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Chaotic Dynamics Analysis for a Class of Delay Nonlinear Finance Systems

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Abstract. This article focuses on a class of nonlinear chaotic finance model with feedback control problem. The dynamic responses of the delayed finance system were analyzed and the chaos control problems were considerd. The main work consists of three steps: (i) for a financial system model with the delayed feedback control, the fixed point was obtained, and a new system was obtained by shifting the fixed point to the coordinate origin; (ii) the delayed term was added to the new system, the characteristic equation of the new system was solved, and the distribution of the characteristic equation roots was analyzed. Since the system with time delay undergoes Hopf bifurcation at the equilibrium point under certain conditions, and the fixed point exists stability switching phenomenon, then the intervals of the stable and unstable fixed point were specifically given;(iii) the stable periodic solution and the stable fixed point were simulated under a set of specific parameters, therefore the previous theoretical results obtained by numerical simulation were verified.

Introduction

Financial system is composed of many elements,which is an open and extremely complex nonlinear systems. In this non-linear system, delay is an important factor. In the financial system, we can not fully describe some economic phenomenon with the general differential equation because of many uncertain factors. Some scholars added the time delay factor in differential equations when they studied the economic dynamics [1-5]. In these studies, time delay was focused on a fixed point. However, in practical applications, time delay is often distributed over a range, so the introduction of continuous distributed delays can better describe the actual economic problems [6].

This paper studies a class of nonlinear chaotic financial system. Since the system appears chaotic response with certain parameters, the delayed feedback control method is introduced into the financial system. We analyse the dynamic response of the finance system with time delay, and the chaos control problem is considered.

1 The proposal of the model

Since the 1980s, economists found that the chaos phenomenon exists in economic system [7-9] and it has a tremendous impact on the western mainstream economics , because the economic system appear chaotic phenomenon means macroeconomic movement itself is inherently instability. Literature [10-12] developed a chaotic financial system differential equations model composed of the manufacturer sub-blocks, sub-block currencies, securities sub-block and sub-block. The equations can be expressed as follows

$$\begin{cases} \dot{X} = Z + (Y - a)X \\ \dot{Y} = 1 - bY - X^2 \\ \dot{Z} = -X - cZ \end{cases} \tag{1-1}$$

In finance, the above parameter variables have practical significance. X represents the interest rate, Y represents the investment demand, Z represents the price index, a represents the amount of storage, b represents the investment growth, c represents the supply and demand factor. These parameters play an important part in the market supply and demand.

By linear transformation [13]:

$$x(t) = X(t), y(t) = Y(t) - 1/b, z(t) = Z(t),$$

the new system is obtained as follow:

$$\begin{cases} \dot{x} = (1/b - a)x + z + xy \\ \dot{y} = -by - x^2 \\ \dot{z} = -x - cz \end{cases} \tag{1-2}$$

System (1-1) and (1-2) are topologically equivalent, then we can reflect dynamical properties of the system (1-1) by studying the kinetic properties of the system (1-2). Cai *et al.* analyzed the fixed point stability of the system (1-2) in detail and a new class of chaotic attractors are discovered under the particular parameter value. In this paper, we try to control this system to be stable at the fixed points by introducing delayed feedback control system.

2 Stability analysis of equilibrium

2.1 Delay finance chaos feedback control system

First, we add delay to the first system of equations (1-2)and consider the delay feedback control problem of the system. Thus, the finance model can be simplified into the following dynamic system model:

$$\begin{cases} \dot{x} = (1/b-a)x + z + xy + k(x(t-\tau)-x(t)) \\ \dot{y} = -by - x^2 \\ \dot{z} = -x - cz \end{cases} \quad (2\text{-}1)$$

In the following discussions, we analyze the fixed points of the equation (1-1) in three cases.

(1) (0,0,0) is a fixed point of the system (1-1).

(2) If $c-b-abc<0$, the system has only one fixed point $(0,0,0)$.

