of sensor.

This paper is structured as follows. Section 2 develops data model. Section 3 discusses identifiability issues and deals with algorithmic issues. Section 4 presents simulation results, and section 5 summarizes our conclusions.

Denote: $[\cdot]^H$, $[\cdot]^T$, $[\cdot]^*$, $[\cdot]^+$ and $\|\cdot\|_F$ stand for the matrix conjugate transpose, matrix transpose, complex conjugate, pseudo-inverse and Frobenius norm, respectively.

2 Data model

A standard spread spectrum system with direct sequence spread is assumed. Consider the following basic linear mixture model:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \quad (1)$$

where the stochastic vector $\mathbf{x}(t) = [\mathbf{x}_1(t), \mathbf{x}_2(t), \dots, \mathbf{x}_M(t)]^T \in \mathbb{C}^{M \times 1}$ represents multi-antenna observations, the components of the stochastic vector $\mathbf{s}(t) = [s_1(t), s_2(t), \dots, s_F(t)]^T \in \mathbb{R}^{F \times 1}$ correspond to source signals which include direct spread spectrum communication signal and jamming signals, $s_1(t)$ is the desired spread spectrum communication signal, the others are jamming signals. $\mathbf{n}(t) \in \mathbb{C}^{M \times 1}$ denotes additive noise. The a priori unknown mixing matrix $\mathbf{A} \in \mathbb{C}^{M \times F}$ characterizes the way the sources are combined in the observations. In this paper we assume that the sources are individually correlated in time. The covariance matrices of the observations then satisfy:

$$\begin{aligned} \mathbf{X}_1 &= E \left\{ \mathbf{x}_i \mathbf{x}_{i+\tau_1}^H \right\} = \mathbf{A} \mathbf{D}_1 \mathbf{A}^H \\ \mathbf{X}_2 &= E \left\{ \mathbf{x}_i \mathbf{x}_{i+\tau_2}^H \right\} = \mathbf{A} \mathbf{D}_2 \mathbf{A}^H \\ &\vdots \\ \mathbf{X}_K &= E \left\{ \mathbf{x}_i \mathbf{x}_{i+\tau_K}^H \right\} = \mathbf{A} \mathbf{D}_K \mathbf{A}^H \end{aligned} \quad (2)$$

where $\mathbf{D}_k = E \left\{ \mathbf{s}_i \mathbf{s}_{i+\tau_k}^H \right\}$ is diagonal, for $k = 1, 2, \dots, K$. One of the delays τ_k can be equal to zero. For simplicity, we have dropped the noise terms; they can be considered as a perturbation of (2). The problem we want to solve is the estimation of \mathbf{A} from the set $\{ \mathbf{X}_k \}$. For (2), joint diagonalization method[4] can be used for blind signal separation. The mixing matrix \mathbf{A} can be estimated by means of a joint diagonalizer of the matrices $\mathbf{X}_k, k = 1, 2, \dots, K$. In this paper, a novel blind signal separation method is investigated.

Define a matrix $\mathbf{H} \in \mathbb{C}^{K \times F}$ by $(\mathbf{H})_{kf} = (\mathbf{D}_k)_{ff}$, $k = 1, 2, \dots, K, f = 1, 2, \dots, F$. (2) can be denoted as

$$\mathbf{X}_k = \mathbf{A} \mathbf{diag}_k(\mathbf{H}) \mathbf{A}^H, k = 1, 2, \dots, K \quad (3)$$

where $\mathbf{diag}_m(\cdot)$ is understood as an operator that extracts the m th row of its matrix argument and constructs a diagonal matrix out of it. In the presence of noise, the received signal model becomes

$$\tilde{\mathbf{X}}_k = \mathbf{A} \mathbf{diag}_k(\mathbf{H}) \mathbf{A}^H + \mathbf{W}_k, k = 1, 2, \dots, K \quad (4)$$

where \mathbf{W}_k is the received noise. Stack the matrices $\mathbf{X}_k (k = 1, 2, \dots, K)$ in a trilinear model which is shown

