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H∞ Loop Shaping Robust Control For Tractor-semitrailer

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Abstract. The tractor-semitrailer model is general described and analyzed in road reference coordinate system. The lateral error of look-ahead distance has been chosen as nominal plant and a robust controller using H_∞ loop-shaping procedure is designed which can ensure the maximum stability margin meet the performance requirements. The results show that the controller can make the tractor-semitrailer stable under perturbed conditions and can guarantee the look-ahead lateral offset keeping in the specified scope proposed. The target of lateral control is satisfactory.

1 Introduction

When the Tractor-semitrailer meets interference on the road, lateral displacement will increase without limited by time. So, it is necessary to do some manipulations to insure that vehicle system can travel by planned routes. The author [1] has designed H_∞ loop-shaping robust controller to study the tractor-semitrailer response in straight line when the vehicle subjected interference by the lateral force, although the study ends up with good results but it dose not take control for the generalized description vehicle model. WANG [2] make use of this control method to study the vehicle steering, but the degree of freedom (DOF) of body roll in vehicle model is not considered. For previous study, this paper will work further more and be organized as follows. First of all, in order to realize the targets of the control for interference and the tracking output, tractor-semitrailer model which include DOF of body roll will be described in road reference coordinate system as generalized controlled object; Secondly, H_∞ loop-shaping robust controller will be designed and the order of the controller will be reduced; Finally, path tracking control of tractor-semitrailer will be simulated.

2 Analyses of tractor-semitrailer model in road reference coordinate system

2.1 Describe of generalized controlled object

In road reference coordinate system, X_r-axis direction is tangential direction of drive line. Y_r-axis direction is vertical with X_r-axis and goes through the center of mass of tractor, Z_r-axis direction is vertical with X_r-Y_r plane and it pointing upwards. Symbol y_{r1} represents the distance between the center of mass of tractor and the center line of the road in the Y_r-axis direction. Look-ahead distance is d in the direction of X_r, lateral displacement of its observation point to target line is Y_d, just show in Figure 1. Parameters are given in Table 1.

Table 1. Parameters values of tractor-semitrailer

Symbol	Parameter name, value and unit
m_{s1}	sprung mass of tractor (6308kg)
m_{u1}	unsprung mass of tractor (2504kg)
m_1	tractor mass (8812kg)
a_1	the distance between tractor front axle to tractor vehicle center(2.804m)
b_1	the distance between tractor rear axle to the towing vehicle center(2.485m)
c	the distance between traction point to tractor center(1.985m)
C_f	the total cornering stiffness of tractor wheel(430.6682kN/rad)
I_{xs1}	inertia that tractor sprung mass spins around the x-axis(6879kg·m^2)
I_{z1}	inertia that tractor vehicle spins around the z-axis(20277kg·m^2)
I_{xz1}	inertia that tractor sprung mass turns around x and z axis(130kg·m^2)
h_{s1}	the distance between roll center (0.519m)
D_f	roll angle damping of front suspension of tractor(2866.24 N·m·s/rad)
D_t	roll angle damping of rear suspension of trailer (5732.48 N·m·s/rad)
$C_{\varphi r}$	roll rate of rear of suspension of trailer tractor(28662.4N·m/rad)
E_f	tilt steering coefficient of tractor (-0.21°/°)
E_t	shift factor of roll for trailer rear(-0.17°/°)
C_s	damping of Steering wheel around kingpin(600N·m·s/rad)

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i	ratio of transmission of steering system(25)
m_{s2}	sprung mass of trailer (5927kg)
m_{u2}	unsprung mass of trailer (1470kg)
m_2	trailer mass (7397kg)
a_2	the distance between traction point to trailer vehicle center(8.588m)
b_2	the distance between trailer axle to a semi-trailer vehicle center(5.180m)
C_t	cornering stiffness of trailer rear (367.1652kN/rad)
C_r	the total cornering stiffness of trailer wheel(484.1082kN/rad)
I_{xs2}	inertia that trailer sprung mass spins around the x-axis(9960 kg·m^2)
I_{z2}	inertia that trailer vehicle spins around the z-axis(181171 kg·m^2)
I_{xz2}	inertia that trailer sprung mass turns around x and z axis (0 kg·m^2)
h_{s2}	the distance between tractor sprung mass center and roll center(1.061m)
D_r	roll angle damping of tractor(2866.24 N·m·s/rad)
$C_{\varphi f}$	roll rate of front suspension of tractor (28662.4N·m/rad)
$C_{\varphi t}$	roll rate of rear suspension of (573248N·m/rad)
E_r	tilt steering coefficient of tractor(0.11°/°)
I_s	inertia that tractor steering wheel turns around kingpin(0.8kg·m^2)
K_s	comprehensive stiffness of steering system(20000N·m/rad)
ξ_1	arm that front wheel lateral force to kingpin(0.05m)