(3) If $c-b-abc>0$, the system (1-1) has two fixed points:

$$(x_1,y_1,z_1) = (-\sqrt{\frac{c-b-abc}{c}}, \frac{abc+b-c}{bc}, \frac{1}{c}\sqrt{\frac{c-b-abc}{c}})$$

$$(x_2,y_2,z_2) = (\sqrt{\frac{c-b-abc}{c}}, \frac{abc+b-c}{bc}, -\frac{1}{c}\sqrt{\frac{c-b-abc}{c}})$$

This article achieves the purpose of chaos control by introducing delay feedback. we will not consider fixed point (0,0,0) and only analyze the dynamic stability of the other two point when $c-b-abc>0$.

We define

$w_1(t) = x(t)-x_i, w_2(t) = y(t)-y_i, w_3(t) = z(t)-z_i, (i=1,2)$,

the equations are obtained as follows:

$$\begin{cases} \frac{dw_1(t)}{dt} = (1/b-a+y_i)w_1(t) + x_i w_2(t) \\ \quad + w_3(t) + w_1(t)w_2(t) + k(w_1(t-\tau)-w_1(t)) \\ \frac{dw_2(t)}{dt} == -2x_i w_1(t) - bw_2(t) - w_1(t)^2 \\ \frac{dw_3(t)}{dt} == -w_1(t) - cw_3(t) \end{cases} \quad (2\text{-}2)$$

The characteristic equation of the equation (2-2) at the origin of coordinates is as follow.

$$\lambda^3 + R_2^*\lambda^2 + R_1^*\lambda + R_0^* - k(\lambda^2 + S_1^*\lambda + S_0^*)e^{-\lambda\tau} = 0 \quad (2\text{-}3)$$

where,

$$\begin{aligned} R_0^* &= kbc + 2(c-b-abc) \\ R_1^* &= bc - 3b/c + k(b+c) + 2 - 2ab \\ R_2^* &= b + c - 1/c + k \\ S_0^* &= bc, S_1^* = b + c \end{aligned} \quad (2\text{-}4)$$

We found that the models are same for two non-trivial fixed points of the characteristic equation. And the method to discuss this two fixed point is similar. So we just need to discuss one fixed point. When $\tau=0$ the parameters are taken into equation (2-3), and equation (2-4) is obtained.

$$\begin{aligned} &\lambda^3 + (b+c-\frac{1}{c})\lambda^2 + (bc - \frac{3b}{c} + 2 - 2ab)\lambda \\ &+ 2(c-b-abc) = 0 \end{aligned} \quad (2\text{-}5)$$

According to the Routh-Hurwitz criterion, we know that if the following equations are observed,

$$\begin{cases} (b+c-\frac{1}{c}) > 0 \\ (b+c-\frac{1}{c})(bc-\frac{3b}{c}+2-2ab) - 2(c-b-abc) > 0 \end{cases},$$

the real part of the solution of the equation (2-3) are located in the left half plane of the axis. In this case, the fixed point $(x_i,y_i,z_i)(i=1,2)$ of chaotic system (2-1) remains stable.

The purpose of this paper is to control the chaos, which means that the unstable fixed points are controlled to be stable. Therefore, the fixed point is unstable when we discuss the situation of $\tau=0$. So the parameters need to meet the following conditions:

$(b+c-\frac{1}{c}) \le 0$ or

$(b+c-\frac{1}{c})(bc-\frac{3b}{c}+2-2ab) - 2(c-b-abc) \le 0$.

2. 2 Analysis of the characteristic root

Here, we analyze the characteristic root of the system. For equation (2-3), we set $\tau=0$. The characteristic equation of the original system (2-3) has the following form:

$$\lambda^3 + (R_2^*-k)\lambda^2 + (R_1^*-kS_1^*)\lambda + R_0^* - kS_0^* = 0 \quad (2\text{-}6)$$

When $\tau=0$, equations (2-3) and (2-5) are equivalent. When $\tau\neq 0$, the characteristic equation is as follow:

$$\lambda^3 + R_2^*\lambda^2 + R_1^*\lambda + R_0^* - k(\lambda^2 + S_1^*\lambda + S_0^*)e^{-\lambda\tau} = 0$$

