H∞ Loop Shaping Robust Control For Tractor-semitrailer

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Abstract. The tractor-semitrailer model is general described and analyzed in road reference coordinate system. The lateral error of look-ahead distance has been chosen as nominal plant and a robust controller using H∞ loop-shaping procedure is designed which can ensure the maximum stability margin meet the performance requirements. The results show that the controller can make the tractor-semitrailer stable under perturbed conditions and can guarantee the look-ahead lateral offset keeping in the specified scope proposed. The target of lateral control is satisfactory.

1 Introduction

When the Tractor-semitrailer meets interference on the road, lateral displacement will increase without limited by time. So, it is necessary to do some manipulations to insure that vehicle system can travel by planned routes. The author [1] has designed H∞ loop-shaping robust controller to study the tractor-semitrailer response in straight line when the vehicle subjected interference by the lateral force, although the study ends up with good results but it dose not take control for the generalized description vehicle model. WANG [2] make use of this control method to study the vehicle steering, but the degree of freedom (DOF) of body roll in vehicle model is not considered. For previous study, this paper will work further more and be organized as follows. First of all, in order to realize the targets of the control for interference and the tracking output, tractor-semitrailer model which include DOF of body roll will be described in road reference coordinate system as generalized controlled object; Secondly, H∞ loop-shaping robust controller will be designed and the order of the controller will be reduced; Finally, path tracking control of tractor-semitrailer will be simulated.

2 Analyses of tractor-semitrailer model in road reference coordinate system

2.1 Describe of generalized controlled object

In road reference coordinate system, Xr-axis direction is tangential direction of drive line. Yr-axis direction is vertical with Xr-axis and goes through the center of mass of tractor, Zr-axis direction is vertical with Xr-Yr plane and it pointing upwards. Symbol yr₁ represents the distance between the center of mass of tractor and the center line of the road in the Yr-axis direction. Look-ahead distance is d in the direction of Xr, lateral displacement of its observation point to target line is Yd, just show in Figure 1. Parameters are given in Table 1. Parameters values of tractor-semitrailer

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter name, value and unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>ml₁</td>
<td>sprung mass of tractor (6308kg)</td>
</tr>
<tr>
<td>m₁</td>
<td>unsprung mass of tractor (2504kg)</td>
</tr>
<tr>
<td>ml</td>
<td>tractor mass (8812kg)</td>
</tr>
<tr>
<td>a₁</td>
<td>the distance between tractor front axle to tractor vehicle center(2.804m)</td>
</tr>
<tr>
<td>b₁</td>
<td>the distance between tractor rear axle to the towing vehicle center(2.485m)</td>
</tr>
<tr>
<td>c</td>
<td>the distance between traction point to tractor center(1.985m)</td>
</tr>
<tr>
<td>Cf</td>
<td>the total cornering stiffness of tractor wheel(430.6682kN/rad)</td>
</tr>
<tr>
<td>Ix₁</td>
<td>inertia that tractor sprung mass spins around the x-axis(6879kg·m²)</td>
</tr>
<tr>
<td>I₁</td>
<td>inertia that tractor vehicle spins around the z-axis(20277kg·m²)</td>
</tr>
<tr>
<td>Ixz₁</td>
<td>inertia that tractor sprung mass turns around x and z axis(130kg·m²)</td>
</tr>
<tr>
<td>h₁</td>
<td>the distance between roll center (0.519m)</td>
</tr>
<tr>
<td>Df</td>
<td>roll angle damping of front suspension of tractor(2866.24 N·m·s/rad)</td>
</tr>
<tr>
<td>D₁</td>
<td>roll angle damping of rear suspension of trailer (5732.48 N·m·s/rad)</td>
</tr>
<tr>
<td>Cre</td>
<td>roll rate of rear of suspension of trailer (2866.24N·m/rad)</td>
</tr>
<tr>
<td>Ef</td>
<td>tilt steering coefficient of tractor (-0.21°/°)</td>
</tr>
<tr>
<td>E₁</td>
<td>shift factor of roll for trailer rear(-0.17°/°)</td>
</tr>
<tr>
<td>Cs</td>
<td>damping of Steering wheel around kingpin(600N·m·s/rad)</td>
</tr>
</tbody>
</table>

