MATEC Web of Conferences **44**, 02033 (2016)
DOI: 10.1051/matecconf/20164402033


# On the Adjacent Strong Equitable Edge Coloring of $P_n \vee P_n$, $P_n \vee C_n$ and $C_n \vee C_n$

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**Abstract.** A proper edge coloring of graph G is called equitable adjacent strong edge coloring if colored sets from every two adjacent vertices incident edge are different,and the number of edges in any two color classes differ by at most one,which the required minimum number of colors is called the adjacent strong equitable edge chromatic number. In this paper, we discuss the adjacent strong equitable edge coloring of join-graphs about $P_n \vee P_n$, $P_n \vee C_n$ and $C_n \vee C_n$.

## 1 Introduction

The coloring problem of graphs is widely applied in practice. In [1], some conditional coloring problems as introduced. Some network problem can be converted to the strong edge coloring[2–5] and adjacent strong edge coloring[6].

**DEFINITION 1**[2–5] For a graph G(V, E), if a proper coloring f is satisfied with $C(u) \neq C(v)$ for $\forall u, v \in V(G)(u \neq v)$, then $f$ is called*k-strong edge coloring of G*, is abbreviated *k*-SEC, and

$$\chi'_s(G) = \min\{k \mid k - SEC\ of\ G\}$$

is called the strong edge chromatic number of G. And for $\forall u, v \in E(G), C(u) \neq C(v)$, then $f$ is called *k-adjacent strong edge coloring of G*, is abbreviated *k*-ASEC, and

$$\chi'_{as}(G) = \min\{k \mid k - ASEC\ of\ G\}$$

is called the adjacent strong edge chromatic number of G[6].Where

$$C(u) = \{f(uv) \mid uv \in E(G)\}.$$

**DEFINITION 2**Let $f$ is a $k-$ ASEC of Gand satisfied with $\big\| E_i | - | E_j \big\| \leq 1\ i, j = 1,2,\cdots,k$

$f$ is called *the adjacent strong equitable edge coloring of G*, and noted by *k–ASEEC*of*G*, and

$$\chi'_{ase}(G) = \min\{k \mid k - ASEEC\ of\ G\}$$

is called *the adjacent strong equitable edge chromatic number of G*.Where

$$E_i = \{e \mid f(e) = i\}\ \ i = 1,2,\cdots,k.$$

**Conjecture**[6] For a connected graph with order $p \geq 3$ and $G \neq C_5$ (5-cyycle),
Where $p = |V(G)|$, $\Delta(G)$ is maximal degree of*G*.
There are many references proof this conjecture is true, for example[7-8], for $\Delta(G) \leq 3$, this conjecture is true;
For a connected graph with $V(G) \geq 3$.

(1) If $G$ is a bipartite graph with no isolate edges, then

$$\chi'_{as}(G) \leq \Delta(G) + 2$$

(2)If $G$ is a k-chromatic graph with no isolate edges, then

$$\chi'_{as}(G) \leq \Delta(G) + O(\log k).$$

**DEFINITION 3**[9] For graph G and graph H, $V(G) \bigcap V(H) = E(G) \bigcap E(H) = \varnothing$, and

$$\begin{cases} V(G \bigcup H) = V(G) \bigcup V(H) \\ E(G \bigcup H) = E(G) \bigcup E(H) \bigcup \\ \{uv \mid u \in V(G), v \in V(H)\} \end{cases},$$

then $G \vee H$ is called join-graph of G and $H$.

**LEMMA 1**[6] If $G$ is a connected graph with $|V(G)| \geq 3$, and $uv \in E(G), d(u) = d(v) = \Delta(G)$ ,then $\chi'_{as}(G) \geq \Delta(G) + 1$.

**LEMMA 2**[9]If $k \geq \chi'(G)$ ,then*k*-PEC of $G$ has been exist

$$\big\| E_i | - | E_j \big\| \leq 1 \cdot i, j = 1,2,\cdots,k$$

Where $e \in E_i$, $f(e) = i (i = 1,2,\cdots,k)$, $\chi'(G)$ is the

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This studywas supported by Lanzhou City University Ph.D. Research Fund (21265099,41361013,GS[2013]GHB1084, LZCU-BS2013-09 and LZCU-BS2013-12).

chromatic number of G.