We assume the characteristic root of the above equation is $\lambda(\tau) = \alpha(\tau) + \beta(\tau)i$. When $\tau = \tau_*$, if $\alpha(\tau_*) = 0, \beta(\tau_*) \neq 0$, the system (2-1) experiences a Hopf bifurcation at (0,0,0) with $\tau = \tau_*$. Then, we assume that the equation has a pure imaginary root $\lambda = i\omega$ $(\omega > 0)$, and take it into the above equation, the following equations are obtained.

$$\begin{aligned} &-R_2^*\omega^2 + R_0^* + (K\omega^2 + KS_0^*)\cos\omega\tau - KS_1^* w\sin\omega\tau \\ &-(\omega^3 - R_1^*\omega + (K\omega^2 - KS_0^*)\sin\omega\tau + KS_1^* w\cos\omega\tau)i = 0 \end{aligned}$$

$$\begin{cases} -\omega^3 + R_1^*\omega = (K\omega^2 - KS_0^*)\sin\omega\tau + KS_1^* w\cos\omega\tau \\ -R_2^*\omega^2 + R_0^* = -(K\omega^2 - KS_0^*)\cos\omega\tau + KS_1^* w\sin\omega\tau \end{cases} \quad (2\text{-}7)$$

After calculation, the equation (2-7) is given.

$$\begin{aligned} &\omega^6 + (R_2^{*2} - 2R_0^* - K^2)w^4 + (R_1^{*2} - 2R_0^*R_2^* \\ &+ 2K^2S_0^* - K^2S_1^{*2})\omega^2 + R_0^{*2} - K^2S_0^{*2} = 0 \end{aligned} \quad (2\text{-}8)$$

Then, we have to discuss a six-order equation. After observation, we found that we can simplify this equation to a cubic equation. When we assume that $p=\omega^2$, the equation (2-8) is obtained:

$$p^3 + l_2p^2 + l_1p + l_0 = 0 \quad (2\text{-}9)$$

Where,

$$l_0 = R_0^{*2} - K^2 S_0^{*2},$$

$$l_1 = (R_1^{*2} - 2R_0^* R_2^* + 2K^2 S_0^* - K^2 S_1^{*2}), \quad (2\text{-}10)$$

$$l_2 = (R_2^{*2} - 2R_0^* - K^2)$$

If equation $p^3 + l_2 p^2 + l_1 p + l_0 = 0$ has two or three positive roots, the equilibrium point may has stable range. So we just need to discuss the existence of positive root of equation (2-9). With the help of the Matlab software, it is easy to achieve in numerical results.

3 Numerical Simulation

3.1. Numerical simulation of the non-Delay Systems

The main purpose of this subsection is to select the appropriate control intensity k and time delay τ . Thus,we can control the fixed-point into a stable, or branched stable periodic solution in the vicinity of the fixed point.

Considering the system (1-2), Let $a = 0.00001, b = 0.1, c = 1,$ we have:

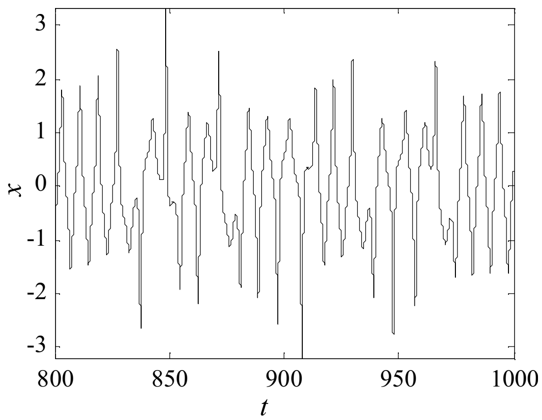
$$\begin{cases} \dot{x} = 9.99999x + z + xy \\ \dot{y} = -0.1y - x^2 \\ \dot{z} = -x - cz \end{cases} \quad (3\text{-}1)$$

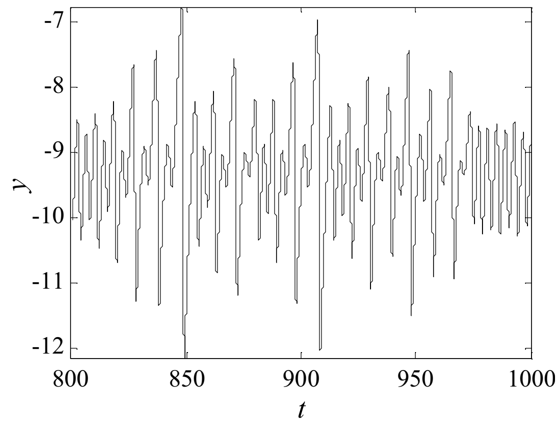
The system (3-1) is chaotic and has three equilibria: (0,0,0),