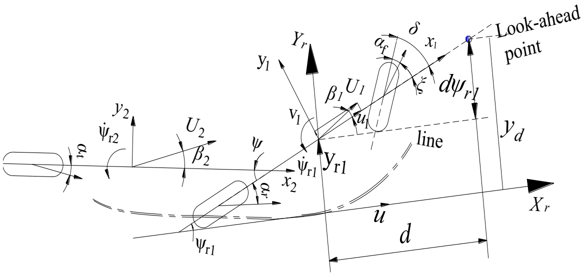

Figure 1. The movement of tractor-semitrailer along target lane

From Figure 1, as for tractor, longitudinal speed is represented by the symbol u_1, yaw angle is represented by the symbol ψ_{r1} and $\dot{\psi}_{r1}$ represent yaw rate. So, the lateral acceleration of tractor in the direction of y_1 could express like Equation 1.

$$\dot{v}_1 = \ddot{y}_{r1} - u_1\dot{\psi}_{r1} \tag{1}$$

To ensure Y_r axis which through the center of mass of tractor is perpendicular to X_r axis, it is necessary to assume coordinates of path to the moving coordinate system, you could know all coordinate system's relations in Figure 2.

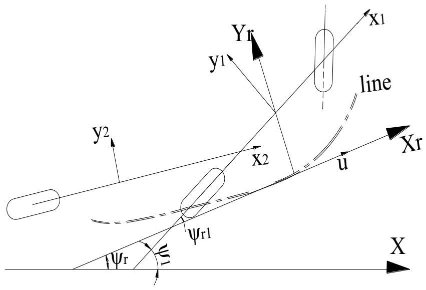

Figure 2. The relationship between fixed and road reference coordinate system

Yaw rate of tractor can be expressed as $\dot{\psi}_1$

$$\dot{\psi}_1 = \dot{\psi}_{r1} + \dot{\psi}_r \tag{2}$$

$\dot{\psi}_r$ is angular velocity that tractor should get in target line when it moves without deviation. Similarly

$$\dot{\psi}_2 = \dot{\psi}_{r2} + \dot{\psi}_r \tag{3}$$

From (2) and (3), $\dot{\psi}_1$ and $\dot{\psi}_2$ are tractor and trailer's raw rate $\dot{r}_1$ and $\dot{r}_2$ under the model of the vehicle's coordinate system in [6], using Eqution 1- Equation3, the state variables

$$\begin{bmatrix}\dot{v}_1 & \dot{r}_1 & \dot{r}_2 & \dot{\psi} & \dot{p}_1 & \dot{p}_2 & \dot{\varphi}_1 & \dot{\varphi}_2 & \ddot{\delta} & \dot{\delta}\end{bmatrix}^T \tag{4}$$

can be replaced by

$$\begin{bmatrix}\dot{y}_{r1} - u_1\psi_{r1} & \dot{\psi}_{r1} + \dot{\psi}_r & \dot{\psi}_{r2} + \dot{\psi}_r & \psi & p_1 & p_2 & \varphi_1 & \varphi_2 & \dot{\delta} & \delta\end{bmatrix}^T \tag{5}$$