**LEMMA 3** For $n \ge 3$,

$$|E(P_n \vee P_n)| = n^2 + 2n - 2$$
$$|E(P_n \vee C_n)| = n^2 + 2n - 1$$
$$|E(C_n \vee C_n)| = n^2 + 2n$$

**LEMMA 4** For $n \ge 3$, then

*The equation* $\begin{cases} x + y = n + 3 \\ nx + (n-1)y = n^2 + 2n - 2 \end{cases}$

*Have a solution* $y \begin{cases} x = 1 \\ y = n - 2 \end{cases}$

*The equation* $\begin{cases} x + y = n + 3 \\ nx + (n-1)y = n^2 + 2n - 1 \end{cases}$

*Have a solution* $y \begin{cases} x = 2 \\ y = n + 1 \end{cases}$

*The equation* $\begin{cases} x + y = n + 3 \\ nx + (n-1)y = n^2 + 2n \end{cases}$

*Have a solution* $y \begin{cases} x = 3 \\ y = n \end{cases}$

For $m > n \ge 1$, there are many adjacent strong chromatic number of $P_m \vee P_n$, $P_m \vee C_n$ and $C_m \vee C_n$. In this paper we have the adjacent strong equitable chromatic number of $P_n \vee P_n$, $P_n \vee C_n$ and $C_n \vee C_n$, the others terminologies refer to [9-10].

## 2 Adjacent Strong Edge Coloring of $P_n \vee P_n$

**THEOREM 1** For $n \ge 2$, $\chi'_{ase}(P_n \vee P_n) = n + 3$.

**Proof** There are four cases to be considered.

**Case1** When $n = 2$, then $P_2 \vee P_2 = K4$ (*complete graph with order* 4), it's true by [6].

When $3 \le n \le 6$, $\Delta(P_n \vee P_n) = n + 2$. By Lemma 1, we need to prove thatexist a $n = 3 - ASEEC$.

Let $P_n$ and $P_n$ be $u_1 u_2 \cdots u_n$ and $v_1 v_2 \cdots v_n$.

**Case2** When $n = 3$, a mapping $f$ from $E(P_3 \vee P_3)$ to {1, 2, 3, 4, 5, 6} is defined as follows:

$$f(u_2v_2) = f(u_3v_1) = 1;\ f(u_2v_3) = f(v_1v_2) = 2;$$
$$f(u_1v_3) = f(u_2u_3) = 3;$$
$$f(u_1u_2) = f(u_3v_2) = 4;\ f(u_1v_2) = f(u_2v_1) = f(u_3v_3) = 5;$$
$$f(u_1v_1) = f(v_2v_3) = 6.$$

Obviously, $f$ is $6 - ASEEC$ of $P_3 \vee P_3$.

**Case3** When $n = 4$, a mapping $f$ from $E(P_4 \vee P_4)$ to {1, 2, 3, 4, 5, 6, 7} is defined as follows:

$$f(u_1v_i) = i,\ i = 1,2,3,4;$$
$$f(u_iv_j) = i + j + 1, i = 2,3; j = 1,2,3,4;$$
$$f(u_4v_1) = 5; f(u_4v_2) = 3;$$
$$f(u_4v_3) = 7; f(u_4v_4) = 1;$$
$$f(u_1u_2) = f(v_1v_2) = 7;$$
$$f(u_2u_3) = f(v_2v_3) = 1;$$
$$f(u_3u_4) = f(v_3v_4) = 2。$$

For such f, we have:

$$C(u_1) = \{5,6\}; C(u_4) = \{3,6\};$$
$$C(v_1) = \{2,6\}; C(v_4) = \{3,5\};$$
$$C(u_2) = \{2\}; C(u_3) = \{3\};$$
$$C(v_2) = \{6\}; C(v_3) = \{4\}.$$

Where $C(w) = \{1,2,3,4,5,6,7\} \setminus C(w)$, $w \in \{u_1,u_2,u_3,u_4\} \cup \{v_1,v_2,v_3,v_4\}$.

So $f$ is $7 - ASEEC$ of $P_4 \vee P_4$. The conclusion is true.