$(x_1, y_1, z_1) = (0.9487, -9.0000, -0.9487),$

$(x_2, y_2, z_2) = (-0.9487, -9.0000, 0.9487)$

By the time $\tau = 0, k = 0$, the system (1-2) and the system given in [13] are the same. In this case, the numerical simulations of nonlinear chaotic systems are as follows:





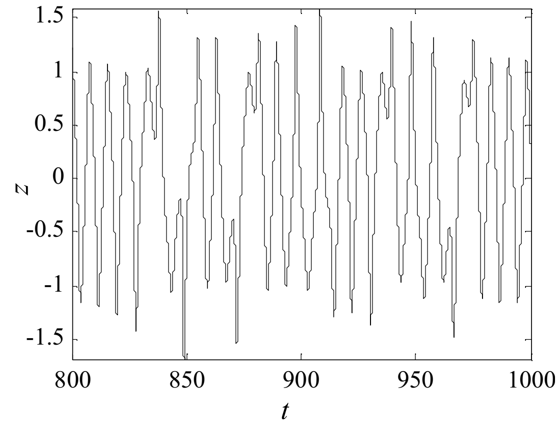


Figure1.when $\tau = 0$,waveforms for (x_1, y_1, z_1) , respectively

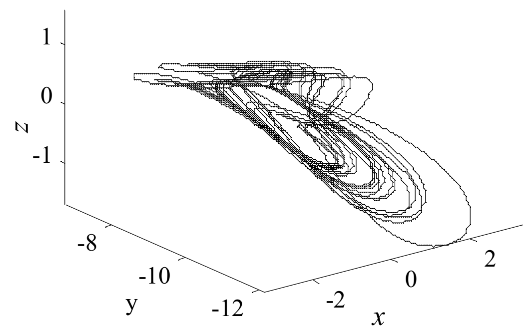


Figure 2.when $\tau = 0$, chaotic motion

It is found that the system is unstable under this condition. According to the discussion in section II, the range of k is obtained when (2-9) has a positive root. Under this situation, the system will produce Hopf bifurcation.

Let $k = 0.33$, we have Substituting $a = 0.00001, b = 0.1, c = 1, k = 0.33$ into （2-4）and（2-10） yields

$R_0^* = 1.8330, R_1^* = 2.1630, R_2^* = 0.4300,$

$S_0^* = 0.1000, S_1^* = 1.1000, l_0 = 3.3588$

$l_1 = 2.9922, l_2 = 3.3588$

Hence, we obtain the positive roots of the equation: $h(p) = p^3 + l_2 p^2 + l_1 p + l_0 = 0$.

In this case, this equation has two positive roots and one negative root:

$p_1 = 2.2171, p_2 = 2.6127, p_3 = -0.5798$

Discuss the stability of the equilibria:

$(x_1, y_1, z_1) = (-0.9487, -9.0000, 0.9487)$

At this time:

$p_1 = 2.2171, h(p_1) = -1.1065 < 0, \omega_1 = \sqrt{p_1} = 1.4890,$

$p_2 = 2.6127, h(p_2) = 1.6164 > 0, \omega_2 = \sqrt{p_2} = 1.6164,$

$\tau_0^{(0)} = 1.8664 < \tau_1^{(0)} = 2.1700 < \tau_1^{(1)} = 6.0572 < \cdots$

Next, we will discuss the Hopf bifurcation and stability.

When $\tau = \tau_0^{(0)} = 1.8664$, we find:

$\mathrm{Re}(C_1(0)) = -1.5759 < 0, \mu_2 = -3.1518 < 0,$

$\beta_2 = -3.1518 < 0, T_2 = 51.0390 > 0$

Since there exists the periodic solutions of Hopf bifurcation when $\tau < \tau_0^{(0)}$, then bifurcation periodic solution is stable in the center manifold and the period of the bifurcation periodic solution increases.