$$x_{m,n,k} = \sum_{f=1}^F a_{m,f} a_{n,f}^* h_{k,f}, \quad m = 1, \dots, M, n = 1, \dots, M, k = 1, \dots, K \quad (5)$$

where $a_{m,f}$ stands for the (m,f) element of \mathbf{A} matrix, and similarly for the others. The parafac trilinear model displays the reflection for three kinds of diversity, which is shown in Fig.1. $\mathbf{X}_k = \mathbf{A} \mathbf{diag}_k(\mathbf{H}) \mathbf{A}^H, k = 1, 2, \dots, K$, can be interpreted as slicing the 3-D data in a series of slices (2-D data) along the sensor direction. The symmetry of the trilinear model allows two more matrix system rearrangements,

$$\mathbf{Y}_m = \mathbf{A}^* \mathbf{diag}_m(\mathbf{A}) \mathbf{H}^T + \mathbf{W}_m, m = 1, \dots, M \quad (6)$$

$$\mathbf{Z}_n = \mathbf{H} \mathbf{diag}_n(\mathbf{A}^*) \mathbf{A}^T + \mathbf{W}_n, n = 1, \dots, M \quad (7)$$

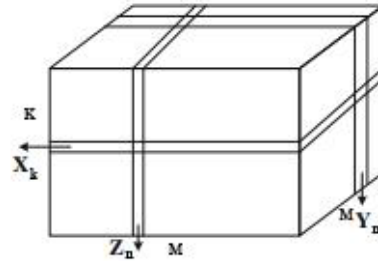


Fig.1 the parafac blind separation model

3 The proposed algorithm description

3.1 Parafac trilinear decomposition

TALS (Trilinear Alternating Least Square) algorithm is the common data detection method for parafac blind decomposition [11]. TALS algorithm is discussed in detail as follows. According to Eq.(3), stack the $\mathbf{X}_k (k = 1, 2, \dots, K)$, we can get

$$\begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \vdots \\ \mathbf{X}_K \end{bmatrix} = \begin{bmatrix} \mathbf{A} \mathbf{diag}_1(\mathbf{H}) \\ \mathbf{A} \mathbf{diag}_2(\mathbf{H}) \\ \vdots \\ \mathbf{A} \mathbf{diag}_K(\mathbf{H}) \end{bmatrix} \mathbf{A}^H \quad (8)$$

The cost function can be constructed via least squares and given by

$$\min_{A, H} \left\| \begin{bmatrix} \tilde{\mathbf{X}}_1 \\ \tilde{\mathbf{X}}_2 \\ \vdots \\ \tilde{\mathbf{X}}_K \end{bmatrix} - \begin{bmatrix} \mathbf{A} \mathbf{diag}_1(\mathbf{H}) \\ \mathbf{A} \mathbf{diag}_2(\mathbf{H}) \\ \vdots \\ \mathbf{A} \mathbf{diag}_K(\mathbf{H}) \end{bmatrix} \right\|_F \quad (9)$$

where $\tilde{\mathbf{X}}_k, k=1,2,\dots, K$ are the noisy slices. Least squares update for \mathbf{A}^* is

$$\hat{\mathbf{A}}^H = \begin{bmatrix} \hat{\mathbf{A}} \mathbf{diag}_1(\hat{\mathbf{H}}) \\ \hat{\mathbf{A}} \mathbf{diag}_2(\hat{\mathbf{H}}) \\ \vdots \\ \hat{\mathbf{A}} \mathbf{diag}_K(\hat{\mathbf{H}}) \end{bmatrix}^+ \begin{bmatrix} \tilde{\mathbf{X}}_1 \\ \tilde{\mathbf{X}}_2 \\ \vdots \\ \tilde{\mathbf{X}}_K \end{bmatrix} \quad (10)$$

where $\hat{\mathbf{A}}$ and $\hat{\mathbf{H}}$ denote previously obtained estimates of \mathbf{A} and \mathbf{H} . Similarly, from the second way of slices: $\mathbf{Y}_m = \mathbf{A}^* \mathbf{diag}_m(\mathbf{A}) \mathbf{H}^T, m=1,\dots, M$. The cost function is

