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Combined Cycle Fatigue Investigation Based on Energy Principle

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Abstract. We present a modified energy-principle based model of fatigue damage accumulation in high temperature alloys usually used in gas turbine engine under combined high cycle fatigue and low cycle fatigue (LCF/HCF) loading conditions. Our model is based on the energy principle which includes a modified approximation formula that describes fatigue crack origin depending on the relative amplitude of stress intensity in the ranges of both high- and low-cycle fatigue under non-isothermal loading. Functional dependence that presents the influence of HCF mechanisms on a fatigue life of our structural material is gradual and it has not breaks of the curve that yields a possibility to rewrite the equation of the S-N curve with taking into account combined cycle fatigue loading. We used the same number of parameters as the initial model. Note, that new parameter interpretation gives clear physical picture. The proposed model is verified by comparing the computed results with the experimental data for one high temperature alloy GH4133.

1 Introduction

The regime named as a combined cycle fatigue (CCF) is one of the main challenges of the modern fatigue damage analysis and its analysis is a rather complex problem. Thus, in spite of a number of life-prediction methods that have been proposed, their improvements are still discussed intensively. The problem of adequate and full description of accumulation damages under the combined cycle fatigue (low and high cycle fatigue) is rather complicated. The existing widely used fatigue life prediction methods are not correctly representing the fatigue damage accumulation. Often practically used conditions, i.e. elastic stress at high strain amplitudes as an input parameter for the fatigue curve, does take into account the relationship between stress and strain (the real characteristics of the test material, the process of characterizing the cyclic elastic-plastic deformation). On the other hand, approaches based on the amplitude of plastic deformation or plastic deformation work per cycle of loading, no longer work in the transition region between LCF and HCF.

One of the methods which seems is very promising and fruitful is the energy-based life prediction approach. Developed last decades this approach uses clear physical picture of crack nucleation both for low-cycle and high cycle regions of fatigue life. It is assumed that the limit state of a material is determined by that value of the mechanical energy which was dissipated irreversible at cycle deformation. Note, cumulative damage theories based on strain energy were mainly developed in the last several decades and some of the energy-based damage parameters have been proposed.We would

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like to discuss one of new methods those attempt to combine LCF and HCF and develop CCF approach based on the special representation of the specific energy of cycle deformation as a damage parameter. The approach is based on works of Ellyin and Kujawski [1], , Troshenko and Fomichev [2], Fatemi and Yang [3], Fomichev [4,5]. The approach developed by these authors allows evaluating the strength and lifetime of gas turbine blades by using mathematical models of damaged medium consisting of elastoplastic constitutive equations, kinetic equations of damage accumulation, and a strength criterion of damaged material. It was shown that under regular cyclic loading, when the amplitude of deformation in a cycle decreases, the transition from low cycle fatigue to high cycle fatigue proceeds gradually and depends on the physical interaction of these mechanisms in the transition zone. We suggest some development of theory and our model includes the combined effects of HCF and LCF loads and it is based on the energy-based approach to the analysis of fatigue failure and “energy” criterion for fatigue failure.The fatigue model developed by us expands upon the works of Bol’shukhin et al. (2010) and Volkov et al. (2012). Let us consider the energy dissipation per cycle. In the energy-based approach it is assumed that the limit state of a material is determined by that value of the mechanical energy which was dissipated irreversible at cycle deformation. The fatigue model developed by us expands upon the works of Bol’shukhin et al. (2010) and Volkov et al. (2012). Let us consider the energy dissipation per cycle. In the energy-based approach it is assumed that the limit state of a material is determined by that value of the mechanical energy which was dissipated irreversible at cycle deformation. Some part of this energy will be spending on the fatigue crack origin. As shown below, we propose a modified formula for the function that describes the mechanism of influence of cycle loading on a fatigue life of our structural material.

