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Applying the Principle of Variables to Solve the Problems of Forced Vibration of the plate with three clamped and the other free with concentrated load

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Abstract. In this paper, with the principle of least action with variables to solve the problems of forced vibration of the Rectangular plate with three clamped and the other free with concentrated load, and the stable solution can be worked out. We can compare the results with the literate; it also can be proved to be true. So the results by calculating not only it have important academic value, but also it can be directly referred in the actual work.

1 Introduction

Curved rectangular sheet has been widely used in engineering practice. when calculating the stability vibration and bending of the sheet based on sheet classical theory, there will be some errors. In the previous solution, it is difficult to find an easy displacement functions to solve it. The principle of least action mixed variables applied in article[1] requires only weak displacement, that is, displacement should meet the requirements of the strain-displacement in advance, instead of boundary conditions. This eliminates the need to make displacement hypothesis. the total potential energy of mixed variables can be set up according to the actual boundary condition of the curved rectangular sheet, thus obtained the steady-state solution of forced vibration can be obtained. This solution overcomes the classical solution of the complex calculation process and the difficulty to solve certain issues and other limitations[2-5].

2 The basic equation

Figure 1 (a) is a rectangular plate with three fixed and one free, undock side *x=0, y=0, y=a* bending moment amplitude constraint substituting $\overline{M}_{xo}$, $\overline{M}_{xa}$, $\overline{M}_{yo}$ as the Figure 1 (b) shows, also assume that the free edge of *y=b* of deflection magnitude $\overline{w}_{yb}$

$$\overline{M}_{xo} = \sum_{n=1,2}^{\infty} A_n \sin \beta_n y \tag{1}$$

$$\overline{M}_{xa} = \sum_{n=1,2}^{\infty} B_n \sin \beta_n y \tag{2}$$

$$\overline{M}_{yo} = \sum_{m=1,2}^{\infty} C_m \sin \frac{m\pi x}{a} \tag{3}$$

$$\overline{w}_{yb} = \sum_{m=1,2}^{\infty} d_m \sin \frac{m\pi x}{a} \tag{4}$$

Wherein $\beta_n = \dfrac{n\pi}{b}$, A_n, B_n, c_m, d_m coefficients to be determined.

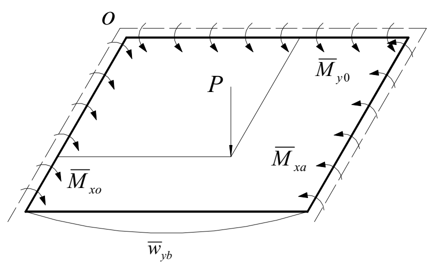

Figure 1(a). The rectangular plate with three clamped and the other free with concentrated load

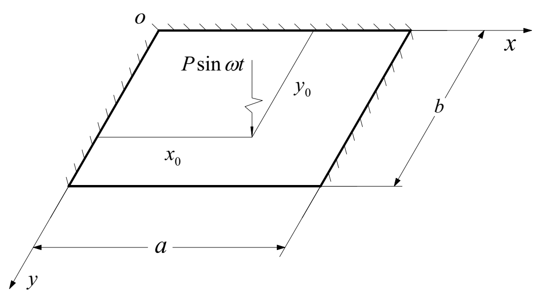

Figure 1(b). The actual system of amplitude

Let any concentrated load

$$F(x, y, t) = P\delta(x - x_0, y - y_0)\sin \omega t$$

Boundary conditions

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$$\left.\begin{aligned}\left(\frac{\partial w}{\partial y}\right)_{y=0}=0,\left(\frac{\partial w}{\partial x}\right)_{x=0,a=0}\\ \left[\frac{\partial^3 w}{\partial y^3}+(2-\nu)\frac{\partial^3 w}{\partial x^2\partial y}\right]_{y=0}=0\end{aligned}\right\} \quad (5)$$

$$\left.\begin{aligned}w_{x=0}=w_{x=a}=w_{y=0}=0\\ \left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right)_{y=b}=0\end{aligned}\right\} \quad (6)$$

