

# Modelling and Design of a 3 kW Permanent Magnet Synchronous Generator suitable for Variable Speed Small Wind Turbines

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**Abstract.** This paper presents the modeling and design of a 3 kW Permanent Magnet Synchronous Generator (PMSG) used for a variable speed wind turbine. Initially, the PMSG is modeled in the d-q reference frame. Different optimized parameters of the generator are extracted from the design and used in simulation of the PMSG. The generator output power is matched with the power of the turbine such that the generator is not either over-sized or under-sized.

## 1 Introduction

Wind turbine is considered as a very promising renewable energy source. The performance of wind turbine depends firstly upon the optimum design of the system and secondly on the control strategies of different possible parameters that can operate efficiently under extreme variations in wind conditions. Generally, both induction and synchronous generators are used in wind power conversion systems. Within an induction generator, three different types of rotors can be used: (a) squirrel cage rotor, (b) wound rotor with slip control and (c) doubly fed induction rotors. From the above three aforementioned technologies, the doubly fed is normally used in wind speed generation as it provides a wide range of speed variation.

Variable speed multi-pole PMSG is normally chosen for small wind turbines as it offers better performance due to its higher efficiency and less maintenance because it does not have any rotor current (no field). PMSG can also be used without a gearbox, which implies a reduction in the weight of the nacelle and cost. Optimum wind energy extraction is achieved by operating the wind turbine generator at variable speed because of the higher energy gain and reduced stresses. The recent advancements in power electronics and control strategies have made it possible to regulate the output voltage of the PMSG, thereby optimizing the generator's power.

## 2 Model of PMSG

The mathematical model of a PMSG is generally based on the following assumptions [1].

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1. The stator windings are positioned sinusoidally along the air-gap as far as the mutual effect with the rotor is concerned.
2. The stator slots cause no significant variations of the rotor inductance with rotor position.
3. Magnetic hysteresis and saturation effects are negligible.
4. The stator windings are considered symmetrical.
5. Damper windings are not considered.
6. Capacitance of all the windings is neglected.
7. Resistances of the coils are assumed to be constant.

The dynamic model of a PMSG is derived from a two-phase synchronous reference, direct (d) and quadrature (q) axis frame in which the q-axis is 90° ahead of the d-axis with respect to the direction of rotation. In the case of a balanced three phase system, the dq reference frame transformation reduces three AC quantities to two DC quantities, allowing simplified calculations to be performed. The dq transformation applied to a three phase system is as shown by equation (1) and its inverse transform is given by equation (2) [1], where F can represent voltages, currents or inductances whose values depend upon the rotor position.

$$\begin{bmatrix} F_d \\ F_q \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \sin\omega t & \sin\left(\omega t - \frac{2\pi}{3}\right) & \sin\left(\omega t + \frac{2\pi}{3}\right) \\ \cos\omega t & \cos\left(\omega t - \frac{2\pi}{3}\right) & \cos\left(\omega t + \frac{2\pi}{3}\right) \end{bmatrix} \begin{bmatrix} F_a \\ F_b \\ F_c \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} F_a \\ F_b \\ F_c \end{bmatrix} = \begin{bmatrix} \sin\omega t & \cos\omega t \\ \sin\left(\omega t - \frac{2\pi}{3}\right) & \cos\left(\omega t - \frac{2\pi}{3}\right) \\ \sin\left(\omega t + \frac{2\pi}{3}\right) & \cos\left(\omega t + \frac{2\pi}{3}\right) \end{bmatrix} \begin{bmatrix} F_d \\ F_q \end{bmatrix} \quad (2)$$

The d and q-axis currents in the frequency (s) domain can be represented as in equations (3) and (4) respectively which can be obtained through equivalent circuit model of PMSG as shown in Figure 1 [1, 2].

$$I_{ds} = \frac{(-V_{ds} - R_s I_{ds} - \omega_r)}{sL_{ds}} \quad (3)$$

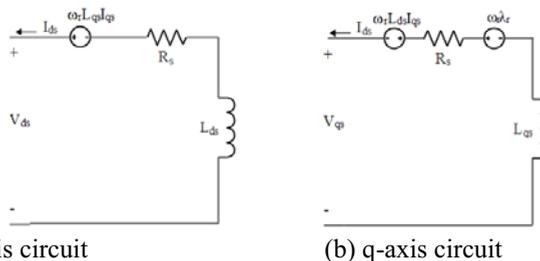
$$I_{qs} = \frac{[-V_{qs} - R_s I_{qs} - \omega_r(L_{ds} + L_{ls})I_{ds} + \omega_r \phi_r]}{sL_{qs}} \quad (4)$$

where,  $V_{ds}$  and  $V_{qs}$  are the d and q-axis stator voltages,  $I_{ds}$  and  $I_{qs}$  are the d and q-axis stator currents,  $R_s$  is the stator resistance and  $\omega_r$  is the angular speed of the generator,  $L_{ds}$  and  $L_{qs}$  are the stator d and q-axis self inductances and  $\phi_r$  is the rotor flux.  $L_{ds}$  and  $L_{qs}$  are given by equations (5) and (6) respectively [2].