**Case4** When $n = 5$, a mapping f from $E(P_5 \vee P_5)$ to {1, 2, 3, 4, 5, 6, 7, 8} is given as follows:

$$f(u_1v_1) = 6; f(u_1v_j) = j, j = 2,3,4; f(u_1v_5) = 5;$$
$$f(u_iv_j) = i + j - 1, i = 2,3,4; j = 1,2,3,4,5;$$
$$f(u_5v_1) = 5; f(u_1u_2) = f(u_5v_2) = 7;$$
$$f(u_2u_3) = f(v_1v_2) = f(u_5v_3) = 8;$$
$$f(u_3u_4) = f(v_2v_3) = f(u_5v_4) = 1; f(u_5v_5) = 4;$$
$$f(u_4u_5) = f(v_3v_4) = 2; f(v_4v_5) = 3.$$

For such f, we have:

$$C(u_1) = \{1,8\}; C(u_i) = \{i-1\}, i = 2,3,4;$$
$$C(u_5) = \{3,6\}; C(v_1) = \{1,7\};$$
$$C(v_i) = \{i+4\}, i = 2,3,4; C(v_5) = \{1,2\}.$$

Where $C(w) = \{1,2,3,4,5,6,7,8\} \setminus C(w)$, $w \in \{u_i \mid i = 1,2,3,4,5\} \cup \{v_j \mid j = 1,2,3,4,5\}$.

So $f$ is $8 - ASEEC$ of $P_5 \vee P_5$. The conclusion is true.

**Case5** When $n = 6$, a mapping $f$ from $E(P_6 \vee P_6)$ to {1, 2, 3, 4, 5, 6, 7, 8, 9} is given as follows:

$$f(u_1v_1) = 5; f(u_1v_2) = 7; f(u_1v_3) = 2;$$
$$f(u_1v_4) = 9; f(u_1v_5) = 4; f(u_1v_6) = 6;$$
$$f(u_2v_1) = 9; f(u_3v_1) = 8; f(u_4v_1) = 4;$$
$$f(u_5v_1) = 3; f(u_6v_1) = 6;$$
$$f(u_iv_j) = i + j - 1, i = 2,3,4,5; j = 2,3,4,5;$$
$$f(u_6v_2) = 8; f(u_6v_3) = 3;$$
$$f(u_6v_4) = 2; f(u_6v_5) = 1; f(u_6v_6) = 7;$$
$$f(u_2v_6) = 8; f(u_3v_6) = 9;$$
$$f(u_4v_6) = 3; f(u_5v_6) = 5;$$
$$f(u_1u_2) = f(u_3u_4) = f(v_1v_2) = f(v_3v_4) = 1;$$
$$f(u_2u_3) = f(u_4u_5) = f(v_5v_6) = 2;$$
$$f(v_4v_5) = 3; f(u_5u_6) = 4; f(v_2v_3) = 9.$$

For the $f$, we have:

$$C(u_1) = \{3,8\}; C(u_2) = \{7\}; C(u_3) = \{3\};$$
$$C(u_4) = \{9\}; C(u_5) = \{1\}; C(u_6) = \{5,9\};$$
$$C(v_1) = \{2,7\}; C(v_2) = \{2\}; C(v_3) = \{8\};$$
$$C(v_4) = \{4\}; C(v_5) = \{5\}; C(v6) = \{1,4\}.$$

So, the $f$ is a $9 - ASEEC$ of $P_6 \vee P_6$, the Theorem 2 is true when n=6.

**Case6** When $n \ge 7$.

**Subcase 6.1** When $n \equiv 0 (\mathrm{mod}\ 2)$. Suppose $n = 2k, k \ge 2$.

From Lemma 5, $P_n \vee P_n$ exists a perfect matching $M_1$;

$(P_n \vee P_n) \setminus \{u_1, u_{k+1}\} \setminus M_1$, exist a prerfect matching $M_2$;

$(P_n \vee P_n) \setminus \{u_2, u_{k+2}\} \setminus \bigcup_{i=1}^{2} M_i$, exist a prerfect matching $M_3$;

$\cdots$

$(P_n \vee P_n) \setminus \{u_k, u_n\} \setminus \bigcup_{i=1}^{k} M_i$, exist a prerfect matching $M_{k+1}$;