3 solving any concentrated harmonic forced vibration under loads with three fixed side of rectangular plates

The total potential energy mix variable assignment

$$\Pi_{mp}=\int_0^a\int_0^b\frac{1}{2}D\left\{\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)^2 -2(1-\nu)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\right]\right\}dxdy$$
$$\int_0^a\int_0^b\left(Pw+\frac{1}{2}\rho\omega^2w^2\right)dxdy+$$
$$\int_0^a\overline{w}_{yb}\left(V_y\right)_{y=b}dx-\int_0^b\overline{M}_{x0}\left(\frac{dw}{dx}\right)_{x=0}dy+$$
$$\int_0^b\overline{M}_{xa}\left(\frac{dw}{dx}\right)_{x=a}dy-\int_0^a\overline{M}_{y0}\left(\frac{dw}{dy}\right)_{y=0}dy \quad (7)$$

Suppose deflection curve equation for curved rectangular plate

$$w(x,y)=\sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty}A_{mn}\sin\alpha_m x\sin\beta_n y$$
$$\left(0\le x\le a\quad 0\le y<b\right) \quad (8)$$

Among them $\alpha_m=\dfrac{m\pi}{a}$

The magnitude of the deflection surface equation (8) into (7) take variational extremum formula calculated, then there are

$$\delta\Pi_{mp}=\frac{Dab}{4}\sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty}A_{mn}\delta A_{mn}K_{dmn}^2-$$
$$q\sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty}\delta A_{mn}\frac{ab}{\pi^2 mn}\left[1-(-1)^m\right]\cdot$$
$$\left[1-(-1)^n\right]-\sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty}\frac{b}{2}A_n\alpha_m\delta A_{mn}$$
$$-\sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty}(-1)^m\frac{b}{2}A_n\alpha_m\delta A_{mn}+$$
$$\sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty}\frac{a}{2}C_m\beta_n\delta A_{mn}+(-1)^n\frac{Da}{2}$$
$$\left[\sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty}\delta A_{mn}d_m\beta_n^3+(2-\nu)\cdot\sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty}\delta A_{mn}d_m\alpha_m^2\beta_n\right]=0 \quad (9)$$

Among them $\lambda^2\equiv\dfrac{1}{D}\rho\omega^2$, $K_{dmn}^2\equiv\left(\alpha_m^2+\beta_n^2\right)^2-\lambda^2$

From (9) can be obtained

$$A_{mn}=\sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty}\frac{4q}{D\pi^2}\frac{1}{mn}\frac{1}{K_{dmn}^2}\left[1-(-1)^m\right]\cdot\left[1-(-1)^n\right]+$$
$$\sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty}\frac{2}{Da}\frac{\alpha_m}{K_{dmn}^2}(A_n)+\sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty}\frac{(-1)^m 2}{Da}\frac{\alpha_m}{K_{dmn}^2}(B_n)$$
$$+\sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty}\frac{2}{Db}\frac{\beta_n}{K_{dmn}^2}(C_m)-\sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty}\frac{2}{b}\frac{(-1)^n}{K_{dmn}^2}\beta_n\cdot$$
$$\left[\beta_n^2+\alpha_m^2(2-\nu)\right](d_m)=0 \quad (10)$$

The magnitude of the deflection surface equation points (10) into (8) represented by triangular series are:

$$w_1=\frac{4P}{Dab}\sum_{m=1.2}^{\infty}\sum_{n=1.2}^{\infty}\frac{\sin\alpha_m x_0\sin\beta_n y_0}{K_{dmn}^2}\cdot\sin\alpha_m x\sin\beta_n y \quad (11)$$

$$w_2=\frac{2}{Da}\sum_{m=1.2}^{\infty}\sum_{n=1.2}^{\infty}\frac{\alpha_m}{K_{dmn}^2}\sin\alpha_m x\cdot\sin\beta_n y(A_n) \quad (12)$$

$$w_3=-\frac{2}{Da}\sum_{m=1.2}^{\infty}\sum_{n=1.2}^{\infty}\frac{(-1)^m\alpha_m}{K_{dmn}^2}\cdot\sin\alpha_m x\sin\beta_n y(B_n) \quad (13)$$

$$w_5=-\frac{2}{b}\sum_{m=1.2}^{\infty}\sum_{n=1.2}^{\infty}\frac{(-1)^n}{K_{dmn}^2}\left[\beta_n^3+\alpha_m^2\beta_n(2-\nu)\right]\sin\alpha_m x\sin\beta_n y(d_m) \quad (14)$$

