

Modeling axial 8-coil system for generating uniform magnetic field in COMSOL

Pavel Baranov^{1,a} and Vitalia Baranova¹

¹National Research Tomsk Polytechnic University, 634050 Tomsk, Russia

Abstract. A changeable source of a uniform magnetic field is necessary for test and calibration of magnetometers. Often the Helmholtz coils is used for generating magnetic uniform field, but uniform field area generating by Helmholtz coils is confined to a small volume in the centre of the coils. The calculation and modelling results of an axial 8-coil system based on Helmholtz coils is described, which enables a magnetic field with a no uniformity of not more than 0.07 % to be produced at a distance of half the radius from the centre of the coil system.

1 Introduction

Measurement of magnetic induction as the characteristics of the magnetic field is one of the most important tasks in the field of creation and operation of navigation systems, orientation and stabilization. A standard of the DC magnetic field requires for test, calibration and graduation of sensors of magnetic fields and magnetometers based on them. As such a standard is applied system of one, two or three coils and a current source. A standard accuracy of this should be at least three times smaller than the error of the calibrated instrument.

The system of solenoid coils or Helmholtz coils [1-3] often used for generating uniform field and calibration magnetic fields sensors. General analysis of the magnetic field in Helmholtz coils and has been presented in works [4-8]. However, it is noticed that the uniform volume of magnetic field in the Helmholtz coil is not large enough for some applications. The calculation of an axial 8-coil system based on Helmholtz coils was described in paper [9].

In this paper we modeled 8-coil system based on Helmholtz coils in COMSOL and compared results.

2 Mathematical description

The Helmholtz coils (Figure 1) consist of two concentric rings of radius R arranged at a distance R from each other as shown in figure 1. The coils are connected in series with the current source to create a uniform field. According to the law of Biot-Savart-Laplace, the resulting field of the two coils is equal to the vector sum of the fields generating by each coil. The axial field of one coil of a predetermined point can be calculated according to the formula (1):

^a Corresponding author : bpf@tpu.ru

$$B(z) = \frac{\mu_0 NIR^2}{2(R^2 + (z-h)^2)^{2/3}}, \quad (1)$$

where μ_0 – magnetic permeability of vacuum, H/m;

N – the number of turns of the each coil;

I – current through the coils, A;

z – coordinate of the point, m;

h – the distance of the coil center from the beginning of the coil center coordinates, m.

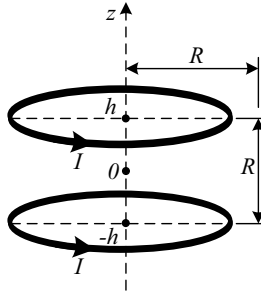


Figure 1. Helmholtz coils.

Axial field of two coils can be calculated by formula (2):

$$B(z) = \frac{\mu_0 NIR^2}{2} \left\{ \left[R^2 + (z+h)^2 \right]^{\frac{3}{2}} + \left[R^2 + (z-h)^2 \right]^{\frac{3}{2}} \right\}. \quad (2)$$

For Helmholtz coils $h=R/2$, then formula (2) take the following form:

$$B(z) = \frac{\mu_0 NIR^2}{2} \left\{ \left[R^2 + \left(z + \frac{R}{2} \right)^2 \right]^{\frac{3}{2}} + \left[R^2 + \left(z - \frac{R}{2} \right)^2 \right]^{\frac{3}{2}} \right\}. \quad (3)$$

Consider the successive derivatives of $B(z)$ in z for estimate the uniform of the field in the center of the coil system. A higher order of the first non-zero derivative at this point provides better uniform of the field in its vicinity.

Equation (3) can be expanded in a Taylor series in the variable z near zero:

$$B(z) = B(0) + \frac{1}{2} B^{(2)}(0)z^2 + \frac{1}{24} B^{(4)}(0)z^4 + \frac{1}{720} B^{(6)}(0)z^6 + \dots \quad (4)$$

Since the Helmholtz coils are arranged at a distance of radius from each other, the second derivative in the central of system becomes zero, i.e. $B^{(2)}(0) = 0$.