$(P_n \vee P_n) \setminus \{v_1, u_{k+1}\} \setminus \bigcup_{i=1}^{k+1} M_i$, exist a prerfect matching $M_{k+2}$;

$(P_n \vee P_n) \setminus \{v_2, u_{k+2}\} \setminus \bigcup_{i=1}^{k+2} M_i$, exist a prerfect matching $M_{k+3}$;

$\cdots$

$(P_n \vee P_n) \setminus \{v_k, u_n\} \setminus \bigcup_{i=1}^{k+1} M_i$, exist a prerfect matching $M_{n+3}$;

Let $f$ be as follows

$$\forall e \in M_i, f(e) = i, i = 1, 2, \cdots, n+3.$$

Obviously, $f$ is a $(n+3) - ASEEC$ of $P_n \vee P_n$.

**Subcase 6.2** When $n \equiv 1(\bmod 2)$ .Suppose $n = 2k+1, k \geq 3$.

From Lemma 3,we can learn $P_n \vee P_n$ exist a perfect matching$M_1$;

$(P_n \vee P_n) \setminus \{u_1, u_{k+1}\} \setminus M_1$, exist a prerfect matching $M_2$;

$(P_n \vee P_n) \setminus \{u_2, u_{k+2}\} \setminus \bigcup_{i=1}^{2} M_i$, exist a prerfect matching $M_3$;

$\cdots$

$(P_n \vee P_n) \setminus \{u_k, u_{n-1}\} \setminus \bigcup_{i=1}^{k} M_i$, exist a prerfect matching $M_{k+1}$;

$(P_n \vee P_n) \setminus \{v_1, u_n\} \setminus \bigcup_{i=1}^{k+1} M_i$, exist a prerfect matching $M_{k+2}$;

$(P_n \vee P_n) \setminus \{v_2, u_{k+1}\} \setminus \bigcup_{i=1}^{k+2} M_i$, exist a prerfect matching $M_{k+3}$;

$(P_n \vee P_n) \setminus \{v_2, u_{k+2}\} \setminus \bigcup_{i=1}^{k+3} M_i$, exist a prerfect matching $M_{k+4}$;

$\cdots$

$(P_n \vee P_n) \setminus \{v_k, u_{n-1}\} \setminus \bigcup_{i=1}^{k+2} M_i$, exist a prerfect matching $M_{n+1}$;

$(P_n \vee P_n) \setminus \{v_k, u_{n-1}\} \setminus \bigcup_{i=1}^{n} M_i$, exist a prerfect matching $M_{n+3}$;

$(P_n \vee P_n) \setminus \{v_1, v_n\} \setminus \bigcup_{i=1}^{n+1} M_i$, exist a prerfect matching $M_{n+2}$;

$(P_n \vee P_n) \setminus \{u_n, v_n\} \setminus \bigcup_{i=1}^{n+2} M_i$, exist a prerfect matching $M_{n+3}$;

Let $f$ be as follows

$$\forall e \in M_i, f(e) = i, i = 1, 2, \cdots, n+3.$$

For $f$ we have

$$C(u_i) = C(u_{k+i}) = \{i+1\}, i = 2, 3, \cdots, k;$$
$$C(u_1) = \{2, k+2\}, C(u_n) = \{k+2, n+3\};$$
$$C(v_i) = C(v_{k+i}) = \{k+1+i\}, i = 2, 3, \cdots, k;$$
$$C(v_1) = \{k+3, n+2\}, C(v_n) = \{k+2, n+3\}.$$

And

$$|E_i| = \begin{cases} n, i = 1 \\ n-1, i = 2, 3, \cdots, n. \end{cases}$$

So, $f$ is a $(n+3) - ASEEC$ of $P_n \vee P_n$.

For above reasons,the theorem 1 is true.

## 3 Adjacent Strong Edge Coloring of $P_n \vee C_n$

**THEOREM 2** For $n \geq 3$,

$$\chi'_{ase}(P_n \vee C_n) = \begin{cases} n+3, if \quad n \equiv 0(\bmod 2) \\ n+4, if \quad n \equiv 1(\bmod 2) \end{cases}$$

**Proof** If $n \equiv 0(\bmod 2)$ ,suppose $n = 2k,\ k \geq 2$.

By lemma 1,we should only proof $P_n \vee C_n$ exist $(n+3)$ $-ASEEC$.