$$\min_{A, H} \left\| \begin{bmatrix} \tilde{\mathbf{Y}}_1 \\ \tilde{\mathbf{Y}}_2 \\ \vdots \\ \tilde{\mathbf{Y}}_M \end{bmatrix} - \begin{bmatrix} \mathbf{A}^* \mathbf{diag}_1(\mathbf{A}) \\ \mathbf{A}^* \mathbf{diag}_2(\mathbf{A}) \\ \vdots \\ \mathbf{A}^* \mathbf{diag}_M(\mathbf{A}) \end{bmatrix} \right\|_F \quad (11)$$

where $\tilde{\mathbf{Y}}_m, m=1,\dots, M$ are noisy slices. And the LS update for \mathbf{H} is

$$\mathbf{H}^T = \begin{bmatrix} \hat{\mathbf{A}}^* \mathbf{diag}_1(\hat{\mathbf{A}}) \\ \hat{\mathbf{A}}^* \mathbf{diag}_2(\hat{\mathbf{A}}) \\ \vdots \\ \hat{\mathbf{A}}^* \mathbf{diag}_M(\hat{\mathbf{A}}) \end{bmatrix}^+ \begin{bmatrix} \tilde{\mathbf{Y}}_1 \\ \tilde{\mathbf{Y}}_2 \\ \vdots \\ \tilde{\mathbf{Y}}_M \end{bmatrix} \quad (12)$$

Finally, from the third way of slices: $\mathbf{Z}_n = \mathbf{H} \mathbf{diag}_n(\mathbf{A}^*) \mathbf{A}^T, n=1,2,\dots, M$. And then LS update for \mathbf{A} is Finally, from the third way of slices: $\mathbf{Z}_n = \mathbf{H} \mathbf{diag}_n(\mathbf{A}^*) \mathbf{A}^T, n=1,2,\dots, M$. And then LS update for \mathbf{A} is

$$\hat{\mathbf{A}}^T = \begin{bmatrix} \hat{\mathbf{H}} \mathbf{diag}_1(\hat{\mathbf{A}}^*) \\ \hat{\mathbf{H}} \mathbf{diag}_2(\hat{\mathbf{A}}^*) \\ \vdots \\ \hat{\mathbf{H}} \mathbf{diag}_M(\hat{\mathbf{A}}^*) \end{bmatrix}^+ \begin{bmatrix} \tilde{\mathbf{Z}}_1 \\ \tilde{\mathbf{Z}}_2 \\ \vdots \\ \tilde{\mathbf{Z}}_M \end{bmatrix} \quad (13)$$

According to (10), (12) and (13), matrices \mathbf{A}^*, \mathbf{H} and \mathbf{A} are updated with conditioned least squares, respectively. The matrix update will stop until convergence. TALS algorithm can be initialized randomly.

TALS is optimal when noise is additive i.i.d. Gaussian. TALS algorithm has several advantages: it is easy to implement, guaranteed to converge and simple to extend to higher order data. But TALS algorithm has slow convergence. In this paper, we use the COMFAC algorithm [11] for trilinear decomposition. COMFAC

algorithm is essentially a fast implementation of TALS, and can speeds up the least square fitting.

3.2 Uniqueness and Identifiability

In this subsection, we first will introduce a key concept, k-rank, and then discuss sufficient condition and necessary condition for uniqueness of parafac blind separation. The k-rank of a matrix is an important concept in the parafac blind source separation.

Definiton 1[8]: Consider the matrix $\mathbf{A} \in \mathbb{C}^{M \times F}$, denote $\text{rank}(\mathbf{A}) = r$ when \mathbf{A} contains r columns which are linear independent. Moreover, if the linear independence in every column of \mathbf{A} exists only for $l \leq F$, which does not hold for more than l columns, note that the k-rank of \mathbf{A} should have $k_A = l$, s.t. $k_A \leq \text{rank}(\mathbf{A}), \forall \mathbf{A}$.