Thus, the change in the field ΔB when moving from the point $z = 0$ is given by (5) and has a fourth order of smallness:

$$\Delta B(z) = \frac{1}{24} B^{(4)}(0)z^4 + \frac{1}{720} B^{(6)}(0)z^6 + \dots \quad (5)$$

The magnetic field at the geometric center of the Helmholtz coils ($z = 0$) is determined from the expression:

$$B(0) = \frac{16}{5\sqrt{5}} \frac{1}{2} \frac{\mu_0 NI}{R}. \quad (6)$$

From the expressions (3) and (6) the formula for the relative nonuniformity of the magnetic field along the axis z is:

$$\delta_z = \frac{B(z) - B(0)}{B(0)} = \frac{5\sqrt{5}}{16} \left\{ \left[1 + \left(\frac{1}{2} + \frac{z}{R} \right)^2 \right]^{\frac{3}{2}} + \left[1 + \left(\frac{1}{2} - \frac{z}{R} \right)^2 \right]^{\frac{3}{2}} \right\} - 1. \quad (7)$$

Figure 2 shows graph of the relative nonuniformity of magnetic field δ_z by the formula (7), where values of the z -axis coordinates are given the coil radius R .

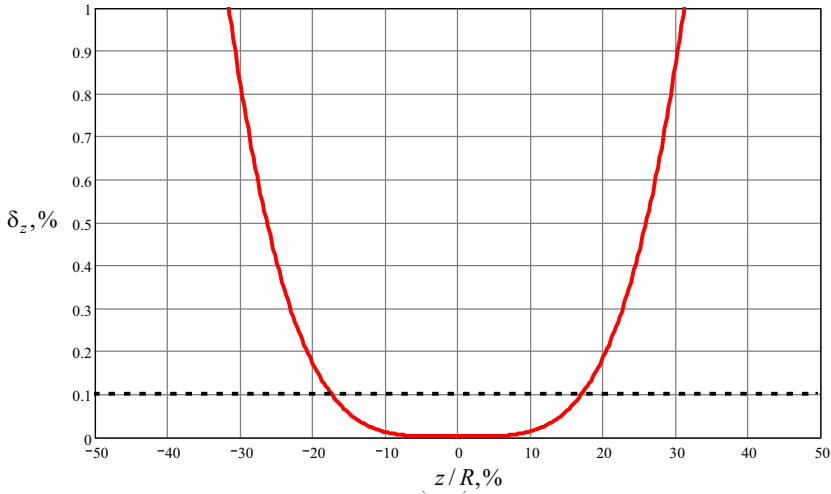


Figure 2. The nonuniformity $\delta_z(z/R)$ of the magnetic field generated by the Helmholtz coils.

Figure 2 shows that the magnetic field nonuniformity $\delta_z \leq 1\%$ can be obtained at a distance from the geometric center $z/R < 31.4\%$. If desired nonuniformity $\delta_z \leq 0.1\%$ (shown by the broken line in Figure 2), the distance z/R should be less than 17.3%.

By analogy with the calculation of magnetic field along the z axis to obtain expression of a cylindrical coordinate system for the axial component $B_z(z, r)$ of the magnetic induction at any point in the volume inside the Helmholtz coils:

$$B_z(z, r) = \frac{\mu_0 NI}{2\pi} \left[\int_0^\pi \frac{R(R - r \cos \alpha)}{[R^2 + r^2 - 2Rr \cos \alpha + (z - R/2)^2]^{3/2}} d\alpha + \int_0^\pi \frac{R(R - r \cos \alpha)}{[R^2 + r^2 - 2Rr \cos \alpha + (z + R/2)^2]^{3/2}} d\alpha \right]. \quad (8)$$

Then axial component relative nonuniformity of magnetic induction at any point in the volume inside the Helmholtz coils is calculated by the formula:

$$\delta_z(z, r) = \frac{B_z(z, r) - B(0)}{B(0)}. \quad (9)$$

Calculation by formula (9) shows that the magnetic field nonuniformity $\delta_z(z, r) \leq 0,1\%$ can be obtained at a distance from the geometric center z/R is less than 17.3 % and r/R is less than 15%.

In the paper [9] we described 8-coil system based on Helmholtz coils with $R = 55$ mm. Parameters of the 8-coil system given in Table 1.

Table 1. Parameters of the 8-coil system to create a uniform magnetic field.

Number coil	The number of turns	The distance from the coil center to the origin, (mm)
1; 5	10	13,5; -13,5
2; 6	12	42,5; -42,5
3; 7	17	80,5; -80,5
4; 8	34	144,5; -144,5

3 Modeling axial 8-coil system

Software COMSOL Multiphysics 4.4 for modeling 8-coil system is used. Figure 3 shows the geometry of the coils system created in COMSOL.

The calculation of the magnetic field produced in the module *mf* as only it allows using the section *Multi-Turn Coil Domain* coils for modeling coils in 3D. The coils have closed geometry with a circular cross-section perpendicular to the axis z . *Circular type* of coil was chosen. The direction of current flow in this case is modeled by specifying the border (reference edge), along which the current flows.