When $\tau = \tau_0^{(0)} = 2.1700$, we find:

$\mathrm{Re}(C_1(0)) = -1.2672 < 0, \mu_2 = 5.4593 > 0,$

$\beta_2 = -2.5344 < 0, T_2 = -44.8958 < 0$

Hopf bifurcation periodic solution exists when $\tau > \tau_1^{(0)}$; the bifurcation periodic solution is stable in the center manifold andthe period of the bifurcation periodic solution is reduced.

3.2 Numerical simulation of systems with time delay feedback

Next, numerical simulation is performed for the parameter of system (2-1) to verify the results discussed above. Simulation results provide a strong basis for the correctness of the conclusions.

We take $a = 0.00001, b = 0.1, c = 1, k = 0.33$, and the initial value is $(0.8, -8, -0.8)$.

Notice that $\tau = 0$ has been discussed before, and shown in Figure 1 and Figure 2 . In the following part, numerical simulation for (x_1, y_1, z_1) are made for the case of the fixed point $\tau = 1.4 < \tau_0^{(0)} = 1.8664$

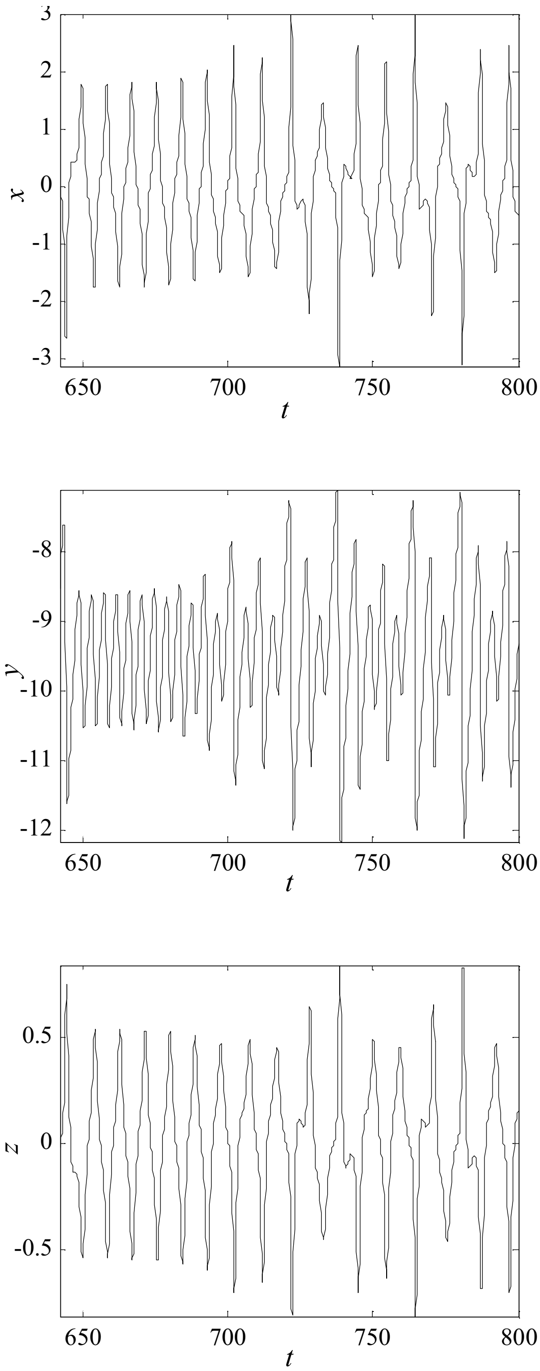


Figure 3.when $\tau = 1.4 < \tau_0^{(0)}$, waveforms for (x_1, y_1, z_1), respectively