By lemma 4, $P_n \vee C_n$ exists two perfect matching $M_1$,$M_2$,and $M_1 \cap M_2 = \varnothing$.

$(P_n \vee C_n) \setminus \{u_1, u_{k+1}\} \setminus \bigcup_{i=1}^{2} M_1$, exist a prerfect matching $M_3$;

$(P_n \vee C_n) \setminus \{u_2, u_{k+2}\} \setminus \bigcup_{i=1}^{3} M_1$, exist a prerfect matching $M_4$;

$\cdots$

$(P_n \vee C_n) \setminus \{u_k, u_n\} \setminus \bigcup_{i=1}^{k+1} M_1$, exist a prerfect matching $M_{n+2}$;

$(P_n \vee C_n) \setminus \{u_1, u_n\} \setminus \bigcup_{i=1}^{k+2} M_1$, exist a prerfect matching $M_{n+3}$;

$(P_n \vee C_n) \setminus \{v_1, u_{k+1}\} \setminus \bigcup_{i=1}^{k+3} M_1$, exist a prerfect matching $M_{n+4}$;

$(P_n \vee C_n) \setminus \{v_2, u_{k+2}\} \setminus \bigcup_{i=1}^{k+4} M_1$, exist a prerfect matching $M_{n+5}$;

$(P_n \vee C_n) \setminus \{v_k, v_n\} \setminus \bigcup_{i=1}^{n+2} M_1$, exist a prerfect matching $M_{n+3}$;

$\cdots$

Let $f$ be as follows

$$\forall e \in M_i, f(e) = i, i = 1, 2, \cdots, n+3.$$

For $f$ we have

$$\bar{C}(u_i) = \bar{C}(u_{k+i}) = \{i+1\}, i = 1, 2, \cdots, k;$$
$$\bar{C}(v_i) = \bar{C}(v_{k+i}) = \{k+1+i\}, i = 1, 2, \cdots, k.$$

And

$$|E_i| = \begin{cases} n, i = 1 \\ n-1, i = 2, 3, \cdots, n+2. \end{cases}$$

So, $f$ is a $(n+3) - ASEEC$ of $P_n \vee C_n$.

If $n \equiv 1(\bmod 2)$ ,we need proof $\chi'_{ase}(P_n \vee C_n) \geq n+4$ first.By lemma 3, we have $\chi'_{ase}(P_n \vee C_n) = n+3$.

Let $f$ is a $(n+3) - ASEEC$ of $P_n \vee C_n$. By lemma 3, $P_n \vee C_n$ must have perfect matching after dismiss two vertices they are not adjacent in $v_1 v_2 \cdots v_n$. For $n \equiv 1(\bmod 2)$, there must have a perfect matching.Let'sone vertex in $u_2 u_3 \cdots u_{n-1}$ and another vertex in $v_1 v_2 \cdots v_n$.Thus

$$C(u_{i0}) = C(v_{j0}), 2 \leq i_0 \leq n-1, 1 \leq j_0 \leq n.$$

$f$ is also a $(n+3) - ASEEC$ of $P_n \vee C_n$, It is contradictory.So $\chi'_{ase}(P_n \vee C_n) \geq n+4$, let we give a $(n+3) - ASEEC$ of $P_n \vee C_n$.

Let f be as follows

$$f(u_i v_j) = i + j - 1(\bmod\ n+4),$$
$$i = 1, 2, \cdots, n-1; j = 1, 2, \cdots, n;$$
$$f(u_n v_i) = n + i + 1(\bmod\ n+4),$$
$$i = 1, 2, \cdots, n;$$
$$f(u_i u_{i+1}) = n + i + 2(\bmod\ n+4), i = 1, 2, \cdots, n-2;$$
$$f(u_{n-1} u_n) = n-3; f(u_n u_1) = n+1;$$
$$f(v_i v_{i+1}) = n + i + 3(\bmod\ n+4), i = 1, 2, \cdots, n-1;$$

