

A numerical simulation on high prandtl number liquid bridge

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Abstract. The free surface characteristics of high prandtl number liquid bridge with dynamic deformation have been investigated under microgravity based on the Navier-Stokes equations coupled with the energy conservation equation on a staggered grid. The free surface deformation and the effects of ambient air were considered by using level set method of mass conservation to capture two phase interfaces in numerical calculations. The variation of surface velocity is presented and discussed.

1 Introduction

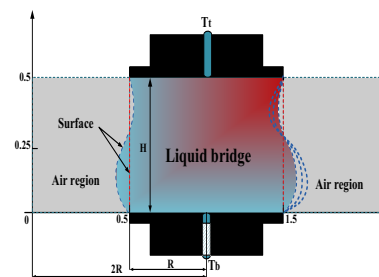
The temperature gradient-induced convection is often called thermocapillary convection [1-2]. The floating zone convection is one kind of thermocapillary convection. Many researches and applications of crystal growing by floating zone method were carried out on the ground. Meanwhile, the size of the crystal growing in floating zone is limited on the ground, which is improved under space environment.

Despite a lot of experiments and numerical simulations on the half-floating zone method have been carried out, almost all studies did not consider the effects of ambient air on thermocapillary convection mechanism. Recently, some scholars conducted studies to investigate flow and temperature fields in liquid bridges involving the effects of ambient air. The experiment carried out by Schwabe and Scharmann [3] demonstrated that the surface tension flow plays an important role on mass and heat transport in crystal growing. Cheng and Kou [4] directly measured the temperature oscillation of a liquid bridge by using optical pyrometer. Levenstam and Amberg [5] investigated the lost steady process of the surface thermocapillary convection in a cylindrical liquid bridge for low Pr number (Pr=0.01) fluid with aspect ratio $H/R=1$. They proposed that the two-dimension steady axisymmetric convection firstly changed into three dimensional steady convection with Reynolds number increasing (Re=1960). When Re number was 6250, the three dimensional steady convection further changed into oscillation convection. Bazzi et al. [6] investigated the stability of the thermocapillary convection in a liquid bridge of low Pr number (Pr=0.016) fluid with aspect ratio $H/R=0.7$. Their research demonstrated that when Marangoni number was 48, the first time instability happened, and then the second time instability happened

corresponding to Ma=80. For high Pr number (Pr>15) fluid, there is still contention on the flow characteristics of liquid bridges.

A mass conserving level set method monitoring the two-phase interface is used to carry out a numerical simulation on the liquid bridge of high prandtl number in the present paper. The surface deformation of liquid bridge and effects of ambient air are all taken into consideration in the numerical calculation.

2 Physical model



- R : Radius of liquid bridge.
- H : Height of liquid bridge.
- T_t, T_b : Temperature of upper and bottom disks, respectively.
- $\Delta T = T_t - T_b$
- $2R$: Outer diameter of gas-phase region.

Fig 1. Schematic of a half-zone liquid bridge model

The simplified model of floating-zone crystal growing is adopted in the present paper as shown in Fig. 1. The diameter of upper and bottom discs is $D=5.00$ mm, and

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the aspect ratio (H/R) is 1. Other main physical parameters are presented in Fig. 1.

3 Controlling equations

The surface tension on the free interface is a linear function of temperature and decreases linearly with the temperature increasing. The basic governing equations of the half-floating zone thermocapillary convection include mass conservation, momentum conservation, and energy conservation equations. The non-dimensional governing equations are given as following:

$$\mathbf{u}_t + (\mathbf{u} \cdot \nabla) \mathbf{u} = \frac{1}{\rho} \left(-\nabla p + \frac{1}{\text{Re}} \nabla \cdot (2\mu \mathbf{D}) + \left(\frac{1}{\text{We}} - \frac{\text{Ca}}{\text{We}} \Theta \right) \kappa \delta(d) \mathbf{n} \right) \quad (1)$$

$$\nabla \cdot \mathbf{u} = 0 \quad (2)$$

$$\frac{\partial \Theta}{\partial t} + \nabla \cdot (\mathbf{u} \Theta) = \frac{1}{\text{RePr}} \nabla^2 \Theta \quad (3)$$

where $\mathbf{u} = (u, v)$ is the fluid velocity, $\rho = \rho(\mathbf{x}, t)$ is the fluid density, $\mu = \mu(\mathbf{x}, t)$ is the fluid viscosity, \mathbf{D} is the viscous stress tensor, κ is the curvature of the interface, d is the normal distance to the interface, δ is the Dirac delta function, \mathbf{n} is the unit normal vector at the interface, t is the time, p is the pressure.