$$L_{ds} = L_{ls} + L_{dm} \quad (5)$$

$$L_{qs} = L_{ls} + L_{qm} \quad (6)$$

where,  $L_{dm}$  and  $L_{qm}$  are the magnetizing inductances in d and q-axis, and  $L_{ls}$  is the leakage inductance. For a non-salient pole PMSG, d and q-axis magnetizing inductances are equal (i.e.  $L_{dm} = L_{qm}$ ), whereas for a salient pole PMSG, d-axis magnetizing inductance is normally lower than the q-axis magnetizing inductance (i.e.  $L_{dm} < L_{qm}$ ) [2]. The simplified dq-axis model in the rotor-field synchronous frame is as shown in Figure 1.



**Figure 1.** Simplified dq-axis model of PMSG in the rotor field synchronous reference frame

The electromagnetic torque ( $T_e$ ) and the rotor speed ( $\omega_r$ ) of the PMSG are calculated as shown in equations (7) and (8) respectively [2].

$$T_e = \frac{3N_{pp}}{2} [\varphi_r i_{qs} - (L_{ds} - L_{qs}) i_{ds} i_{qs}] \quad (7)$$

$$\omega_r = \frac{N_{pp}}{J_S} (T_e - T_m) \quad (8)$$

where,  $N_{pp}$  is the number of pole pairs of the rotor,  $J$  is the rotational inertia of the generator and  $T_m$  is the mechanical torque for the generator (in the case of the PMSG connected to a wind turbine,  $T_m$  is the torque from the turbine).

Using equations from (1) to (7), a PMSG model is developed in Matlab/Simulink. Inputs for the model are d-q axis voltages and mechanical torque. Rotor speed ( $\omega_r$ ), is calculated through equation (8) and is used as feedback to the system. Number of pole pairs, d and q-axis inductances, leakage inductance, magnetic flux of the rotor magnets, stator resistance and moment of inertia of rotor and load can be initially set for a particular size of the generator. The d-axis current ( $I_{ds}$ ), q-axis current ( $I_{qs}$ ) and electromagnetic torque ( $T_e$ ) are the outputs from the system as calculated by equations (3), (4) and (7), respectively.

### 3 Design Considerations for PMSG

Axial flux coreless PMSG is regarded as the suitable generator for small wind turbine because of its simplicity of design and manufacture. The absence of cogging torque for coreless design eliminates any magnetic pull between stationary and rotating parts [3]. Most of the axial flux machines have two sided rotor configuration rather than the single sided rotor to balance the axial forces [4]. Coreless design reduces the mass and increases the efficiency as compared to the conventional design [5].

Matching the generator size with power captured by the blade from wind is most important. A small rotor blade with big generator will not generate the full power while small generator with big rotor blades will generate full power below rated speed. Hence the generator is designed such that it extracts maximum power from the wind. Tip Speed Ratio (TSR) is the vital factor in matching generator power with the wind power. The optimum TSR depends upon number of rotor blades i.e. fewer the number of blades, faster should be the turbine speed to extract the available wind power. For small value of TSR (i.e. for improper design of rotor blades), the rotor will turn too slowly or even stall. For high value of TSR, rotor will spin very fast through turbulent air, optimum power cannot be extracted, and the turbine will be at great risk to have structural failure.

The value of TSR for a well designed three bladed wind turbine system will be around 6 to 7 [6]. The system considered here is geared three bladed systems and hence the generator rotors will have speed higher than that of the blades. Considering TSR value as 6 for wind speed of 7 m/s and the blade with radius 4 m (for 3 kW wind turbine), the blade speed of around 100 rpm is achieved. With gear ratio 1:4, the generator parameters are optimized such that it produces power output of around 3 kW at around 400 rpm. The general optimized parameters of the generator are type and size of the magnet, number of rotor poles, air gap flux density, number of coil turns, air gap distance and stator coil diameter. The overall system parameters for the generator design are as shown in Table 1.

