MATEC Web of Conferences 40, 02020 (2016)
DOI: 10.1051/matecconf/20164002020

Establishment and Numerical Analysis of Lubrication Model of Double Involute Gear

Fan Zhimin [1] , Xu Jun [1]

[1] Qingdao University of Science & Technology, Qingdao, Shandong, China

Abstract. The purpose of this paper is to derive the simplified elastohydrodynamic (EHD) lubrication model of the double involute gear (DIG) and study the effect of the segmented tooth waist on the lubricating properties of DIG. Geometric analysis of the DIG gear was performed on the simplified EHD lubrication model based on the engagement theory, and then the line contact steady isothermal Reynolds' equation was established. Numerical analysis of EHD lubrication of the new gear drive was conducted by using the multi-grid method.

1 Introduction

DIG drive is a new type of gear drive, which combines the advantages of involute gear and sub-order double arc gear. Its tooth profile consists of two segmented in-volutes which are connected by a transition curve. The tooth root involute and the addendum involute are arranged with a segmented manner[1]. Figure 1 is the transverse tooth profile of the DIG, where the dotted line is the transverse tooth profile of the common involute gear. The left side of the tooth profile of the common involute gear consists of dedendum transition *a′b′* and involute *b′e′*, and the left side of the tooth profile of the double involute gear consists of the de-dendum transition curve *ab*, the lower tooth involute *bc*, the tooth waist transition curve *cd* and the above tooth involute *de*. The two working tooth profiles of the DIG are connected by the tooth waist transition curve, in which *cd* is tangent to the above tooth at point *d* and intersects with the lower tooth at point *c*.

Based on finite element analysis and photoelasticity test, Xu et al. [2] proved that the bending strength of the DIG was significantly higher than that of the ordi-nary involute gear. Fan et al. [3] found that the vibration and noise of the DIG sig-nificantly reduced by the comparative tests with the ordinary involute gear which had the same parameters and working conditions. Also contact carrying capacity did not show reduction. Feng et al.'s [4] study showed that using conventional hobbing cutter could achieve the processing of double involute gear by using or-thogonal bidirectional linkage deflection method, which reduced the processing cost of the new gear. Recent studies have demonstrated that this new gear has a good application prospect and research value [5].

At present, researches on the bending strength calculations of the DIG are far-reaching at home and abroad. However, there are few application studies with respect to its carrying capacity, dynamics, lubrication performance and so on. It is very important to conduct a profound and systematic study of the lubrication performance of the DIG, in term of its application future and practical value.

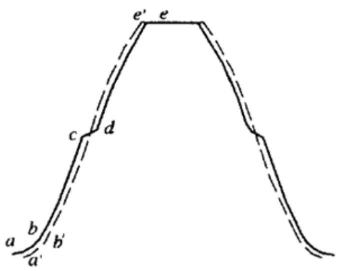

Figure 1. Transverse tooth profile of the DIG

2 Geometric model of EHD lubrication and entrainment velocity

The tooth profile of the DIG is helical and its contact line is discontinuous in the place of the tooth waist transition curve, which is different from the ordinary involute gear [1-3]. But it has the same engagement theory with the ordinary involute gear[6].

The geometrical properties of two reverse segment circular cone rollers can be described in cartesian coordinate system in which the x-y plane is perpendicular to the paper flat surface and the x-axis points to the outside of paper surface (Fig. 2).

Entrainment velocity is written as:

$$\begin{cases} u = (u_a + u_b)/2 \\ \upsilon = (\upsilon_a + \upsilon_b)/2 \end{cases} \tag{1}$$

u_a, υ_a, u_b and υ_b are the speeds of any point of the solid body a and solid body b in the x and y direction, respectively. Where u and υ are entrainment velocities in the x direction and y direction, respectively[6].

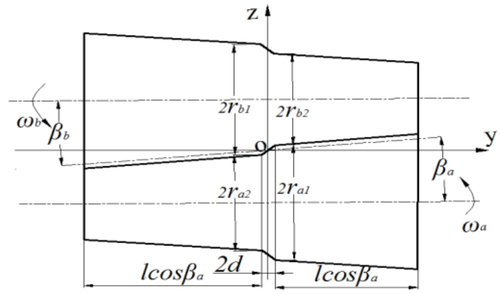

Figure 2. Geometric characteristics of the two reverse tapered roller

3 MATHEMATICAL MODEL

Compared with the common involute gear, the study of EHD lubrication problems of the new gear drive is more complex and difficult, as the radius of curvature, surface velocity and load of every point along the tooth surface contact of the DIG are different and the involute tooth profile is ladder in the place of the tooth waist. Based on the simplified model of EHD lubrication and the meshing process of the DIG and the formation mechanism of the lubricating oil film, and then the numerical calculation formula is deduced[6].