To speed up the convergence rate and eliminate deflection and moment magnitude of the triangular series representation of the boundary that appears in the first category discontinuity, it also must points deflection surface equation for the amplitude of the triangular series and hyperbolic functions expressed mixed form.

In the case of $\alpha_m^2<\lambda$ and $\beta_n^2<\lambda$, then there are

$$w_1 = \frac{2P}{Db}\sum_{n=1.2}^{\infty}\frac{1}{\alpha_n^{'2}-\beta_n^{'2}}\left[-\frac{sh\alpha_n^{'}(a-x_0)sh\alpha_n^{"}x}{\alpha_n^{'}sh\alpha_n^{'}\alpha}+\frac{sh\beta_n^{'}(a-x_0)sh\beta_n^{'}x}{\beta_n^{'}sh\beta_n^{'}a}\right]\cdot \sin\beta_n y_0 \sin\beta_n y \quad (0\le x\le x_0) \tag{15}$$

$$w_1 = \frac{2P}{Db}\sum_{n=1.2}^{\infty}\frac{1}{\alpha_n^{'2}-\beta_n^{'2}}\left[-\frac{sh\alpha_n^{'}x_0 sh\alpha_n^{'}(a-x)}{\alpha_n^{'}sh\alpha_n^{'}\alpha}+\frac{sh\beta_n^{'}x_0 sh\beta_n^{'}(a-x)}{\beta_n^{'}sh\beta_n^{'}a}\right]\cdot\sin\beta_n y_0\sin\beta_n t \quad x_0\le x\le a \tag{16}$$

$$w_1 = \frac{2P}{Da}\sum_{m=1.2}^{\infty}\frac{1}{\alpha_m^{'2}-\beta_m^{'2}}\left[-\frac{sh\alpha_m^{'}(b-y_0)sha_m^{'}y}{\alpha_m^{'}sh\alpha_m^{'}b}+\frac{sh\beta_m^{'}(b-y_0)sh\beta_m^{'}y}{\beta_m^{'}sh\beta_m^{'}b}\right]\cdot\sin\alpha_m x_0\sin\alpha_m x \quad 0\le y\le y_0 \tag{17}$$

$$w_1 = \frac{2P}{Da}\sum_{m=1.2}^{\infty}\frac{1}{\alpha_m^{'2}-\beta_m^{'2}}\left[-\frac{sh\alpha_m^{'}y_0 sha_m^{'}(b-y)}{\alpha_m^{'}sh\alpha_m^{'}b}+\frac{sh\beta_m^{'}y_0 sh\beta_m^{'}(b-y)}{\beta_m^{'}sh\beta_m^{'}b}\right]\cdot\sin\alpha_m x_0\sin\alpha_m x \quad y_0\le y\le b \tag{18}$$

Among them $\alpha_n^{'2}=\sqrt{\beta_n^2+\lambda}$, $\beta_n^{'2}=\sqrt{\beta_n^2-\lambda}$

$\alpha_m^{'2}=\sqrt{\alpha_m^2+\lambda}$, $\beta_m^{'2}=\sqrt{\alpha_m^2-\lambda}$

$$w_2 = \frac{1}{D}\sum_{n=1.2}^{\infty}\frac{1}{\alpha_n^{'2}-\beta_n^{'2}}\left[-\frac{sh\alpha_n^{'}(a-x)}{sh\alpha_n^{'}\alpha}+\frac{sh\beta_n^{'}(a-x)}{sh\beta_n^{'}a}\right]\sin\beta_n y(A_n) \tag{19}$$