The key parameters are $\text{Re} = \rho_l U L / \mu_l$, Reynolds number, where L is the characteristic length defined as $L = 2R$, where R is the initial radius of the liquid bridge. U is the characteristic velocity defined as $U = \sigma_T \Delta T / \mu_l$ in microgravity conditions, where σ_T is the temperature dependency of surface tension, $\Delta T = T_t - T_b$ is the temperature difference between the top and bottom disks; $\text{We} = U^2 L \rho_l / \sigma$, Weber number; $\text{Pr} = \mu_l / \rho_l a$, Prandtl number, where a is the thermal diffusivity; $\text{Ca} = \mu_l U / \sigma$, Capillary number; We denote σ as the surface tension, $\sigma = \sigma_c - \sigma_T (T - T_b)$, where σ_c is a reference value of surface tension, and Θ as the excess temperature, $\Theta = (T - T_b) / \Delta T$. Here ρ_l and μ_l are the dimensional liquid density and viscosity, respectively, and the dimensionless density (ρ_l / ρ_l) and viscosity (μ_l / μ_l) inside the liquid bridge are equal to 1. ρ_g and μ_g are the dimensional density and viscosity of ambient air, respectively.

Table 1: Physical parameters

Element	Size
Temperature of upper disk, (K)	325
Temperature of bottom disk, (K)	300
Density ratio, ρ_l / ρ_g	722.68

Density ρ_l , (kg/m^3)	873
Viscosity ratio, μ_l / μ_g	97.22
Viscosity μ_l , (Pa·s)	1.75×10^{-3}

4 Results and discussion

In the following, the analysis of velocity and temperature fields in liquid bridge of high Pr number fluid under microgravity is given. The computational conditions are given as follows: the Marangoni number is 73915.6, $Ma = \text{RePr}$, the Prandtl number is 27.9, the Weber number is 268.7, the Capillary number is 0.1015, and gravity level is $g = 0.1g_0$ ($g_0 = 9.81 \text{ m/s}^2$). Other main calculation parameters are shown in table 1.

Figure 2 shows the spatial distributions of axial velocity (v) on the free surface. The shape of spatial distributions of axial velocity (v) on the free surface is unimodal and the peak of axial velocity (v) on the free surface changes over time. The position of the peak of axial velocity (v) also changes with the development of thermocapillary convection. It is close to the hot disk at first and then gradually moves toward the cold. Meanwhile, the overall level of axial velocity is improved and the peak exists fluctuation phenomena with the increasing of temperature difference between two disks. The free surface changes dramatically near the hot and cold disks. The structure of lateral velocity (u) along the interface presents a single peak and the peak is near the hot disk in the initial development stage of thermocapillary convection as shown in Fig.3. However, the peak exists near the hot and cold disk with the influence scope of

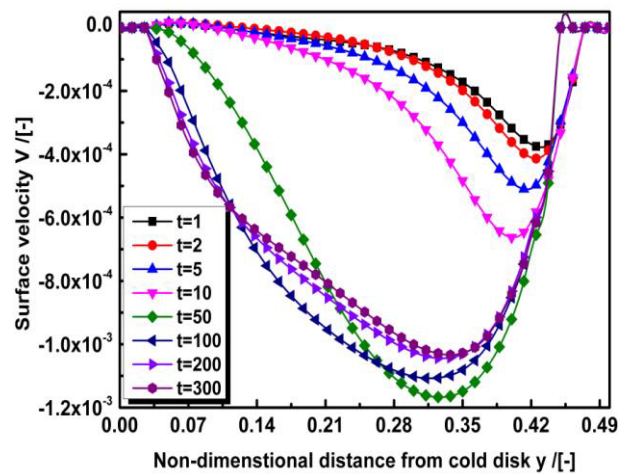


Fig 2. Variation of surface velocity in Y axis direction at different times ($Pr=27.9$, $Ma=73915.6$, $We=268.7$, $Ca=0.1015$, $D=5.0\text{mm}$, $H/R=1.0$, $g=0.1g_0$)