3.1 Line contact steady isothermal Reynolds' equation

The Reynolds' equation between two reverse segment circular cone rollers can be expressed as [7-9]

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{\eta}\cdot\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\rho h^3}{\eta}\cdot\frac{\partial p}{\partial y}\right)=12\left[\frac{\partial}{\partial x}(U\rho h)+\frac{\partial}{\partial y}(V\rho h)+2\rho\left(w_h-w_0\right)\right] \tag{2}$$

The line contact steady isothermal Reynolds' equation can be expressed as:

$$\frac{d}{dx}\left(\frac{\rho h^3}{\eta}\cdot\frac{dp}{dx}\right)=12u\frac{d(\rho h)}{dx} \tag{3}$$

where u denotes the entrainment velocity and $u = (u_a + u_b)/2$. u_a and u_b are the tangential velocity along tooth profile of the gear a and gear b in meshing point. x represents the coordinate variables. p is the oil film pressure, h is the oil film thickness, ρ is the lubricating oil density, η is the lubricating oil viscosity.

3.2 Film thickness equation

The film thickness equation can be expressed as:

$$h=h_0+\frac{x^2}{2\mathrm{R}_a}+\frac{x^2}{2\mathrm{R}_b}=h_0+\frac{x^2}{2}\left(\frac{1}{\mathrm{R}_a}+\frac{1}{\mathrm{R}_b}\right) \tag{4}$$

where R_a and R_b can be obtained as follows:

When $y \geqslant d$, $\mathrm{R}_a = r_{a1} + (y - d) \tan\beta_a$, $\mathrm{R}_b = r_{a2} + (y - d) \tan\beta_b$;

When $y \leqslant -d$, $\mathrm{R}_a = r_{a2} + (y + d) \tan\beta_a$, $\mathrm{R}_b = r_{b1} - (y + d) \tan\beta_b$;

The total deformation caused by elastic deformation can be described as:

$$\delta(x)=-\frac{2}{\pi E'}\int_{s_1}^{s_2}P(s)\ln(x-s)^2\,ds+C \tag{5}$$

Based on Eqs. (4) and (5), the geometric equation of lubrication film thickness can be derived as follow:

$$h=h_0+\frac{x^2}{2\mathrm{R}}-\frac{2}{\pi E'}\int_{-\infty}^{x}P(s)\ln(x-s)^2\,ds \tag{6}$$

3.3 Load equation

$$w=\int_{x_{in}}^{x_{out}}p\,dx \tag{7}$$

where x_{in} and x_{out} are the position coordinates of the entrance and exit of the lubrication region, respectively.

3.4 Oil pressure viscosity equation

Roelands' pressure viscosity temperature relationship can be expressed as

$$\eta=\eta_0\exp\left\{\left(\ln\eta_0+9.67\right)\left[\left(1+5.1\times10^{-9}p\right)^z-1\right]\right\} \tag{8}$$

where $Z = \alpha / (5.1\mathrm{x}10^{-9} [\ln (\eta_0) + 9.67])$. p and η_0 denote the pressure and lubricant viscosity under the initial temperature and pressure, respectively.

3.5 Oil sealing pressure equation

According to Dowson-Higginson consolidation equation, density can be expressed as:

$$\rho=\rho_0\left(1+\frac{Ap}{1+Bp}\right) \tag{9}$$

where ρ_0 is the density of lubricant at room temperature. A and B represent the experimental constants, the values of which are usually, $6\times10^{-10}\mathrm{m}^2$/N and 1.7×10^{-9} m^2/ N, respectively.

3.6 Boundary condition

Equation with Reynolds' boundary condition is shown as

$$\begin{cases} p=0 & x=x_{in} \\ p=\dfrac{dp}{dx} & x=x_{out} \end{cases} \tag{10}$$

4 Numerical Analysis

Table 1. Major design parameters of DIG

parameters	Value
Number of teeth of the pinion (z_1)	32
Number of teeth of the gear (z_2)	32
Elastic modulus of the pinion (E_1)	960GPa
Elastic modulus of the gear (E_2)	206GPa
Module (m)	4
Pressure angle (α)	20。
Speed of the pinion (n_1)	960r/min
Load (W)	1. 2×10^3N/mm
Poisson's ratio (μ)	0. 3
Lubricant viscosity (η_0)	0. 075pa · s
Pressure viscosity index (α)	2. 2×$10^{-8}$$m^2$/N
Number of teeth of the pinion (z_1)	32

Table 1 shows the major design parameters of the DIG drive. Figure 3 depicts the distribution of the dimensionless load along the meshing line. In the theoretical analysis, the DIG drive is equivalent to the contact of two reverse segment circular cone rollers. Simplicity makes line contact become static contact, and the changes of parameters of the area of contact have no influence on the pressure and thick-ness of lubricating oil film. the changes of parameters caused by the time influ-ence on the follow-up analysis were ignored [10,11]. Based on the multi-grid me-thod and the multi-grid integration method, the EHD lubrication problems of the double involute gear were figured out through the programming interface between program and software.