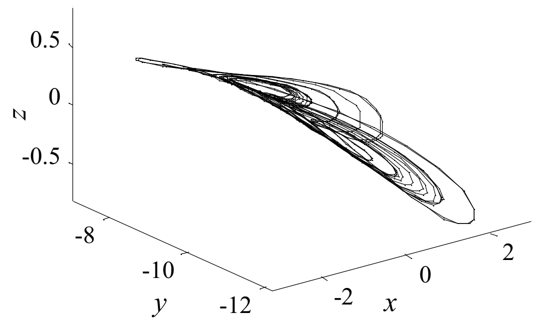
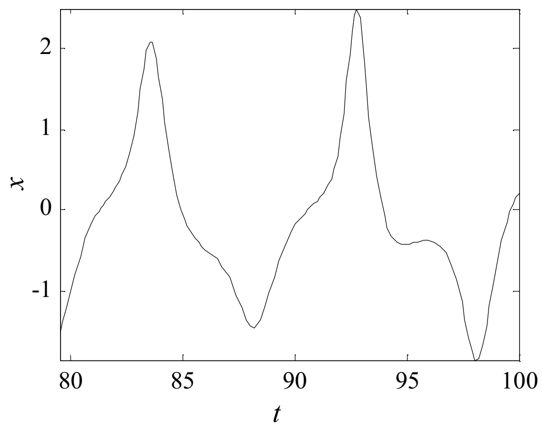
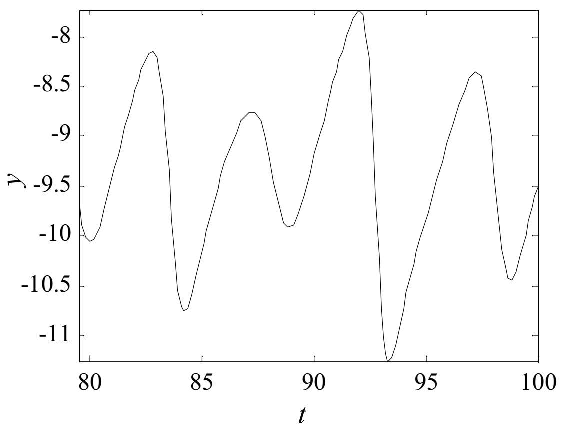


Figure 4. when $\tau = 1.4 < \tau_0^{(0)}$, the chaotic motion

According to Figure 3and Figure 4 , when $0 < \tau = 1.4 < \tau_0^{(0)}$, we can clearly see that the system is still chaotic at a fixed point (x_1, y_1, z_1) .

Next, we will analyse the situation of (x_1, y_1, z_1) when $\tau_0^{(0)} < \tau < \tau_1^{(0)}$





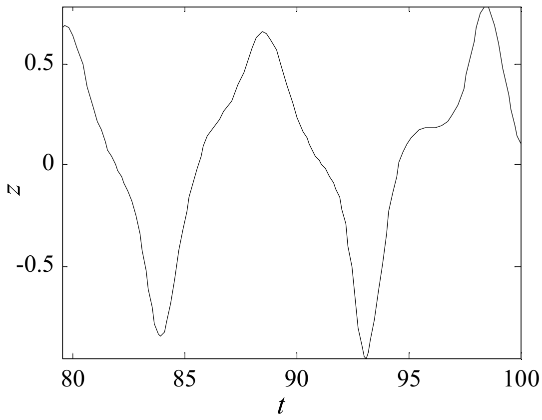


Figure 5. when $\tau_0^{(0)} < \tau = 1.9 < \tau_1^{(0)}$,waveforms for (x_1, y_1, z_1) , respectively

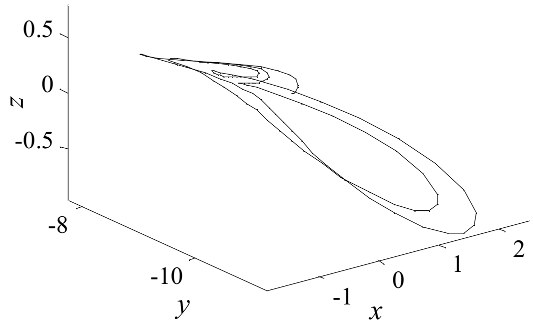
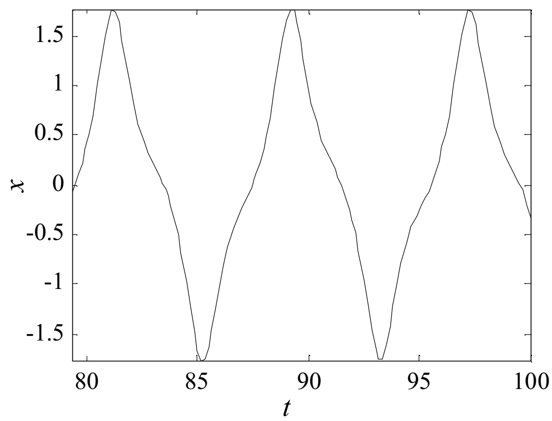