For $f$, we have

$$\bar{C}(u_1)=\{n+2,0\};\bar{C}(u_i)=\{i-1,n+i(\mathrm{mod}(n+4))\},$$
$$i=2,3,\cdots,n-2;$$
$$\bar{C}(u_{n-1})=\{n-3,2n-1(\mathrm{mod}(n+4)\};\bar{C}(u_n)=\{n-1,n\};$$
$$\bar{C}(v_1)=\{n,\ n+1,n+3\};$$
$$\bar{C}(v_n)=\{n-1,2n-1(\mathrm{mod}(n+\ 4)),2n(\mathrm{mod}(n+\ 4))\};$$
$$\bar{C}(v_i)=\{n+i-1(\mathrm{mod}(n+\ 4)),$$
$$n+i(\mathrm{mod}(n+\ 4))\},i=2,3,\cdots,n-1.$$

So $f$ is a $(n+4)-ASEEC$of $P_n\vee C_n$.

Hence the conclusion is true.

## 4 Adjacent Strong Edge Coloring of $C_n \vee C_n$

**THEOREM 3**For $n\ge 3$,then

$$\chi'_{ase}(C_n\vee C_n)=\begin{cases}n+3,n=3 \quad or \quad n\equiv 0(\mathrm{mod}2)\\ n+4,n\ge 5 \quad or \quad n\equiv 1(\mathrm{mod}2)\end{cases}$$

**Proof**Supposing the two cycles are $u_1u_2\cdots u_nu_1$ and $v_1v_2\cdots v_nv_1$ with separately.

When $n=3$, $C_n\vee C_n=K_6$ (complete graph with order 6), can be seen in appendix.

**Case1** $n\equiv 0(\mathrm{mod}\,2)$ ,by lemma 1,2,4, $C_n\vee C_n$ exist three perfect matching $M_1$,$M_2$,$M_3$and $M_1\cap M_2=M_2\cap M_3=M_3\cap M_1=\varnothing$.

When $n=4$, let's $f$ as be follows:

$$f(u_1u_4)=f(u_2u_3)=f(v_1v_4)=f(v_2v_3)=7;$$
$$f(u_1v_4)=f(u_2v_1)=f(u_3v_3)=f(u_4v_2)=6;$$
$$f(u_1v_2)=f(u_2v_4)=f(u_3v_1)=f(u_4v_3)=5;$$
$$f(u_1v_1)=f(u_2v_3)=f(u_3u_4)=4;$$
$$f(u_1u_2)=f(u_3v_2)=f(u_4v_4)=3;$$
$$f(u_1v_3)=f(u_3v_4)=f(v_1v_2)=2;$$
$$f(u_2v_2)=f(u_4v_1)=f(v_3v_4)=1.$$

So $f$ is a $7-ASEEC$of $C_4\vee C_4$.

When $n=6$,

$(C_6\vee C_6)\setminus\{u_1,u_3\}\setminus\bigcup_{i=1}^{3}M_i$, exist a prerfect matching $M_4$;

$(C_6\vee C_6)\setminus\{u_1,u_3\}\setminus\bigcup_{i=1}^{3}M_i$, exist a prerfect matching $M_4$;

$(C_6\vee C_6)\setminus\{u_2,u_5\}\setminus\bigcup_{i=1}^{4}M_i$, exist a prerfect matching $M_5$;

$(C_6\vee C_6)\setminus\{u_4,u_6\}\setminus\bigcup_{i=1}^{5}M_i$, exist a prerfect matching $M_6$;

$(C_6\vee C_6)\setminus\{v_1,v_3\}\setminus\bigcup_{i=1}^{6}M_i$, exist a prerfect matching $M_7$;

$(C_6\vee C_6)\setminus\{v_2,v_5\}\setminus\bigcup_{i=1}^{7}M_i$, exist a prerfect matching $M_8$;

$(C_6\vee C_6)\setminus\{v_4,v_6\}\setminus\bigcup_{i=1}^{8}M_i$, exist a prerfect matching $M_9$;

Let $f$ be as follows

$$\forall e\in M_i, f(e)=i.$$

So $f$ is a $9-ASEEC$of $C_6\vee C_6$.

Similarly we can proof $C_n\vee C_n$ exist $(n+3)-ASEEC$when $n\equiv 0(\mathrm{mod}\,2)$ and $n\ge 8$.