$$w_3 = \frac{1}{D}\sum_{m=1.2}^{\infty}\frac{1}{\alpha_m^{'2}-\beta_m^{'2}}\left[\frac{sha_n^{'}x}{sh\alpha_m^{'}a}-\frac{sh\beta_m^{'}x}{sh\beta_m^{'}a}\right]\sin\beta_n y(B_n) \tag{20}$$

$$w_4 = \frac{1}{D}\sum_{m=1.2}^{\infty}\frac{1}{\alpha_m^{'2}-\beta_m^{'2}}\left[-\frac{sha_m^{'}(b-y)}{sh\alpha_m^{'}b}+\frac{{}_0 sh\beta_m^{'}(b-y)}{sh\beta_m^{'}b}\right]\sin\alpha_m x(C_m) \tag{21}$$

$$w_5 = \sum_{m=1.2}^{\infty}\frac{1}{\alpha_m^{'2}-\beta_m^{'2}}\left\{\left[\alpha_m^{'2}-\alpha_m^2(2-\nu)\right]\frac{sh\alpha_m^{'}y}{sh\alpha_m^{'}b}-\left[\beta_m^{'2}-\alpha_m^2(2-\nu)\right]\frac{sh\beta_m^{'}y}{sh\beta_m^{'}b}\right\}\cdot\sin\alpha_m x(d_m) \tag{22}$$

When $\alpha_m^2<\lambda$ and $\beta_n^2<\lambda$, easy to get the corresponding points deflection surface amplitude equation, due to limited space no longer given.

Investigation boundary conditions.

Boundary condition formula (5) is automatically satisfied, the following boundary condition investigated formula (6).When $\alpha_m^2>\lambda$ and $\beta_n^2>\lambda$, executive boundary condition $\left(\frac{\partial w}{\partial y}\right)_{y=0}=0$, then get

$$\frac{2P}{Da}\left[-\frac{sh\alpha_m^{'}(b-y_0)}{sh\alpha_m^{'}b}+\frac{sh\beta_m^{'}(b-y_0)}{sh\beta_m^{'}b}\right]\cdot \sin\alpha_m x_0+\frac{4\lambda}{Da}\sum_{n=1.2}^{\infty}\frac{\alpha_m\beta_n}{K_{dmn}^2}(A_n)- \frac{4\lambda}{Da}\sum_{n=1.2}^{\infty}\frac{(-1)^m\alpha_m\beta_n}{K_{dmn}^2}(B_n)+ \frac{1}{D}(\alpha_m^{'}cth\alpha_m^{'}b-\beta_m cth\beta_m^{'}b)(C_m)+ \left\{\left[\alpha_m^{'2}-\alpha_m^2(2-\nu)\right]\frac{\alpha_m^{'}}{sh\alpha_m^{'}b} \left[\beta_m^{'2}-\alpha_m^2(2-\nu)\right]\frac{\beta_m^{'}}{sh\beta_m^{'}b}\right\}(d_m)=0 \tag{23}$$

Executive boundary condition $\left(\frac{\partial w}{\partial x}\right)_{x=a}=0$, then get

$$\frac{2P}{Db}\left(-\frac{sh\alpha_n^{'}(a-x_0)}{sh\alpha_n^{'}a}+\frac{sh\beta_n^{'}(a-x_0)}{sh\beta_n^{'}a}\right)\sin\beta_m y_0 +\frac{1}{D}\left(\alpha_n^{'}cth\alpha_n^{'}a-\beta_n^{'}cth\beta_n^{'}a\right)(A_n)- \frac{1}{D}\left(\frac{\alpha_n^{'}}{sh\alpha_n^{'}a}-\frac{\beta_n^{'}}{sh\beta_n^{'}a}\right)(B_n)+\frac{4\lambda}{Db}\cdot \sum_{m=1.2}^{\infty}\frac{\alpha_m\beta_n}{K_{dmn}^2}(C_n)-\frac{4\lambda}{b}\sum_{m=1,2}^{\infty}\frac{(-1)^n\alpha_m\beta_n}{K_{dmn}^2} \left[\beta_n^2+\alpha_m^2(2-\nu)\right](d_m)=0 \tag{24}$$