#### 3.1 Design Parameters Calculations

The turbine will start generating useful power above the cut in speed. The cut in turbine speed can be calculated using equation (9).

$$TSR = \frac{\omega_{cutin} R}{V_{cutin}} \quad (9)$$

where,  $\omega_{cutin}$  is the turbine cut in speed in rad/s,  $R$  is radius of the blade in metre,  $V_{cutin}$  is the cut in speed for the wind in m/s. Assuming that the turbine starts at 3 m/s wind speed with Tip speed Ratio (TSR) value of 6 and blade radius of 4 m, turbine cut in speed can be calculated as 4.5 rad/s (or 43 rpm). The cut in frequency can be calculated from equation (10) [7].

$$f_{cutin} = \frac{N_{cutin} P}{120} \quad (10)$$

where,  $N_{cutin}$  is the generator cut in speed;  $P$  is the total number of rotor poles. Considering gear box ratio of 1:4, the generator cut in speed is approximately 130 rpm. With 20 number of rotor poles (i.e. total number of magnets), cut in frequency is obtained as 20 Hz. The total number of turns per phase ( $N_{turns/phase}$ ) for the stator winding can be obtained from equation (11) [7].

$$N_{turns/phase} = \frac{V_p}{4.44f_{cutin}\phi_g K_w} \tag{11}$$

where,  $V_p$  is the phase voltage at generator cut in speed,  $\Phi_g$  is the magnetic flux in air gap and  $K_w$  is the winding factor which is normally taken as 0.85 to 0.95.

The inductance of the stator coil can be obtained considering size of the coil and distance between the two coils. The most efficient shape is to keep the ratio of the winding depth to length close to unity so that the winding have square cross section. The square shape will keep minimum distance between the turns. There will be a high level of magnetic coupling when the turns are kept together and hence inductance of a coil increases with the square of the number of turns. Similarly, resistance of the coil can be determined after determining total length and cross sectional area of the coil. The length of the coil is calculated taking average diameter of inner and outer dimension of the coil. The inner diameter is chosen such that it has same dimension as that of magnetizing face of the magnet.

**Table 1.** Overall system parameters

Parameters	Values
Rated wind turbine power (P)	3kW
Rated wind speed ( $V_w$ )	7 m/s
Density of wind ( $\rho$ )	1.2 kg/m <sup>3</sup>
Power Coefficient ( $C_p$ )	0.3
Number of blades	2
Tip Speed Ratio (TSR)	6
Area of the blade (A)	48.6 m <sup>2</sup>
Diameter of the blade (D)	8 m
Rated turbine speed ( $N_t$ )	102 rpm
Cut in wind speed	3 m/s
Cut in turbine speed	43 rpm
Gear ratio	1:4
Cut in generator speed ( $N_{cutin}$ )	130 rpm

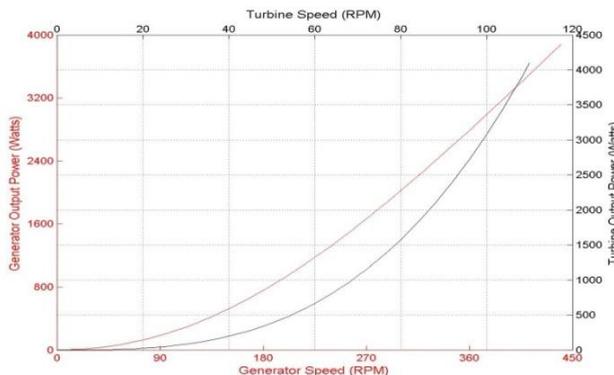
**Table 2.** Optimized generator parameters

Parameters	Values
Number of rotor pole pairs ( $N_{pp}$ )	10
Number of stator coils	9
Number of turns per phase ( $N_{turns/phase}$ )	442
Flux density in the air gap ( $\phi_g$ )	4.84X10 <sup>-4</sup> Tesla
Magnet Dimensions	55X55X20 mm <sup>3</sup>
Air Gap	2.3 mm
Generator outer diameter	500 mm
Stator coil diameter	1.628mm
Inductance of the coil/Phase ( $L_{coil/phase}$ )	0.405 mH
Resistance of the coil/phase ( $R_{coil/phase}$ )	0.9067 $\Omega$
Moment of inertia	0.00062J Kg.m <sup>2</sup>
Viscous damping	0.0003035 N.m.s

### 3.2 Optimization of Generator Parameters

The choice of phase number in the stator is directly related to the pole pair number which determines the output frequency as well as the total number of coils. The nominal frequency at 400 RPM should be around 50-60 Hz and each phase should have equal number of coils. The thickness of the stator support, that is able to provide enough mechanical strength, must not exceed that of the coil thickness. The diameter of the wire used in the stator should be as small as possible leading to high current density at the nominal point. Short circuit current would be the maximum current that might flow in the stator coil. Assuming that the system is designed to be stalled after 500 RPM, a coil size of 1.628 mm is recommended which can carry maximum enclosed 17 A current with current density of 8.2 A/mm<sup>2</sup> [8]. A large number of coils lead to large number of stator supports, thus increasing magnet and winding thickness. To compromise all these parameters, a phase number of 9 and pole pair of 10 are chosen.