By lemma 4, $C_n\vee C_n$ exist three perfect matching$M_1$,$M_2$,$M_3$and $M_1\cap M_2=M_2\cap M_3=M_3\cap M_1=\varnothing$.Suppose $n=2k,\ k\ge 2$.

$(C_n\vee C_n)\setminus\{u_1,u_{k+1}\}\setminus\bigcup_{i=1}^{3}M_i$, exist a prerfect matching $M_4$;

$(C_n\vee C_n)\setminus\{u_2,u_{k+2}\}\setminus\bigcup_{i=1}^{4}M_i$, exist a prerfect matching $M_4$;

…

$(C_n\vee C_n)\setminus\{u_k,u_n\}\setminus\bigcup_{i=1}^{k+3}M_i$, exist a prerfect matching $M_{k+4}$;

$(C_n\vee C_n)\setminus\{v_1,v_{k+1}\}\setminus\bigcup_{i=1}^{k+1}M_i$, exist a prerfect matching $M_{k+5}$;

…

$(C_n\vee C_n)\setminus\{v_k,v_n\}\setminus\bigcup_{i=1}^{k+4}M_i$, exist a prerfect matching $M_{n+3}$;

Let $f$ be as follows

$$\forall e\in M_i, f(e)=i,i=1,2,\cdots,n+3.$$

For $f$ we have

$$\bar{C}(u_i)=\bar{C}(u_{k+i})=\{i\},i=1,2,\cdots,k;$$
$$\bar{C}(v_i)=\bar{C}(v_{k+i})=\{k+i+1\},i=1,2,\cdots,k.$$

And

$$|E_i|=\begin{cases}n,i=1,2,3;\\ n-1,i=4,5,\cdots,n+3.\end{cases}$$

So, $f$ is a $(n+3)-ASEEC$of $C_n\vee C_n$.

**Case2** $n\equiv 1(\mathrm{mod}\,2)$ and $n\ge 5$, We proof $\chi'_{as}(C_n\vee C_n)\ge n+4$

If $\chi'_{as}(C_n\vee C_n)=n+3$, there are at least 3 colors represented at every vertex. For $|E(C_n\vee C_n)|=n^2+2n$ ,andeach vertex lack just one color and the vertices lack the same color in a same cycle and not adjacent,and $n$is an odd, so it is not possible that odd number vertices lack same color. For the edge number of $C_n\vee C_n$,the number of vertices lack same color just an even. So there must have two vertices in two different cycle lack same color. It is contradictory. Thus $\chi'_{ase}(C_n\vee C_n)=n+4$ when $n\equiv 1(\mathrm{mod}\,2)$.

Let $f$ be as follows: the edges of $u_1u_2,u_2u_3,\cdots,u_{n-1}u_n$ are coloring colors with $n+3,n+4$ rotate, the edges of $v_1v_2,v_2v_3,\cdots,v_{n-1}v_n$ are coloring colors with $n$+4,$n$+3 rotate;

$$f(u_1u_n)=f(v_1v_n)=n+1;f(u_1v_j)=j,\ j=1,2,\cdots,n;$$
$$f(u_iv_j)=i+j(when\ i+j>n+2,$$
$$then\ \mathrm{mod}\ n+\ 2),i=2,3,\cdots,n-1;j=1,2,\cdots,n;$$
$$f(u_nv_1)=n+\ 2;f(u_nv_j)=j-1,j=2,3,\cdots,\ n.$$

For the $f$, we have

$$\bar{C}(u_1)=\{n+2,n+4\};\bar{C}(u_i)=\{i-1,i\},$$
$$i=2,3,\cdots,n-1;\bar{C}(u_n)=\{n,\ n+3\}(n\equiv 1(\mathrm{mod}\,2));$$
$$\bar{C}(v_1)=\{2,n+3\};\bar{C}(v_i)=\{i+1,n+i\}(when\ i+j>n+2,$$
$$take\ \mathrm{mod}\ n+\ 2;\bar{C}(v_n)=\{n,\ n+4\}(n\equiv 1(\mathrm{mod}\,2)).$$

So, the f is a $n+4-ASEEC$ of $C_n\vee C_n$, the theorem 3 is true.

From all of above, the theorem 3 is true.

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