Figure 6. when $\tau_0^{(0)} < \tau = 1.9 < \tau_1^{(0)}$,the fixed point (x_1, y_1, z_1) is stable

When the fixed point near the phase diagram is as follows:



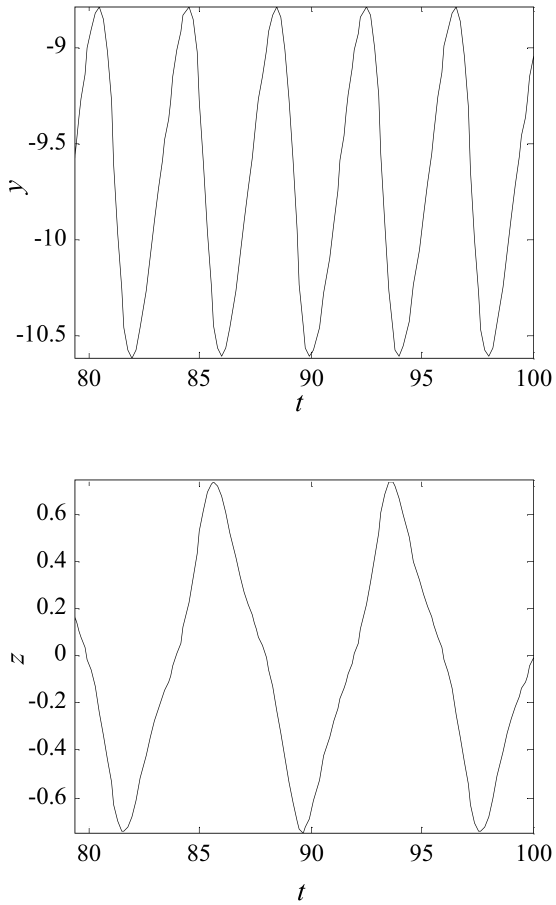


Figure 7. when $\tau = 2.8000 > \tau_1^{(0)}$, waveforms for (x_1, y_1, z_1) respectively

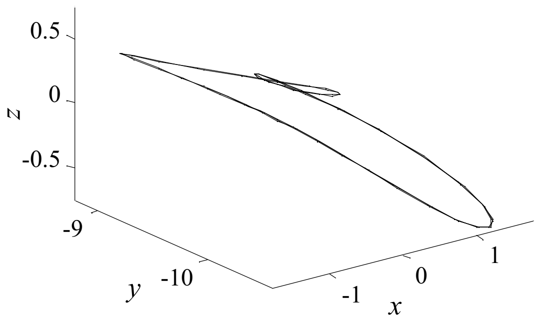


Figure 8. when $\tau = 2.8000 > \tau_1^{(0)}$, period solutions of Fixed Points (x_1, y_1, z_1)