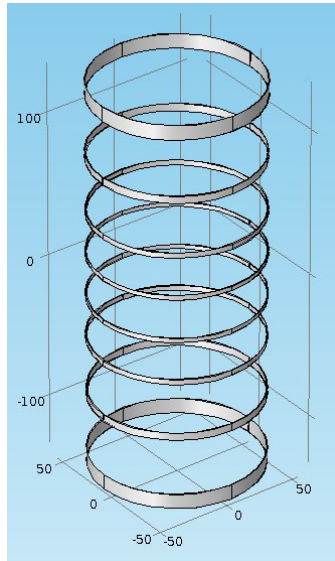


Figure 3. The axial 8-coil system in COMSOL.

Modeling results of the coils system at the DC current of 0.001 A are shown in figure 4 and figure 5.

$$\delta_{z,cae}(z) = \frac{B(z) - B(0)}{B(0)} \cdot 100\% \quad (10)$$

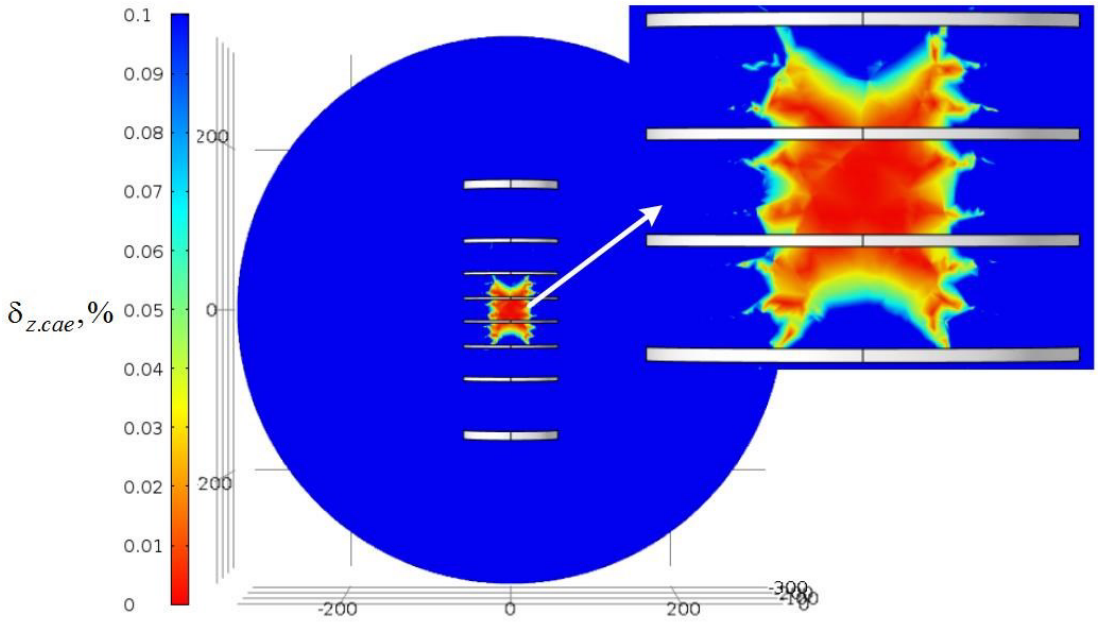


Figure 4. The distribution of the magnetic field in the axial direction.