2 Combined Cycle Fatigue Model

Below we would like to discussed one of new methods those attempt to combine LCF and HCF and develop CCF approach based on the special representation of the specific energy of cycle deformation as a damage parameter. It’s considering the energy dissipation per cycle. It is supposed the summary energy dissipated per cycle can be divided on two parts, i.e. i) the so-called “dangerous” part which leads to the fatigue crack origin and ii) a “non-dangerous” part which does not be spent on fatigue failure. Thus, the “dangerous” part the summary strain energy, W_{df},could be written as the difference between total strain energy, W_t, and , its “non-dangerous” part, W_n, see Troshenko and Fomichev [2],

$$W_{df} = W_t - W_n \tag{1}$$

Then, the fatigue failure criterion can be written in the following manner

$$W_{df} = const \tag{2}$$

There are many phenomenological formulas have been proposed to describe the behavior of W_{df}. As it was pointed by Troshenko [6], the analysis of experimental data for a wide range of constructional materials demonstrates that a good result (i.e. when W_{df} is constant) provides the following equation

$$W_{df} = \sum_1^{N_f} \left[\Delta W_{st} - \Delta W_{sc} \left(\frac{\Delta W_{st}}{\Delta W_{sc}}\right)^{\alpha}\right] \tag{3}$$

Here W_{df} corresponds to the fatigue crack origin in the given volume of a metal after N_f cycles, ΔW_{sc} is a specific work of the stress deviators per cycle which is corresponded to the fatigue limit, ΔW_{st} is the total specific work per cycle, α is a parameter of the given material.

In the case of symmetric regular cyclic loading following works we write the fatigue failure criterion as

$$W_{df} = \sum_1^{N_f} \Delta W_{sf} = const \tag{4}$$

where W_{df} corresponds to the fatigue crack origin in the given volume of a metal after N_f cycles.

The total specific work of the stress deviators on the deviators of elastic strain accumulated after N_f cycles is given

$$W_t = \sum_1^{N_f} \Delta W_{st} = \sum_1^{N_f} \sigma_{ij}^{'} \Delta e_{ij}^{'e}. \tag{5}$$

The "non-dangerous" part W_n is given by the following formulae, see Troshenko and Fomichev [2],

$$W_n = \sum_1^{N_f} \Delta W_{sc} \left(\frac{\Delta W_{st}}{\Delta W_{sc}}\right)^{\alpha} \tag{6}$$

Here ΔW_{sc} is specific work of the stress deviators per cycle which is corresponded to the fatigue limit. Thus, combining these formulas we have the following fatigue failure criterion (comp. Eq. (4))

$$\sum_1^{N_f} \Delta W_{sf} = \sum_1^{N_f} \left[\Delta W_{st} - \Delta W_{sc} \left(\frac{\Delta W_{st}}{\Delta W_{sc}}\right)^{\alpha}\right] = const \tag{7}$$

In the case of irregular cyclic loading we can rewrite Eq. (3) for the loading step $\Delta t = \mathrm{t_{n+1}} - \mathrm{t_n}$, in following form, see Volkov and Korotkikh [7],

$$\Delta W_0 = \Delta W_e \big(1 - f(\gamma)\big) \tag{8}$$

where $\gamma = \sigma_u/\sigma_{uy}$, $\Delta W_e = \sigma_{ij}^{'} \Delta e_{ij}^{'e}/2$; $\sigma_u = \sqrt{\sigma_{ij}^{'} \sigma_{ij}^{'}}$ is the intensity of stress tensors; $\Delta e_{ue} = \sqrt{e_{ij}^{'e} e_{ij}^{'e}}$ is the incrementing intensity of plastic deformation, σ_{uy} is the intensity of stress tensor which is correspondent to endurance limit stress; ΔW_0 is the "dangerous" part of the specific energy ΔW_e; the "non-dangerous" part under the loading is written as $\Delta W_e f(\gamma)$.

Function $f(\gamma)$ describes the mechanism of influence of HCF on a fatigue life of our structural material. Here we would like to discuss some approximation of the function $f(\gamma)$. Usually (see e.g. Bol'shukhin [8]), the following approximation is used

$$f(\gamma) = \begin{cases} 1 & if \gamma \leq 1 \\ 1 - b^* \left(\frac{\gamma - 1}{\gamma^* - 1}\right)^m & if\ 1 \leq \gamma \leq \gamma^* \\ 1 - b^* & if \gamma \geq \gamma^* \end{cases} \tag{9}$$

Here b^* is an asymptotic value of the $f(\gamma)$ at $\gamma \to \gamma^*, \sigma_u \to \sigma_u^*$ and $0 < b^* < 1$. Fig. 1 represents the expected behavior $f(\gamma)$ as a function of a variable γ.