Executive boundary condition

$$\left[\frac{\partial^3 w}{\partial y^3}+(2-\nu)\frac{\partial^3 w}{\partial x^2\partial y}\right]_{y=b}=0,$$

then get

$$\frac{2P}{Da}\left\{\left[\alpha_m^{'2}-\alpha_m^2(2-\nu)\right]\frac{sh\alpha_m^{'}}{sh\alpha_m^{'}b}-\right.$$

$$\left.\left[\beta_m^{'2}-\alpha_m^2(2-\nu)\right]\frac{sh\beta_m^{'}y_0}{sh\beta_m^{'}b}\right\}\sin\alpha_m x_0-$$

$$\frac{4\lambda}{Da}\sum_{n=1.2}^{\infty}\frac{(-1)^n\alpha_m\beta_n}{K_{dmn}^2}\left[\alpha_m^2+\beta_n^2(2-\nu)\right](A_n)+$$

$$\frac{4\lambda}{Da}\sum_{n=1.2}^{\infty}\frac{(-1)^{m+n}\alpha_m\beta_n}{K_{dmn}^2}\left[\alpha_m^2+\beta_n^2(2-\nu)\right](B_n)+$$

$$\frac{1}{D}\left\{\left[\alpha_m^{'2}-\alpha_m^2(2-\nu)\right]\frac{\alpha_m^{'}}{sh\alpha_m^{'}b}-\right.$$

$$\left.\left[\beta_m^{'2}-\alpha_m^2(2-\nu)\right]\frac{\beta_m^{'}}{sh\beta_m^{'}b}\right\}(C_m)+$$

$$\left\{\left[\alpha_m^{'2}-\alpha_m^2(2-\nu)\right]^2\alpha_m^{'}cth\alpha_m^{'}b-\left[\beta_m^{'2}-\alpha_m^{'2}(2-\nu)\right]^2\right.$$

$$\left.\beta_m^{'}cth\beta_m^{'}b\right\}=0 \tag{25}$$

When $\alpha_m^2<\lambda$ and $\beta_n^{'}<\lambda$, the boundary condition (27) - (29)

$$\frac{2P}{Da}\left[-\frac{sh\alpha_m^{'}(b-y_0)}{sh\alpha_m^{'}b}+\frac{sh\beta_m^{"}(b-y_0)}{sh\beta_m^{'}b}\right]\cdot\sin\alpha_m x_0$$

$$+\frac{4\lambda}{Da}\sum_{n=1.2}^{\infty}\frac{\alpha_m\beta_n}{K_{dmn}^2}(A_n)-\frac{4\lambda}{Da}\sum_{n=1.2}^{\infty}\frac{(-1)^m\alpha_m\beta_n}{K_{dmn}^2}(B_n)$$

$$+\frac{1}{D}(\alpha_m^{'}cth\alpha_m^{'}b-\beta_m^{"}ctg\beta_m^{"}b)(C_m)+$$

$$\left\{\left[\alpha_m^{'2}-\alpha_m^2(2-\nu)\right]\frac{\alpha_m^{'}}{sh\alpha_m^{'}b}+\right.$$

$$\left.\left[\beta_m^{"2}-\alpha_m^2(2-\nu)\right]\frac{\beta_m^{"}}{sh\beta_m^{"}b}\right\}(d_m)=0 \tag{26}$$

$$\frac{2P}{Db}\left(-\frac{sh\alpha_n^{'}(a-x_0)}{sh\alpha_n^{'}a}+\frac{sh\beta_n^{"}(a-x_0)}{sh\beta_n^{"}a}\right)\cdot$$

$$\sin\beta_n y_0+\frac{1}{D}\left(\alpha_n^{'}cth\alpha_n^{'}a-\beta_n^{"}cth\beta_n^{"}a\right)(A_n)-$$

$$\frac{1}{D}\left(\frac{\alpha_n^{'}}{sh\alpha_n^{'}a}-\frac{\beta_n^{"}}{sh\beta_n^{'}a}\right)(B_n)+\frac{4\lambda}{Db}\cdot$$