Theorem1[8] $\mathbf{X}_k = \mathbf{A} \mathbf{diag}_k(\mathbf{H}) \mathbf{A}^H, \text{ for } k=1,2,\dots, K$

where $\mathbf{A} \in \mathbb{C}^{M \times F}, \mathbf{H} \in \mathbb{C}^{K \times F}$. Considering the matrix is full rank and full k-rank, if

$$2 \min(M, F) + \min(F, K) \geq 2F + 2. \quad (14)$$

then \mathbf{A} and \mathbf{H} are unique up to permutation and scaling of columns, that is to say, any other $\bar{\mathbf{A}}$ and $\bar{\mathbf{H}}$ that construct $\mathbf{X}_k (k=1,2,\dots, K)$ is related to \mathbf{A} and \mathbf{H} via

$$\bar{\mathbf{A}} = \mathbf{A} \mathbf{\Pi} \mathbf{\Delta}_1, \bar{\mathbf{H}} = \mathbf{H} \mathbf{\Pi} \mathbf{\Delta}_2. \quad (15)$$

where $\mathbf{\Pi}$ is a permutation matrix, and $\mathbf{\Delta}_1, \mathbf{\Delta}_2$ are diagonal scaling matrices satisfying $\mathbf{\Delta}_1 \mathbf{\Delta}_1^* \mathbf{\Delta}_2 = \mathbf{I}$.

When $M \geq F (M$ is number of sensors; F is the number of sources), $F \geq 2$, then identifiable condition is $K \geq 2$.

When $M \leq F, F \geq 2, K > F$, then identifiable condition is $F/2 + 1 \leq M \leq F$. So our proposed algorithm works well in the underdetermined condition, where the number of sources is larger than the number of sensors. It should be pointed out that the joint diagonalization methods require stronger conditions in terms of the number of sensors as compared to the our proposed algorithm. Indeed, $M \geq F$ is required for the joint diagonalization algorithms [4], whereas this constraint is not needed for our proposed algorithm.

3.3 Blind signal Separation using parafac decomposition

Blind signal separation algorithm using parafac blind separate techniques is proposed in this paper. This algorithm firstly uses parafac decomposition to estimate the mixing matrix $\hat{\mathbf{A}}$, and the source signal is $\hat{\mathbf{S}} = \hat{\mathbf{A}}^+ \mathbf{X}$.

Based on the above analysis, the Parafac blind source separation for jammer suppression in DS-CDMA system is summarized as follows:

Step1: Random initialization for the matrices for \mathbf{A} and \mathbf{H} ;

Step 2: LS update for matrix \mathbf{A}^* according to (10);

Step 3: LS update for matrix \mathbf{H} according to (12);

4 Simulation and analysis

In this section, we present the simulation results to show the performance. A system with $K=3$ signals was considered. Signal-to-noise ratio (SNR) and the signal to jammer ratio (SJR) were defined with respect to the desired signal.

Simulation 1: There are 3 sources and 5 sensors to receive signal. The mixing matrix is $\mathbf{A}=[\mathbf{a}(\theta_1) \ \mathbf{a}(\theta_2) \ \mathbf{a}(\theta_3)]$, where $\mathbf{a}(\theta)=[1, \exp(-j\pi \cos \theta), \dots, \exp(-j4\pi \cos \theta)]^T$. And $\theta_1 = 10^\circ$, $\theta_2 = 20^\circ$ and $\theta_3 = 30^\circ$. $s_1(t)$ is the desired direct sequence signal, the source rate is 1 kbit/s, using BPSK modulation mode, the length of PN code is 16, the amplitude is 1, the carrier frequency is 600kHz. $s_2(t) = 5\sin(350t)$ is the tone jamming signal. $s_3(t) = 4\sin(1500t^2)$ is the linear frequency modulation jamming signal. The source signals $s_1(t)$, $s_2(t)$, $s_3(t)$ are shown in Fig.2. The received signal of sensors is shown in Fig.3. Fig.4 shows the separated signal with our proposed algorithm at SNR=15dB and SJR=-16 dB. But there is a little change in the amplitude and order of the signals, it is common in the BSS techniques and does not affect the estimation of the signals. In order to qualify BSS performance, the similar coefficient between the separated signal and source signal is defined as