Generally speaking, this function f depends on four variables but three of them, i.e. $b^*,\ \gamma^*,\ m$, are considered as parameters. The values of these parameters should be determined from experimental data. Here we should note the curve that presented at Fig. 1, has a convex type in the region $\gamma \in [1, \gamma^*]$. It is clear that this convexity is determined by the following demand $m \geq 1$. It is the case that experimentally supported and discussed in literature last time (see, e.g. [8-10]).

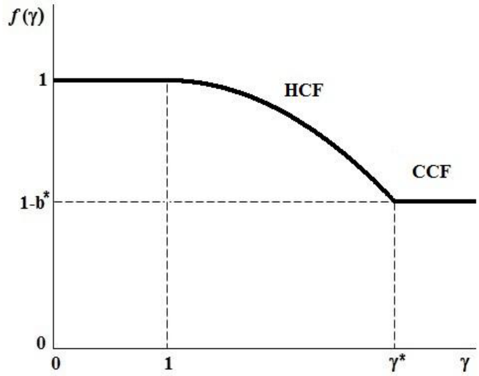

Figure 1. Function f that describes the mechanism of influence of HCF on a fatigue life of a structural material vs the normalized intensity of stress tensorsγ.

Let us discuss the physical picture of material behavior that is described by this model of the function $f(\gamma)$. There is no expected the appearing of a fatigue crack in the interval $\gamma \in [0,1]$. But, fatigue cracks may appear due to HCF mechanisms in the region $\gamma \in [1, \gamma^*]$. At last, mutual act of combined HCF and LCF mechanisms of a fatigue crack origin is expected in the region where $\gamma > \gamma^*$.

It is seems clear, that such hard separation of the regions is quite approximate in real. We propose here to use another equation for the function $f(\gamma)$ in hope that it is more gradual. Thus, it gives us possibility to write some analytical formulas.

Let us write the following equation

$$f_n(\gamma, b^*, \mu, T_m) = \frac{1-b^*+e^{\frac{\mu-\gamma}{T m}}}{1+e^{\frac{\mu-\gamma}{T m}}} \tag{10}$$

Our function, $f_{n,}(\gamma)$ depends on three additional parameters too. We use for them the following symbols: b^*, μ and T. The functional dependence is a modified version of a logistic function which is the well-known Fermi-Dirac distribution. It is supposed that T_m is small as to compare with μ. Parameters μ and T_m in this case have a well-defined meaning. Thus, μ is the middle point of the transitional region of the width T_m. Meanwhile, Eq. (10) gives more gradual approximation than the Eq. (9), see Fig. 2.

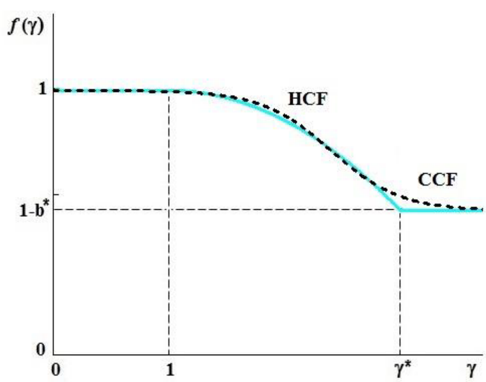

Figure 2. Comparison between functional dependences of the modifiedfunction $f_n(\gamma)$ (dashed line) and an usual approximation of the function $f(y)$ (solid line) at the same value of the parameter b^*.

Note, that in our model there is no sharp boundary between the region where fatigue cracks are caused by the HCF mechanisms and the region where CCF mechanisms is expected. It seems to us, that this physical picture is quite realistic as to compare with the sharp change of the fatigue crack origin mechanism. Then, parameter b^* has the same physical meaning that it is in the model which is described by Eq. (9). Namely, it bound the minimal value of the “non-dangerous” part of the specific energy under the loading. But now, the process of fatigue crack appearing is described by the smooth transition from HCF to CCF mechanisms. The ”middle point” of the transition between HCF and CCF processes is described now by the parameter μ. The parameter T_m specifies the width and sharpness of the transition curve.