$$\sum_{m=1.2}^{\infty}\frac{\alpha_m\beta_n}{K_{dmn}^2}(C_n)-\frac{4\lambda}{b}\sum_{m=1.2}^{\infty}\frac{(-1)^n\alpha_m\beta_n}{K_{dmn}^{'2}}$$

$$\left[\beta_n^2+\alpha_m^2(2-\nu)\right](d_m)=0 \tag{27}$$

$$\frac{2P}{Da}\left\{\left[\alpha_m^{'2}-\alpha_m^2(2-\nu)\right]\frac{sh\alpha_m^{'}y_0}{sh\alpha_m^{'}b}-\right.$$

$$\left.\left[\beta_m^{"2}-\alpha_m^2(2-\nu)\frac{sh\beta_m^{"}y_0}{sh\beta_m^{"}b}\right]\right\}\sin\alpha_m x_0-$$

$$\frac{4\lambda}{Da}\sum_{n=1.2}^{\infty}\frac{(-1)^n\alpha_m\beta_n}{K_{dmn}^2}\left[\alpha_m^2+\beta_n^2(2-\nu)\right](A_n)+$$

$$\frac{4\lambda}{Da}\sum_{n=1.2}^{\infty}\frac{(-1)^{m+n}\alpha_m\beta_n}{K_{dmn}^2}\left[\alpha_m^2+\beta_n^2(2-\nu)\right](B_n)+$$

$$\frac{1}{D}\left\{\left[\alpha_m^{'2}-\alpha_m^2(2-\nu)\right]\frac{\alpha_m^{'}}{sh\alpha_m^{'}b}-\right.$$

$$\left.\left[\beta_m^{"2}-\alpha_m^2(2-\nu)\right]\frac{\beta_m^{"}}{sh\beta_m^{"}b}\right\}(C_m)+$$

$$\left\{\left[\alpha_m^{'2}-\alpha_m^2(2-\nu)\right]^2cth\alpha_m^{'}b-\right.$$

$$\left.\left[\beta_m^{"2}-\alpha_m^2(2-\nu)\right]^2cth\beta_m^{"}b\right\}(d_m)=0 \tag{28}$$

4 Numerical Analysis

So we get four sets of infinite simultaneous equations (23)-(25) or (26)-(28). Remove restricted item, Solutions for the A_n, B_n, C_m and d_m. And then according to the formula (1) to (4) to obtain the moment and deflection magnitude of the amplitude. In particular, take each items of A_n, B_n,C_m and d_m, and assuming harmonic loads concentrated on midpoint board programming calculated for different values obtained are shown in table 1 deflection magnitude of moment magnitude and fixed side edge moment , and as shown in Figure 2 and Figure 3.

Discuss:

(1) The issue of convergence coefficient. Since the load and structural balance in the direction axis for symmetric, so there are $A_n=B_n, C_m=d_m=0(m=2,4...)$, C_m and d_m that is actually only take eight. And calculated that, An convergence of magnitude from 10^0 to 10^{-2} magnitude, C_m magnitude from 10^0 to converge to 10^{-4} magnitude, d_m magnitude from 10^{-2} to 10^{-5} convergence of magnitude, than the uniform load harmonic convergence is better.

(2) With regard to the distribution of M_{x0}. It is found, When $\omega/\omega_{11}=0.8$, M_{x0} larger value of the free edges will appear near the end. This suggests that, as the load frequency close to the natural frequency and a significant increase in the magnitude of the impact on its adjacent sides of the free edge of the moment.