$$\rho_{ij} = \frac{\left| \sum_{t=0}^T a_i(t)b_j(t) \right|}{\sqrt{\sum_{t=0}^T a_i^2(t) \sum_{t=0}^T b_j^2(t)}}$$

where $a_i(t)$ and $b_j(t)$ are separated signal and source signal, respectively. The similar coefficients between the separated signal in Fig.4 and source signal in Fig.2 at SNR=15dB are 0.9987, 1.0000 and 0.9967.

Simulation 2: The proposed algorithm and joint diagonalization method [4] firstly estimate the mixing matrix, and then separate signal. The mixing matrix \mathbf{A} estimation performance determines the signal separation performance. We present Monte Carlo simulations that are to assess the mixing matrix estimation performance of the proposed algorithm. The number of Monte Carlo trials is 500. Define $MSE = \frac{1}{500 \times M \times F} \sum_{m,f} \sum_{i=1}^{500} \left\| \hat{a}_{m,f,i} - a_{m,f} \right\|^2$,

where $\hat{a}_{m,f,i}$ is (m, f) element of the estimated mixing matrix of the i th Monte Carlo simulation, and $a_{m,f}$ is (m, f) element of the perfect mixing matrix. $M=5$ is the number of sensors, and $F=3$ is the number of the sources. $K=4$ in this simulation. Their performance under different SNR is shown in Fig.5. From Fig.5 we find that our proposed algorithm has better the mixing matrix estimation performance than joint diagonalization method.

So our proposed algorithm has better signal separation performance.

Simulation 3: The parameters of spread spectrum signal and interference signal remains as simulation 1. Fig. 6 shows the error rate of spread spectrum signal with the increase of SNR while keeping SJR fixed to -10dB. Fig.7 gives the error rate curve with the change of SJR at SNR equal to 6dB. In order to compare performance, no jamming, not separation jamming, joint diagonalization blind separation and the proposed algorithm based on parafac blind separation are given in Fig.6-7. From Fig.6-7 we can see that the performance can be improved after blind source separation. The performance of the proposed algorithm is better than other conditions.

Simulation 4: Conventional blind signal separation methods including joint diagonalization method cannot work in the underdetermined condition, where the number of sources exceeds the number of sensors. Our proposed algorithm has no this constrain. Blind signal separation in the underdetermined condition is investigated in this simulation. The number of sources is 5, and the number of sensors is 4. K is set 20 in this simulation. Fig.8 shows the mixing matrix estimation performance in the underdetermined condition. From Fig.8 we find that our proposed algorithm works well in the underdetermined condition. Our proposed algorithm overcomes the shortcoming of conventional BSS methods.