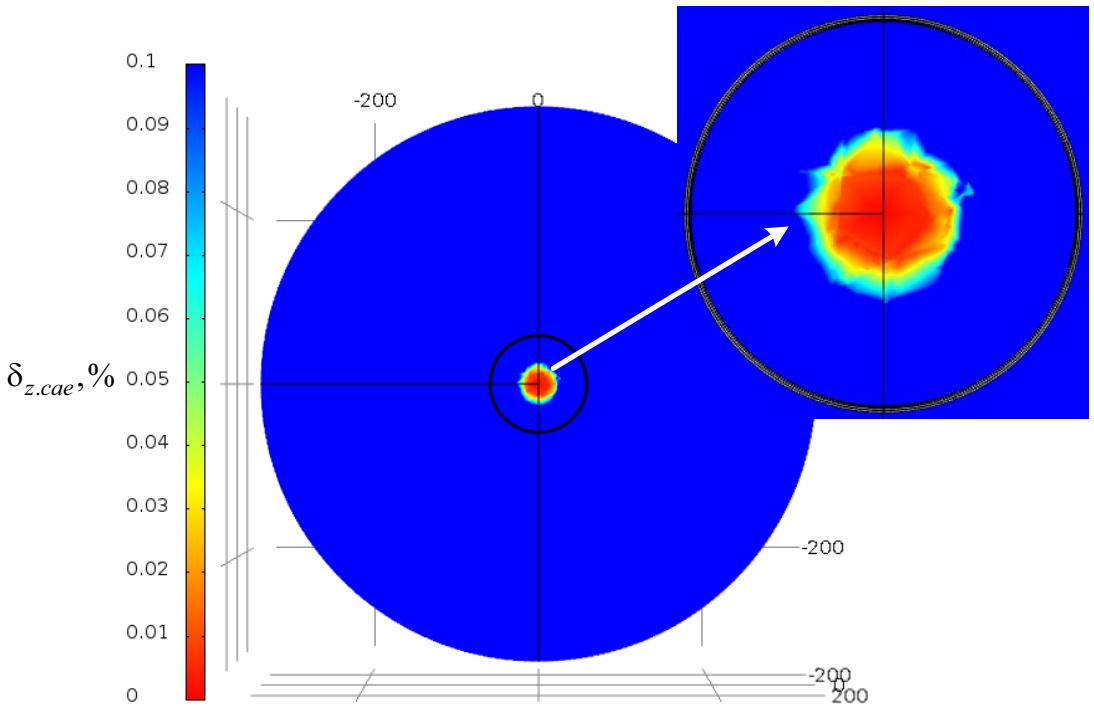


Figure 5. The distribution of the magnetic field in the radial direction.

Nonuniformity of the magnetic field was calculated by formula (10) and is shown in Figure 6.

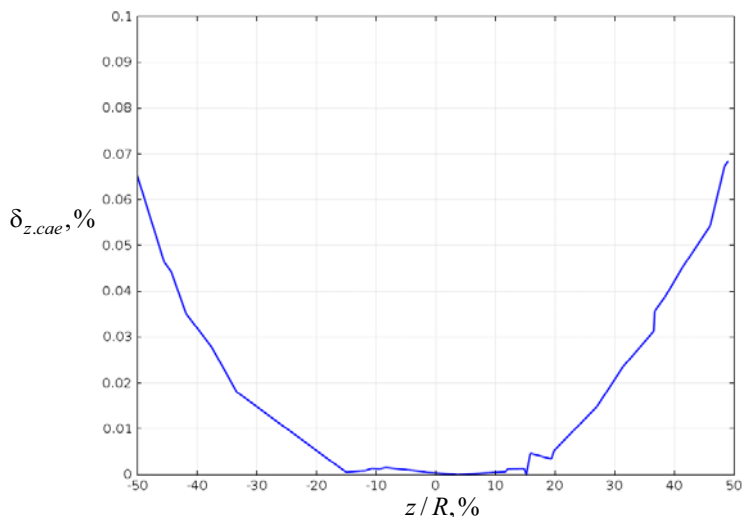


Figure 6. The nonuniformity of the magnetic field.

4 Conclusion

A standard of the DC magnetic field requires for test, calibration and graduation of sensors of magnetic fields and magnetometers based on them.

As a result of work modeling 8-coil system to generating uniform magnetic field with increased volume in comparison Helmholtz coils was described. Parameters of axial 8-coil system for generating uniform magnetic field are described. Modeling results of axial 8-coil system showed that the uniformity of the field production by a system of 8 coils of 55 mm radius is not less than 0.07 % at distances up to $R/2$ from the center of the system.

The obtained results show that such a system can be used to test, calibrate and graduate magnetic sensors and magnetometers based on them.

Acknowledgement

The work has been supported in part by the grant of the RFBR, research project No. 15-08-01007.

References

1. R.A. Schill, and H. Karin, *Review of Scientific Instruments* **72**, 2769 (2001)
2. Po Gyu Park, et al., *CPEM*, 456 (2008)
3. Po Gyu Park, et al., *CPEM*, 312 (2010)
4. D. Lorrain, D. Corson (Freeman, San Francisco, 1979)
5. J. D. Kraus (McGraw–Hill, New York, 1973)
6. P.M. Fishbane, S. Gasiorowicz, S.T. Thornton (Prentice Hall, New York, 1996)
7. M. Hosoya, E. Goto, *Review of Scientific Instruments* **62**, 2472 (1991)
8. V.E. Baranova, P.F. Baranov, *Dynamics of Systems, Mechanisms and Machines* (2014)
9. V.E. Baranova et al., *Measurement Techniques* **58**, 550 (2015)