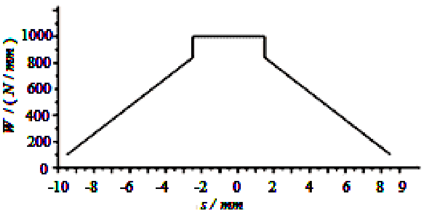

Figure 3. Variation diagram of dimensionless load along the meshing line

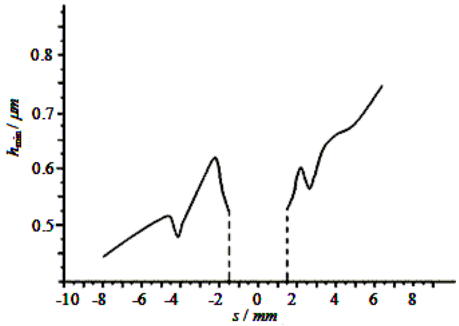

Figure 4. Change of minimum film thickness along the meshing line

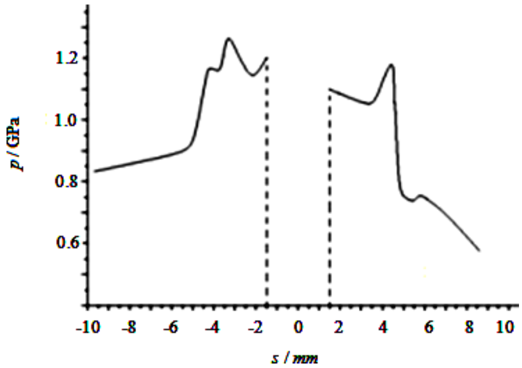

Figure 5. Change of center pressure along the meshing line

Figure 4 shows the change pattern of the minimum thickness of the lubricant film along the meshing line. The oil film thickness is at the thinnest when it is at the beginning of meshing point resulting from the meshing impact. At the same instant, the load tends towards stabilization after the gear contacts. As the pinion rolls, when double tooth pair is in contact, the load of single tooth bearing gradual-ly becomes smaller, and the minimum film thickness becomes larger. The mini-mum film thickness has a sudden reduction in the meshing line at -4mm place, as the tooth surface loads suddenly become larger when the another meshing inter-rupts at the tooth waist. The analysis of line interrupts in (-1.5,1.5) section the tooth meshing line is segmented at the tooth waist, where lubricant film thickness is not considered. While meshing point is leaving the contact, the central film thickness reaches a maximum value.

Figure 5 shows that the change of center pressure along the meshing line generally increases first and then decreases except for two fluctuations in the meshing process, which is caused by the segmented waist meshing.

5 Conclusion

The geometric model and mathematical model of EHD lubrication of the DIG drive was established. Based on the deduced oil film thickness equation, the load equation and the lubricating oil pressure equation etc, the problem of double invo-lute gear lubrication was solved using the multi-grid method.

Analysis of the results can be concluded that the tooth surface under load has the circumstances of sudden change caused by DIG tooth waist discontinuity. Therefore, favorable for smooth drive of double involute gear to increase the coincidence degree. The research has an important significance in further explor-ing transmission performance and bearing capacity of the DIG under EHD lubri-cation condition. Meanwhile, it can provide an theoretical basis for applying tri-bology theory to lubrication design and optimization design of the gear drive.

Acknowledgments The work described in this paper was supported by the Shandong Provincial Natural Science Foundation of China (ZR2014EEM003).

References

1. Xu, H.B, Zhang,G.H.,Journal of Chongqing University.**22**, 5 (1999)
2. Xu, H.B, Zhang,G.H.,Chinese Journal of Mechanical Engineering. **36**, 4 (2000)
3. Fan, Z.M, Zhang,G.H.,Mechanical science and technology. **21**, 4 (2002)
4. Feng, X.Y, Zhang, C. R, L,C. Y, Ai, X.,Journal of Mechanical drive. **22**, 3 (1998)
5. Fan,Z.M, Zhang,G.H.,Chinese Journal of Mechanical Engineering. **38**, 4 (2002)
6. Fan, Z.M, Zhang Z.Q.,Materials Processing and Manufacturing III.**4** (2012)
7. Huang, L, Hu,Y.Z,Wang,W.Z.,Lubrication and Sealing. **34**, 5 (2009)
8. Shi,G.W,Wang,Y.Q.,Lubrication and Sealing.35, 4 (2010)
9. Tian, Q., Sun, Y.L., Liu, C., Hu, H., Flores, P.,Comput. Struct. **2**, 5 (2013)
10. Lu,H.,Yang,P.R.,Lubrication and Sealing. **35**, 5 (2010)
11. Ye, Y.D, Zhou,Z.B.,Journal of Mechanical drive. **36**, 4 (2012)