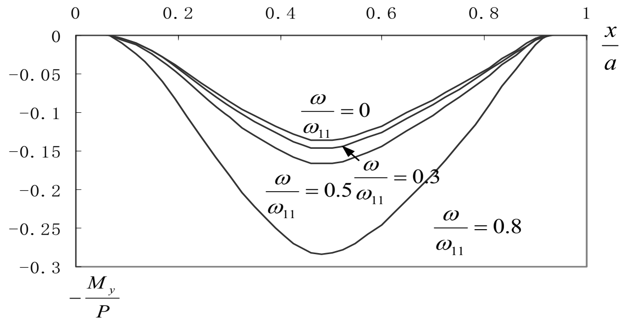

Figure 2. The amplitudes a/b=1 of the moment of clamped edge y=0

(3) With regard to the distribution of M_{x0}.When the ω/ω_{11} value is not the same,When the value is not the same, M_{y0} along $y=0$ edge was smooth symmetrical, and at both ends of a slight reverse moment appear.

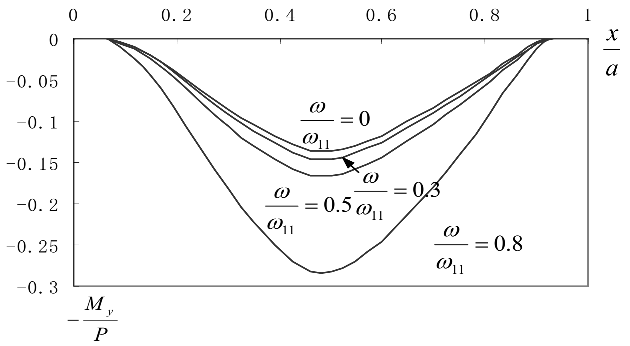

Figure 3. The amplitudes a/b=1 of the moment of clamped edge y=b

(4) The impact of load frequency. When the ω/ω_{11} from 0.0 to 0.8, the free side of the midpoint deflection of about 4.46 times the amplitude increases, y=0 along the side of the midpoint fixed maximum moment magnitude approximately double.

Table 1. The amplitudes（P）of the moment and the maximum deflection （Pa2/D） a/b=1at the clamped ends

ω/ω_{11}	$x/a(y/b)$ / $M(w)$	0.05	0.15	0.35
0.0	M_{x0}	0.000647	-0.02006	-0.1066
	M_{y0}	0.001470	-0.01934	-0.1045
	w_{yb}	0.000044	0.000508	0.002086
0.3	M_{x0}	0.000359	0.022110	-0.1142
	M_{y0}	0.001633	-0.02097	-0.1122
	w_{yb}	0.000062	0.000634	0.002515
0.5	M_{x0}	-0.000416	-0.02682	-0.13110
	M_{y0}	0.002043	-0.02459	-0.12920
	w_{yb}	0.000106	0.000957	0.003601
0.8	M_{x0}	-0.00697	-.05466	-0.22210
	M_{y0}	0.00512	-0.04353	-.2211
	w_{yb}	0.000457	0.003413	0.01171

continued

ω/ω_{11}	$x/a(y/b)$ / $M(w)$	0.5	0.7	0.9	0.95
0.0	M_{x0}	-0.1497	-0.13490	-0.08149	-0.06926
	M_{y0}	-0.1355	-0.08397	-0.00485	0.00147
	w_{yb}	0.002634	0.001712	0.000216	0.000044
0.3	M_{x0}	-0.16140	-0.14970	-0.09592	-0.08865
	M_{y0}	-0.1451	-0.09023	-0.00534	0.001633
	w_{yb}	0.00316	0.002074	0.000278	0.000062
0.5	M_{x0}	-0.1879	-0.18500	-0.13200	-0.13840
	M_{y0}	-0.1664	-0.1042	-0.00641	0.002043
	w_{yb}	0.004489	0.002989	0.000435	0.000106
0.8	M_{x0}	-0.34180	-0.4176	-0.3949	-0.5209
	M_{y0}	-0.2824	-0.17911	-0.01179	0.00512
	w_{yb}	0.01438	0.009844	0.001648	0.000457

5 Conclusion

(1) In this paper, mixed variables method for solving a rectangular plate with three fixed and one free vibration in any concentrated harmonic loads obtain the steady-state solution by forced vibration.

(2) Mix variables method is a simple vibration, universal, effective method to solve problem of forced vibration of bending a rectangular sheet.

(3) The results obtained by mixed variables method is correct, it may be practical engineering directly.

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