Then, motion equations of tractor-semitrailer are written about $y_r(t)$ two order form

$$\ddot{y}_r(t) + M^{-1}N\dot{y}_r(t) + M^{-1}Py_r(t) = M^{-1}Q\delta_{sw} + M^{-1}D_1\dot{\psi}_r(t) + M^{-1}D_2\ddot{\psi}_r(t) \tag{6}$$

where

$$x_1(t) = y_r(t) = \begin{bmatrix}y_{r1} & \psi_{r1} & \psi_{r2} & \varphi_1 & \varphi_2 & \psi & \delta\end{bmatrix}^T \tag{7}$$

$$x_2(t) = \dot{y}_r(t) = \begin{bmatrix}\dot{y}_{r1} & \dot{\psi}_{r1} & \dot{\psi}_{r2} & \dot{\varphi}_1 & \dot{\varphi}_2 & \dot{\psi} & \dot{\delta}\end{bmatrix}^T \tag{8}$$

so equations of motion express as

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = \begin{bmatrix}\boldsymbol{0} & \boldsymbol{I}\\ -M^{-1}P & -M^{-1}N\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + \begin{bmatrix}\boldsymbol{0}\\ M^{-1}Q\end{bmatrix}\delta_{sw} + \begin{bmatrix}\boldsymbol{0}\\ M^{-1}D_1\end{bmatrix}\dot{\psi}_r + \begin{bmatrix}\boldsymbol{0}\\ M^{-1}D_2\end{bmatrix}\ddot{\psi}_r \tag{9}$$

In (9), ***0*** is 7-order zero matrix, ***I*** is 7-order unit matrix, which write as the form of generalized description

$$\begin{bmatrix}\dot{x}\\ \hdashline z\\ y\end{bmatrix} = \left[\begin{array}{c:cc}A & B_1 & B_2\\ \hdashline C_1 & D_{11} & D_{12}\\ C_2 & D_{21} & D_{22}\end{array}\right]\begin{bmatrix}x\\ \hdashline w\\ u\end{bmatrix} \tag{10}$$

where

$$A = \begin{bmatrix}\boldsymbol{0} & \boldsymbol{I}\\ -M^{-1}P & -M^{-1}N\end{bmatrix} \quad B_1 = \begin{bmatrix}\boldsymbol{0} & \boldsymbol{0}\\ M^{-1}D_1 & M^{-1}D_2\end{bmatrix} \quad B_2 = \begin{bmatrix}\boldsymbol{0}\\ M^{-1}Q\end{bmatrix}$$

$$w = \begin{bmatrix} \dot{\psi}_r \\ \ddot{\psi}_r \end{bmatrix} \quad u = \delta_{sw}$$

$$M = \begin{bmatrix} m_1 + m_2 & -m_2 c & -m_2 a_2 & -m_{s1} h_1 & -m_{s2} h_2 & 0 & 0 \\ m_{s1} h_1 & -I_{xz1} & 0 & -I_{xs1} & 0 & 0 & 0 \\ m_{s2} h_2 & -m_{s2} h_2 c & -m_{s2} h_2 a_2 - I_{xz2} & 0 & -I_{xs2} & 0 & 0 \\ m_1 c & I_{z1} & 0 & -m_{s1} h_1 c + I_{xz1} & 0 & 0 & 0 \\ m_2 a_2 & -m_2 a_2 c & -m_2 a_2^2 - I_{z2} & 0 & -I_{xz2} - m_{s2} h_2 a_2 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I_s \end{bmatrix}$$

$$N = \begin{bmatrix} -\dfrac{C_f + C_r + C_t}{u_1} & -\dfrac{C_f a_1 - C_r b_1 - C_t c}{u_1} & \dfrac{C_t (a_2 + b_2)}{u_1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(D_f + D_r) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -D_t & 0 & 0 \\ \begin{matrix} -\dfrac{(a_1 + c)C_f}{u_1} \\ \underline{+(c - b_1)C_r} \end{matrix} & \begin{matrix} -\dfrac{a_1 (a_1 + c)C_f}{u_1} \\ \underline{-b_1 (c - b_1)C_r} \end{matrix} & 0 & 0 & 0 & 0 & 0 \\ -\dfrac{(a_2 + b_2)C_t}{u_1} & \dfrac{(a_2 + b_2)cC_t}{u_1} & \dfrac{(a_2 + b_2)^2 C_t}{u_1} & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & -1 & 0 \\ \dfrac{C_f \xi_1}{u_1} & \dfrac{C_f a_1 \xi_1}{u_1} & 0 & 0 & 0 & 0 & C_s \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & C_f + C_r & C_t & C_f E_f + C_r E_r & C_t E_t & 0 & C_f \\ 0 & 0 & 0 & m_{s1} g h_1 - C_{\varphi f} - C_{\varphi r} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_{s2} g h_2 - C_{\varphi t} & 0 & 0 \\ 0 & \begin{matrix} (a_1 + c)C_f \\ +(c - b_1)C_r \end{matrix} & 0 & \begin{matrix} (a_1 + c)C_f E_f \\ +(c - b_1)C_r E_r \end{matrix} & 0 & 0 & (a_1 + c)C_f \\ 0 & 0 & (a_2 + b_2)C_t & 0 & (a_2 + b_2)C_t E_t & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & -1 & 0 \\ 0 & -C_f \xi_1 & 0 & -C_f E_f \xi_1 & 0 & 0 & k_s - C_f \xi_1 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \dfrac{k_s}{i} \end{bmatrix}^T$$