Let us look what are the consequences. Following [8-10] we write the structure of the evolution equation of damage accumulation for the HCF region in a manner like the LCF region (for details see Mitenkov et. al. [10]). Integrating this equation for the given case of loading process give us the equation which bounds the total damage accumulation in an elementary volume,ω, with the total relative specific “dangerous” energy, z_s, that has been accumulated as a result of loading.

For the sake of simplicity, we limit ourselves here the following model. Let the function $F_v(\beta)$, that describes the influence of stress conditions to the process of damage accumulation, is constant. Then, we may write the following equation

$$\omega = 1 - [1 - F_v(\beta) z_s^{\alpha+1}]^{1/(r+1)} \tag{11}$$

Firstly, we discuss the case of cyclic symmetrical loading of a cylindrical sample at uniaxial tensile and compression test with the amplitudeof the intensity of stress tensors σ_{ua} (which is corresponded to the intensity of the strain tensor $e_{ua} = \sigma_{ua}/(2G)$, where$G$is the shear modulus). The specific energy per cycle be written

$$\Delta W_{st} = \frac{e_{ua}\,\sigma_{ua}}{2} \cdot 4 = \frac{\sigma_{ua}}{G} \tag{12}$$

The “dangerous” part of the specific energy per cycle, ΔW_{sf}, is given by

$$\Delta W_{sf} = \Delta W_{st}\big(1 - f_n(\gamma_a)\big) = \frac{\sigma_{ua}^2}{G} \cdot \frac{b^*}{1+e^{\frac{\mu-\gamma_a}{T_m}}} \tag{13}$$

Here $\gamma_a = \sigma_{ua}/\sigma_{cu}$, σ_{cu} is the intensity of stress tensor which is corresponded to the fatigue limit The relative specific “dangerous” energy per cycle is given by (compare with Eq. (12) in paper of Bol’shukhin et al. [8])

$$\Delta z_{st} = \frac{\Delta W_{st}}{\Delta W_{fe}}\big(1 - f_n(\gamma_a)\big) = \frac{\sigma_{ua}^2}{G W_{fe}} \cdot \frac{b^*}{1+e^{\frac{\mu-\gamma_a}{T_m}}} \tag{14}$$

Here W_{fe} is the critical value of the specific “dangerous” energy in HCF regime.

Thus, the total relative specific “dangerous” energy which has been accumulated after *N*cycles is given by

$$z_s = \sum_1^N \Delta z_{st} = \frac{\sigma_{ua}^2}{G W_{fe}} \cdot \frac{b^*}{1+e^{\frac{\mu-\gamma_a}{T_m}}} \cdot N \tag{15}$$

The total damage accumulation, ω_f, after *N*cycles, as it was written in [1], we obtain in the following form

$$\omega_f = 1 - [1 - F_v(\beta) z_s^{\alpha+1}]^{1/(r+1)} \tag{16}$$

where the values of parameters should be determined by fitting experimental data, $\beta = \sigma/\sigma_u$, $\sigma_u = \sqrt{\sigma_{ij}^{'}\sigma_{ij}^{'}}$, $\sigma = \sigma_{ii}/3$. At $N = N_f$ a fatigue crack appears and we have $\omega_f(N_f) = 1$.

As it follows from Eqs. (15-16), we obtain the following equation for the S-N curve

$$N_f = \left(\frac{1}{F_{vm}(\beta)}\right)^{\frac{1}{1+\alpha}} \cdot \frac{GW_{fe}}{\sigma_{ua}^2} \cdot \frac{1}{b^*} \cdot \left(1 + e^{\frac{\mu - \gamma_a}{T_m}}\right) \tag{17}$$

Here $F_{vm}(\beta)$ is the middle value of our function F_v per cycle: $F_{vm}(\beta) = \left(F_v(\beta_c) + F_v(\beta_p)\right)/2$, $\beta_c = \sigma_t/\sigma_{ua}$, $\beta_{pc} = \sigma_c/\sigma_{ua}$.