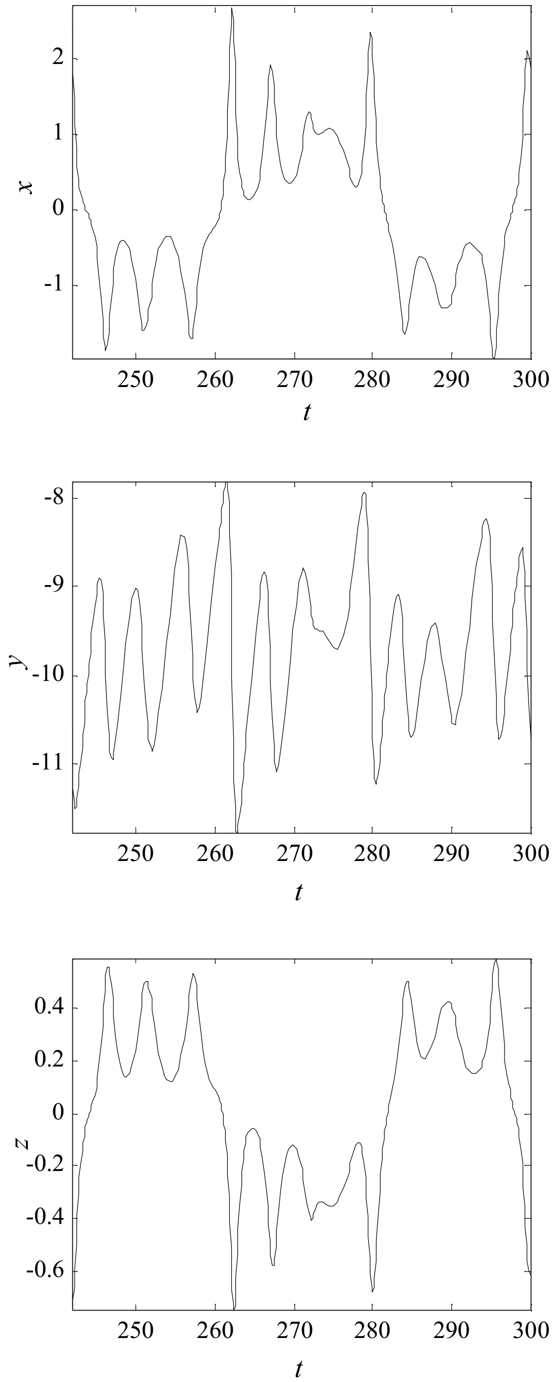


Figure 9. when $\tau = 10 > \tau_1^{(0)}$, waveforms for (x_1, y_1, z_1) respectively

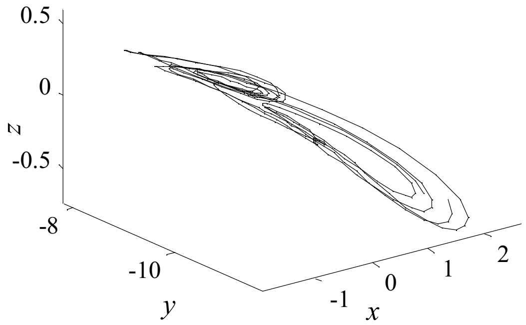


Figure 10. when $\tau = 10 > \tau_1^{(0)}$, the chaotic motion

We can see that there will be no large-scale solutions near the point of the cycle. We can not explain this phenomenon in theory, but from the numerical simulation, we can clearly find that there exists a similar periodic solutions.

It can be found that under the same set a,b,c of parameters and the same initial value, if we take the value k as the positive root of equation $h(p) = p^3 + l_2 p^2 + l_1 p + l_0 = 0$ With the increase of τ , the system will produce a stable or unstable periodic solution.

In this section, numerical simulations are performed by using the DDE application of Matlab toolkit. The Matlab software has a powerful computing capabilities and graphics capabilities. Using the data discussed in the finance model provided in [13], we successfully make the chaotic system stable in the vicinity of the fixed point. At the same time, we discovered the Hopf phenomenon and verified the correctness of the theoretical derivationbyusing the powerful DDE45 toolkit of Matlab, . Thereby, through combining theory and practice, we achieve the purpose of illustrating this problem.

4 Conclusions

This paper discusses a class of finance models with chaotic characteristics and the delay feedback control problem for these models. According to the finance chaos model which was discussed in the literature [13] by Cai, when the delay feedback was added into the equation of the system, the unstable fixed point of chaos system was controlled into a stable fixed point, or the stable periodic solution was branched near the fixed point. The numerical simulation are performed with the assistance of the computer. It is found that the unsuitable parameter combination is the root of the appearance of chaos for the system.Therefore, the study of the stability of this control system as well as the analysis of the characteristics of the internal dynamics for the system are very necessary. We also found that the financial system with nonlinear chaotic characteristic and delay feedback has rich dynamic responses. It is indicated that the delay feedback item is a sensitive factor in the financial system. This study has an important practical value which can provide a theoretical reference for the relevant departments to regulate the economic activity.

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