$$D_1 = \begin{bmatrix} -(m_1 + m_2)u_1 + \dfrac{C_f a_1 - C_r b_1 - C_t (a_2 + b_2 + c)}{u_1} \\ -m_{s1} h_1 u_1 \\ -m_{s2} h_2 u_1 \\ -m_1 u_1 c + \dfrac{a_1 (a_1 + c)C_f - b_1 (c - b_1)C_r}{u_1} \\ -m_2 u_1 a_2 - \dfrac{C_t (a_2 + b_2)(a_2 + b_2 + c)}{u_1} \\ 0 \\ -\dfrac{C_f a_1 \xi_1}{u_1} \end{bmatrix}$$

$$D_2 = \begin{bmatrix} m_2 (c + a_2) \\ I_{xz1} \\ m_{s2} h_2 (a_2 + c) + I_{xz2} \\ -I_{z1} \\ m_2 a_2 (c + a_2) + I_{z2} \\ 0 \\ 0 \end{bmatrix} \quad (11)$$

A is 14 order matrix, B_1 is 14×2 order matrix, other factors will be determined. Designing H_∞ robust controller can make use of (10).

2.2 The analysis of transfer functions

In the control generalized system by disturbance Fig.3, $G_0(s)$ is nominal controlled object, K_s is controller, y is measuring output, interference signal w is as the system disturbance input d_s which come from transfer function $W(s)$, control output z is same with the measuring output y from system.

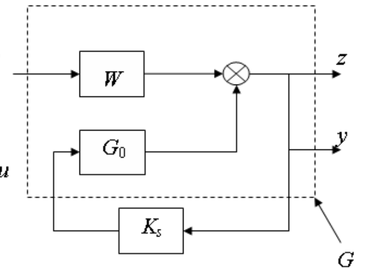

Figure 3. Diagram of generalized interference control system

From the Figure 3

$$\begin{bmatrix} z(s) \\ y(s) \end{bmatrix} = \begin{bmatrix} W(s) & G_0(s) \\ W(s) & G_0(s) \end{bmatrix} \begin{bmatrix} w(s) \\ u(s) \end{bmatrix} = G(s) \begin{bmatrix} w(s) \\ u(s) \end{bmatrix} \quad (12)$$

so the transfer function matrix is

$$G(s) = \begin{bmatrix} W(s) & G_0(s) \\ W(s) & G_0(s) \end{bmatrix} \quad (13)$$

Setting y_d as the look-ahead distance d (meter) point offset, express as

$$y_d = y_{r1} + d \cdot \psi_{r1} \quad (14)$$

the transfer function $G_0(s)$ from input of steering wheel δ_{sw} to offset y_d of target point is

$$G_0(s) = \frac{Y_d(s)}{\delta_{sw}(s)} = G_{Y1}(s) + d \cdot G_{\psi 1}(s) = C_{21}(sI - A)^{-1} B_2 + d \cdot C_{22}(sI - A)^{-1} B_2 \quad (15)$$

where

$$C_{21} = [1 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0] \quad (16)$$

$$C_{22} = [0 \; 1 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0] \quad (17)$$

Transfer function $W(s)$ of $w = \begin{bmatrix} \dot{\psi}_r \\ \ddot{\psi}_r \end{bmatrix}$ to y_d can express like

$$W(s) = \frac{Y_d(s)}{w(s)} = G_{Y1}(s) + d \cdot G_{\psi 1}(s) = C_{11}(sI - A)^{-1} B_1 + d \cdot C_{12}(sI - A)^{-1} B_1 \quad (18)$$

Because of that controlled output z and measuring output y from system is same, so that $C_{11}=C_{21}$， $C_{12}=C_{22}$. Putting the data from Appendix A table into (10), (15) and (18), look-ahead distance is 10metter, and then the transfer function matrix (13) can be gotten.