In order to reduce eddy current losses, the air gap flux density should be kept low. Hence, magnet is chosen with flux density of around 0.2-0.4 T. It is better to increase the wire thickness and decrease the air gap flux density in order to reduce copper and eddy current losses. The generator outer diameter should not exceed 10 to 15 % of the blade diameter [9]. There are different parameters which affects output of the generator at the particular speed. Parameters such as flux density of the magnet; magnet size; wire diameter and phase number were varied in order to obtain generator output power of 3 kW when rotating at 400 RPM. A Mat lab program was developed and the output voltage, Current and Power was plotted varying different parameters until the required power output was obtained at the desirable speed. Output powers at various turbine speed and generator speed is plotted as shown in Figure 2 which shows that wind turbine power of 3 kW is achieved at turbine speed of nearly 100 rpm. Similarly, generator output power of 3 kW is achieved at the speed of nearly 400 rpm. Different optimized generator parameters, in order to obtain 3 kW output at 400 RPM, are summarized in Table 2.



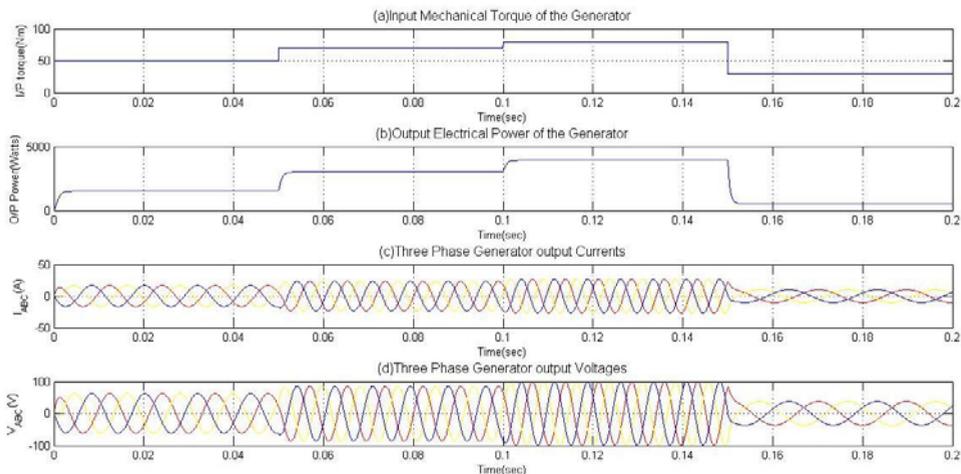
**Figure 2.** Matching wind turbine and generator power outputs

## 4 Simulation Results

The dq axis stator currents,  $i_{ds}$  and  $i_{qs}$ , in the synchronous frame rotating at the synchronous speed ( $\omega_r$ ), is calculated within the PMSG model. They are then transformed into the abc-axis stator currents,  $i_{as}$ ,  $i_{bs}$ , and  $i_{cs}$ , in stationary frame through dq/abc transformation. The calculated load voltages  $v_{as}$ ,  $v_{bs}$ , and  $v_{cs}$ , which are also the stator voltages, are transformed to the dq- axis voltages  $v_{ds}$  and  $v_{qs}$  in the synchronous frame, which are then fed back to the PMSG model.

The simulation was performed using optimized generator parameters as shown in Table 2. The generator output power, three phase voltages and currents are as shown in Figure 3 for different values of mechanical torque which represents the wind turbine torque. As shown in Figure 3, high torque represents high wind speed and low torque represents low wind speed. The output power of the generator also varies according to the input torque. The generator is designed to produce nearly 3000

W at the rated wind speed i.e. 7 m/s. The generator speed at 7 m/s is calculated to be 400 rpm with TSR value of 6 and hence the rated torque is calculated to be around 70 Nm. As shown in Figure 3 (b), the output of the generator is around 3000 W at this value of mechanical torque.



**Figure 3.** Electrical output power, voltages and currents at different generator mechanical input torque

## 5 Conclusions

The 3 kW PMSG is designed for variable speed wind turbine. Different Generator parameters were optimized in order to obtain 3 kW power output at wind speed of 7 m/s. The power output from optimized generator parameters were initially matched with the turbine output power in order to avoid under-sizing or over-sizing of the generator. These generator parameters were used within PMSG model in order to estimate performance of the generator. Result shows that the generator almost matches with the turbine output power at the rated wind speed. The simulation is performed at different input torques, which resembles different wind speed out of wind turbine. Result shows that the generator output voltages, currents and power varies with the wind speed and output power of the generator is around 3 kW at rotor speed of 400 rpm (i.e. turbine speed of around 100 rpm).

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