Let's introduce a new constant B_1

$$B_1 = \left(\frac{1}{F_{vm}(\beta)}\right)^{\frac{1}{1+\alpha}} \frac{GW_{fe}}{b^*} \tag{18}$$

Then Eq. (17) we rewrite in the following manner

$$N_f = B_1 \frac{1 + \exp\left\{\frac{\mu - \left(\frac{\sigma_{ua}}{\sigma_{cu}}\right)}{T_m}\right\}}{\sigma_{ua}^2} \tag{19}$$

Generally speaking, the dependence $N_f(\sigma_{ua})$, that is described by Eq. (19) in the presented model, is not so sharp as to compare with the power law of Eq. (27) in the paper of Bol'shukhin et al. [8].

Here we should note, that at large values of stresses σ_{ua}, Eq. (19) in the main approximation provides the formulae of the following type

$$2N_f = B \frac{1}{\sigma_{ua}{}^2} \tag{20}$$

where constant B one may calculate with taking into account Eqs. (18) and (19).

However, to compare the model results with experimental data it is better to use Eq. (19). In this case we have to solve it numerically to reveal S-N curve in coordinates σ_{ua} and N_f.

The asymptotic value of the relative specific dangerous energy per cycle, b^*, is given as

$$b^* = \frac{\Delta W_{sf}^*}{\Delta W_{st}} \tag{21}$$

Parameters $\mu$and T_m in Eq. (19) should be determined in a such way that function $f_n(\gamma_a, b^*, \mu, T_m)$ possess the value which is equal to $(1 - b^*)$ with the given accuracy to the given material.

Usually, the shift along the axis γ on $4T_m$ from the value of μ yields the deviation from the value of $(1 - b^*)$which is of the order of 1%. This value of the $\mu + 4T_m$ should be corresponded to the technological yield stress of the given material.

The developed model is verified by comparing the data of experiments on uni-axial tensile testing for high-temperature GH4133 alloy at 300 °C with the fitted computed fatigue curve. Figure 3 presents the comparison between tests data for high temperature alloy GH4133 at temperature T= 300 °C and data which has been obtained as s result of numerical calculation of our modified model using Eq. (19).

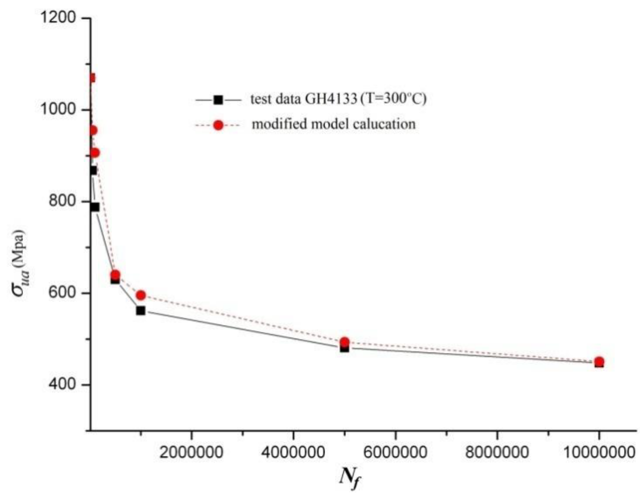

Figure 3. Comparison between curve $\sigma(N)$ predicted by the proposed model (circles) and those tested for GH4133 at 300 ℃ under symmetric loading (squares).

It can be seen from Fig. 3 that calculated curve is close enough to the experimental results for GH4133 at temperature T= 300 °C. Temperature increasing change slightly this picture.The expanded scope of this approach with comparing results of calculations using different models of CCF will be published shortly.

3 Conclusion

The aim of current research was to present a modified energy-principle based model of fatigue damage accumulation in high temperature alloys usually used in gas turbine engine under combined high cycle fatigue and low cycle fatigue loading conditions. In this paper, model is based on the energy principle which includes a modified approximation formula that describes fatigue crack origin depending on the relative amplitude of stress intensity in the ranges of both high- and low-cycle fatigue under non-isothermal loading. Functional dependence that presents the influence of HCF mechanisms on a fatigue life of our structural material is gradual and it has not breaks of the curve that yields a possibility to rewrite the equation of the S-N curve with taking into account combined cycle fatigue loading. We used the same number of parameters as the initial model. Note, that new parameters specify the width and sharpness of the transition region between HCF and CCF mechanisms.The proposed model is verified by comparing the computed results with the experimental data for one high temperature alloy GH4133.

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