3 Design of H∞ loop shaping controller

From McFarlane and Glover [3,4,5] use the steps of the designing of H_∞ loop-shaping controller, we can obtain results of the loop-shaping and the controller calculates.

Choosing $W_1 = 0.5\dfrac{s+80}{s+2.8}$, $W_2 = 1$, through *MATLAB* robust and algorithm toolbox obtaining $\gamma = 2.7812$ （$1 < \gamma < 10$）, making $\varepsilon_{\max}$ <1, satisfy the goal of robust performance.[4,5,6]

Figure 4 is each singular value curve after shaping, we can see that after shaping the singular value curves of $W_2 G_0 W_1$ and open loop function $G_0 K_s$ (red solid lines and red dotted line) are higher than G_0 (Black dotted line) in low frequency values, it shows that tracking capability and anti-low-frequency interference have improved with the use of controller. As the same time, $W_2 G_0 W_1$ singular value's slope of the curve is littler than G_0 (Curve is dropping fast) in high frequency, it means that after shaping the power to control noise is strengthen, slope of the $G_0 K_s$ curve is same with nominal controlled object G_0 , there is no weaken in control noise after shaping. The result of γ is in a reasonable range, loop shaping is good to satisfy the robust stability's index.

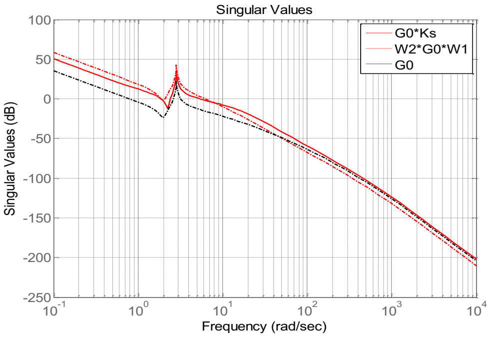

Figure 4. Singular values

The order of positive feedback controller K_s which is original obtained in this paper is 13. It is not good for application in engineering when order is too high. Therefore, we choose *Hankel norm approximation method for model* to reduced order. [7] [8]

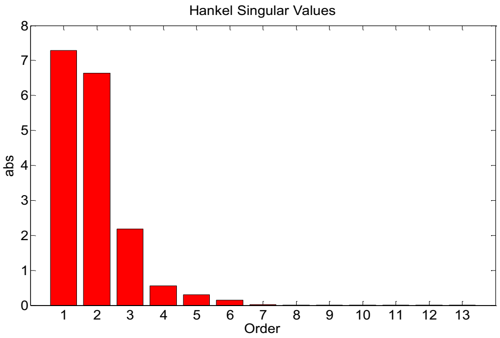

Figure 5. Hankel singular values

Figure 5 shows that the obtaining Hankel singular values which use this algorithm, abscissa represents original controller system order, ordinate represents singular value or "state energy" in every order, bigger "state energy" means that additional error $\left\| K_s - \hat{K}_s \right\|_\infty$ is bigger. Finally, 7 reduced-order controller is chosen, additional error is 0.0298, controller is

$$\hat{K}_s = \left[\begin{array}{c|c} \hat{A}_{Ks} & \hat{B}_{Ks} \\ \hline \hat{C}_{Ks} & \hat{D}_{Ks} \end{array}\right] = \left[\begin{array}{ccccccc|c} -14.8192 & -15.5582 & -7.5199 & -5.4253 & 1.8744 & 2.004 & -5.2654 & -8.4571 \\ 7.6835 & -14.8192 & 4.4934 & -4.4143 & 0.568 & 0.4127 & -3.1816 & -4.1418 \\ 0 & 0 & -4.0416 & -7.1796 & 0.1501 & 0.642 & -2.6266 & -1.6811 \\ 0 & 0 & 1.6861 & -4.0416 & -0.0901 & 0.7211 & -3.7316 & -1.4222 \\ 0 & 0 & 0 & 0 & -0.1236 & 1.9457 & 0.0841 & 1.6405 \\ 0 & 0 & 0 & 0 & -2.0505 & -0.1236 & 0.4914 & 0.2506 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1.1539 & -1.1516 \\ \hline 5.2332 & 7.3436 & 1.4877 & 2.9988 & 0.0872 & 0.6469 & 2.1567 & -1.2976 \end{array}\right] \tag{19}$$