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From Figure 1, as for tractor, longitudinal speed is represented by the symbol $u_1$, yaw angle is represented by the symbol $\psi_{r1}$, and $\dot{\psi}_{r1}$ represent yaw rate. So, the lateral acceleration of tractor in the direction of $y_1$ could express like Equation 1.

$$\ddot{y}_1 = \dot{y}_{r1} - u_1\dot{\psi}_{r1}$$  (1)

To ensure $Y_1$ axis which through the center of mass of tractor is perpendicular to $X_1$ axis, it is necessary to assume coordinates of path to the moving coordinate system, you could know all coordinate system’s relations in Figure 2.

$$\dot{\psi}_1 = \dot{\psi}_{r1} + \psi_r$$  (2)

$\dot{\psi}_r$ is angular velocity that tractor should get in target line when it moves without deviation. Similarly

$$\dot{\psi}_2 = \dot{\psi}_{r2} + \dot{\psi}_r$$  (3)

From (2) and (3), $\dot{\psi}_1$ and $\dot{\psi}_2$ are tractor and trailer’s raw rate $\dot{r}_1$ and $\dot{r}_2$ under the model of the vehicle’s coordinate system in [6], using Equation 1- Equation 3, the state variables

$$\begin{bmatrix} \ddot{y}_1 \end{bmatrix} = \begin{bmatrix} \ddot{y}_{r1} + \ddot{y}_r \end{bmatrix}$$  (4)

Then, motion equations of tractor-semi trailer are written about $y_i(t)$ two order form

$$\ddot{y}_i(t) + M^{-1}N\dot{y}_i(t) + M^{-1}P\dot{y}_i(t) = M^{-1}Q\ddot{\delta}_s + M^{-1}D\dot{\psi}_i(t) + M^{-1}D\dot{\psi}_i(t)$$  (5)

where

$$x_1(t) = y_{11}(t) = \begin{bmatrix} y_{r1} & \psi_{r1} & \psi_{r2} & \varphi_1 & \varphi_2 & \dot{\psi}_r & \dot{\phi}_1 & \dot{\phi}_2 & \delta_s & \delta \end{bmatrix}^T$$  (7)

$$x_2(t) = \dot{x}_2(t) = \begin{bmatrix} \dot{y}_{r1} & \dot{\psi}_{r1} & \dot{\psi}_{r2} & \dot{\varphi}_1 & \dot{\varphi}_2 & \dot{\psi}_r & \dot{\phi}_1 & \dot{\phi}_2 & \dot{\delta}_s & \dot{\delta} \end{bmatrix}^T$$  (8)

so equations of motion express as

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} \theta & I \\ -M^{-1}P & -M^{-1}N \end{bmatrix} x_1(t) + \begin{bmatrix} \theta \\ M^{-1}Q \end{bmatrix} \dot{\delta}_s + \begin{bmatrix} \theta \\ M^{-1}D \end{bmatrix} \dot{\psi}_r + \begin{bmatrix} \theta \\ M^{-1}D \end{bmatrix} \dot{\psi}_r \end{bmatrix}$$  (9)

In (9), $\theta$ is 7-order zero matrix, $I$ is 7-order unit matrix, which write as the form of generalized description

$$\begin{bmatrix} \dot{x} \\ \dot{z} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} x \\ w \\ u \end{bmatrix}$$  (10)

where

$$A = \begin{bmatrix} \theta & I \\ -M^{-1}P & -M^{-1}N \end{bmatrix}, B_1 = \begin{bmatrix} \theta & 0 \\ M^{-1}D_1 & -M^{-1}D_2 \end{bmatrix}, B_2 = \begin{bmatrix} \theta \\ M^{-1}Q \end{bmatrix}$$

\[\text{Figure 1. The movement of tractor-semi trailer along target lane}\]

\[\text{Figure 2. The relationship between fixed and road reference coordinate system}\]
In the control generalized system by disturbance Fig.3, $G_0(s)$ is nominal controlled object, $K_i$ is controller, $\gamma$ is measuring output, interference signal $w$ is as the system disturbance input $d_i$ which come from transfer function $W(s)$, control output $z$ is same with the measuring output $y$ from system.