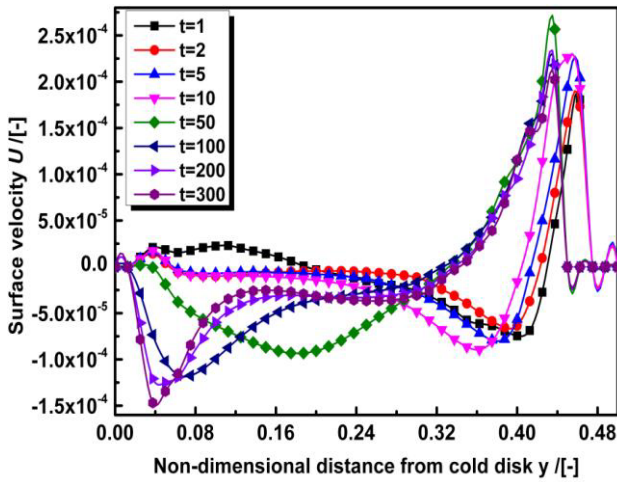


Fig 3. Variation of surface velocity in X axis direction at different times
 ($Pr=27.9, Ma=73915.6, We=268.7, Ca=0.1015, D=5.0\text{mm}, H/R=1.0, g=0.1g_0$)

thermocapillary convection and moves toward the cold disk. At $t=300$ (see Fig.4), the effect of thermalcapillary convection on the curvature of the temperature curves for different radius is obvious. The curvature of the temperature curves near the interface is smaller than that near the center of the liquid bridge.

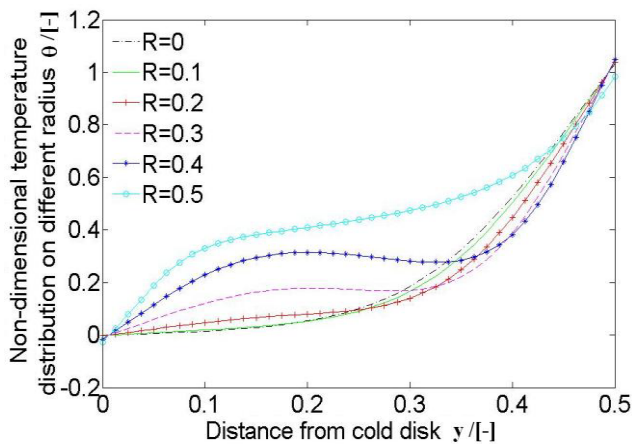


Fig 4. Temperature distribution of liquid bridge for different radius at $t=300$
 ($Pr=27.9, Ma=73915.6, We=268.7, Ca=0.1015, D=5.0\text{mm}, H/R=1.0, g=0.1g_0$)

5 Conclusions

1. The spatial distributions of axial velocity (v) on the free surface is not mirror symmetry at $y=0.25$. The location of the peak of axial velocity (v) on the free surface changes over time. Thermcapillary convection originates in the hot corner at first. The position of the peak of axial velocity (v) is close to the hot disk, and then it gradually moves toward the cold with the development of thermocapillary convection. Meanwhile, thermocapillary convection continues to develop with the increasing of temperature difference between two disks, and the

overall level of axial velocity is improved, but the peak exists fluctuation phenomena.

2. The lateral velocity of free surface presents dramatic change near the hot and cold disks. In the initial development stage of thermocapillary convection, the influence scope of cell stream is small and locates at the hot corner. The structure of lateral velocity along the interface presents a single peak and the peak is near the hot disk. The peak exists near the hot and cold disk with the influence scope of thermocapillary convection and moves toward the cold disk. The peak close to the cold disk increases significantly over time.
3. When $t=300$, the curvature of the temperature curves near the interface is small compared to that near the center of liquid bridge. It explains that the temperature on the interface is sensitive to the change of temperature difference between the two disks and is affected by thermalcapillary convection.

Acknowledgements

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