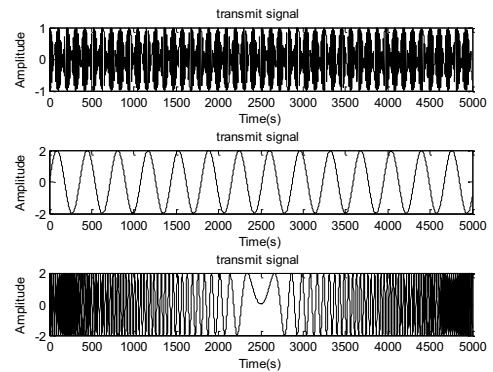


Fig.2 the source signals

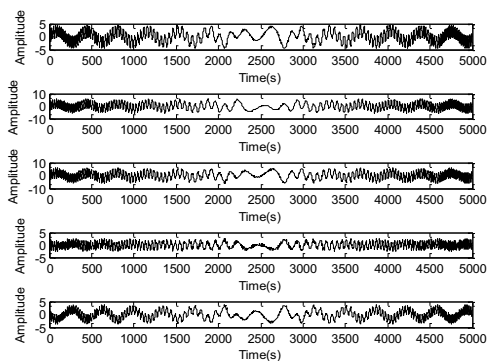


Fig.3 the received mixing signals

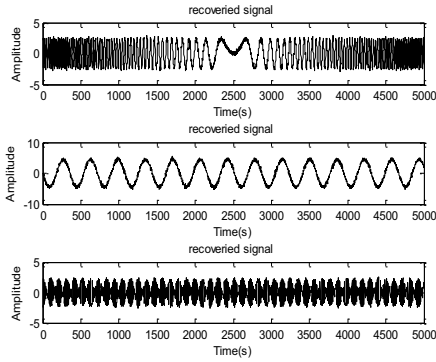


Fig.4 the separated signals based on parafac blind source separation

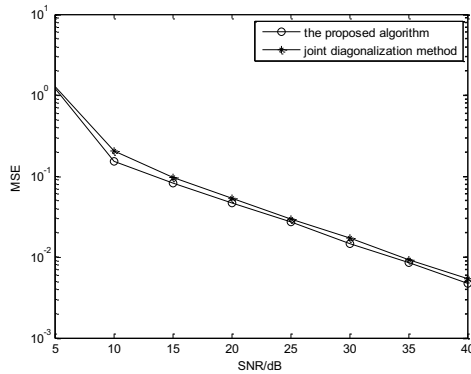


Fig.5 the algorithm performance comparison

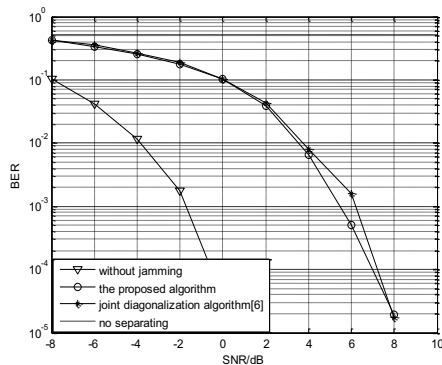


Fig.6 BER performance comparison at SJR=-10dB

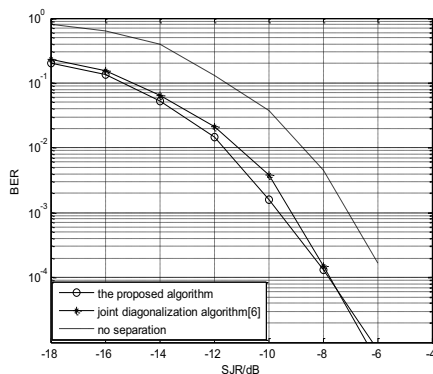


Fig.7 BER performance comparison at SNR=6dB

Jamming problems are important in DS-CDMA communications because both intentional and unintentional jamming can occur in several practical circumstances. In this paper, we have considered suppression of jammer signal in the direct sequence spread spectrum communication system. Our work links the blind signal separation problem to the parafac model and derives a novel blind signal separation algorithm whose performance is better than joint diagonalization method. Our proposed algorithm doesn't need whitening processing. Instead, our proposed algorithm relies on the uniqueness of low-rank three-way array decomposition. Furthermore, our proposed algorithm works well in the underdetermined condition, where the number of sources exceeds the number of sensors. Theory analysis and simulation results show the feasibility and effectiveness of the proposed algorithm and the research work is expected to provide a new thought for the development of communications anti-jamming.

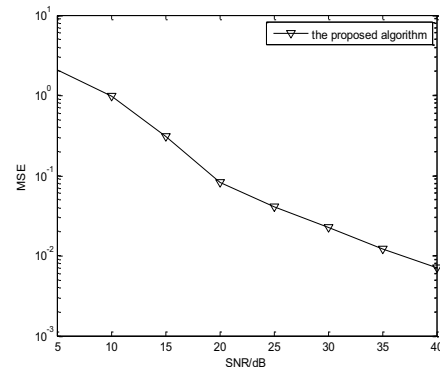


Fig.8 the algorithm performance in the underdetermined condition

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5 Conclusion

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