4 Simulation of tractor-semitrailer route tracking control

4.1. Routes designing

Thinking that widths of road and tractor-semitrailer, when tractor-semitrailer driving, lateral offset is better not more than0.2 to 0.3 meters[2]. So, tractor-semitrailer's control goal is to let vehicle system drive in the road, if it appears lateral deviation, controller should ensure that lateral offset is not beyond the required range. According to control objectives, it structures a driving route that it is continuous multi-curves, the designing route shows in Figure 6. Assuming that tractor-semitrailer drives in 500 meters road with constant advance speed, then goes to curvature radius of 600 meters' circular route which has three sections respectively, then, driving out along 500 meters straight. In the total route, steering wheel keeps zero degree firstly, and then changes three times, at last zero again.

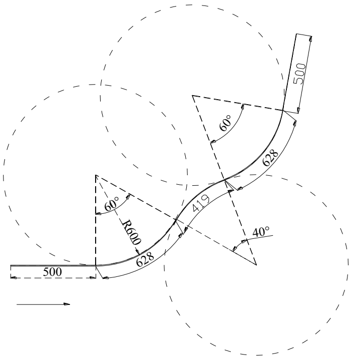

Figure 6. The route of tractor-semitrailer

4.2 Simulate analysis of route tracking control

Figure 7 shows the closed-loop control system of tractor-semitrailer, yaw rate and yaw angular acceleration can be considered as disturbance input in corners. The lateral offset which is 10 meters look-ahead of vehicle can be considered as controllable output. Steering angle initial input of in tractor-semitrailer is set to 0, constant velocity is 60 km/h or 16.67m/s , the initial value of disturbance input $\dot{\psi}_r$ is 0.028 (1/s) and $\ddot{\psi}_r$ is also setting to 0.028 ($1/s^2$) ($\dot{\psi}_r = u / r_0 \approx u_1 / r_0$). Times of each section of route are 30s, 37.67s, 25.1s, 37.67s and 30s.

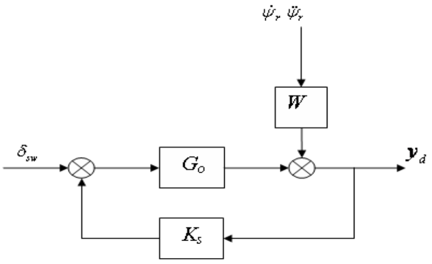

Figure 7. Implementation of the controller

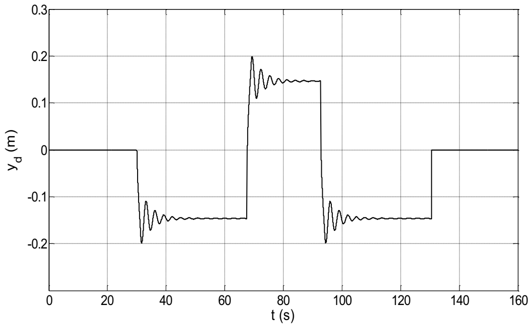

Figure 8. Lateral tracking offset simulation

Figure 8 shows that maximum vibration amplitude of lateral offset is limited in 0.2m and can have stable value at least under the controller. Simulation results show that the tractor-semitrailer closed-loop system has the required robust with the help of controller, and it can keep the robust steady.

5 Conclusions

Tractor-semitrailer model which include DOF of body roll is described in road reference coordinate system as generalized controlled object. H_∞ loop-shaping robust controller is designed and the order of the controller is reduced which could make the tractor-semitrailer stable under perturbed conditions and could guarantee the look-ahead lateral offset keeping in the satisfied range.

Acknowledgments

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