\[ W(s) = \begin{bmatrix} \psi_r \\ \psi_r \end{bmatrix} \quad u = \delta_{nw} \]

\[
M = \begin{bmatrix}
    m_1 + m_2 & -m_2 & -m_2 & -m_2 & -m_2 & -m_2 & -m_2 & -m_2 & 0 & 0 \\
    m_2 & -I_{u_1} & 0 & -I_{u_1} & 0 & 0 & 0 & 0 & 0 & 0 \\
    -m_2 & -m_2 & -m_2 & -m_2 & -m_2 & -m_2 & -m_2 & -m_2 & 0 & 0 \\
    m_2 & I_{u_1} & 0 & m_2 & I_{u_1} & 0 & 0 & 0 & 0 & 0 \\
    m_2 & I_{u_1} & 0 & m_2 & I_{u_1} & 0 & 0 & 0 & 0 & 0 \\
    0 & 1 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & I_j \\
\end{bmatrix}
\]

\[
N = \begin{bmatrix}
    C_1 + C_c & C_1 C_a - C_h C_c & C_1 (u_1 + b_1) \\
    u_1 & u_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    (a_1 + b_1) C_c & u_1 & u_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    (a_1 + b_1) C_c & (a_1 + b_1) C_c & u_1 & u_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
    C_c & C_c & u_1 & u_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
P = \begin{bmatrix}
    0 & C_1 + C_c & C_1 E & C_1 E & C_1 E & 0 & 0 & C_1 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & -C_c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
D_1 = \begin{bmatrix}
    -(m_1 + m_2) u_1 + C_1 a_1 \gamma_c - C_1 a_1 \gamma_c + (a_1 + b_1) c_1 \\
    -m_2 u_1 + C_1 a_1 \gamma_c - C_1 a_1 \gamma_c + (a_1 + b_1) c_1 \\
    -m_2 u_1 + C_1 a_1 \gamma_c - C_1 a_1 \gamma_c + (a_1 + b_1) c_1 \\
    -m_2 u_1 + C_1 a_1 \gamma_c - C_1 a_1 \gamma_c + (a_1 + b_1) c_1 \\
    0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
D_2 = \begin{bmatrix}
    m_2 (c + a_1) I_{u_1} \\
    m_2 a_2 (a_1 + c) + I_{u_2} \\
    0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
A \text{ is 14 order matrix, } B_1 \text{ is 14x2 order matrix, other factors will be determined. Designing } H_o \text{ robust controller can make use of } (10).
\]

### 2.2 The analysis of transfer functions

In the control generalized system by disturbance Fig.3, $G_0(s)$ is nominal controlled object, $K_i$ is controller, $\gamma$ is measuring output, interference signal $w$ is as the system disturbance input $d_i$ which come from transfer function

\[
\begin{bmatrix} z(s) \\ y(s) \end{bmatrix} = \begin{bmatrix} W(s) & G_0(s) \end{bmatrix} \begin{bmatrix} w(s) \\ u(s) \end{bmatrix} = G(s) \begin{bmatrix} w(s) \\ u(s) \end{bmatrix}
\]

so the transfer function matrix is

\[
G(s) = \begin{bmatrix} W(s) & G_0(s) \end{bmatrix} \begin{bmatrix} W(s) & G_0(s) \end{bmatrix}
\]

Setting $y_d$ as the look-ahead distance $d$ (meter) point offset, express as

\[
y_d = y_r + d \cdot \psi_r
\]

the transfer function $G_d(s)$ from input of steering wheel $\delta_{sw}$ to offset $y_d$ of target point is

\[
G_d(s) = \frac{1}{\delta_{sw}(s)} = \frac{G_d(s)}{d \cdot G_d(s)} = C \cdot (sI - A)^{-1} B_t + d \cdot C \cdot (sI - A)^{-1} B_t
\]

where

\[
C_{21} = [1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]
\]

\[
C_{22} = [0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]
\]

Transfer function $W(s)$ of $W = \begin{bmatrix} \psi_r \\ \psi_r \end{bmatrix}$ to $y_d$ can express like

\[
W(s) = \frac{Y_d(s)}{W(s)} = G_d(s) + d \cdot G_d(s) = C \cdot (sI - A)^{-1} B_t + d \cdot C \cdot (sI - A)^{-1} B_t
\]

Because of that controlled output $z$ and measuring output $y$ from system is same, so that $C_{11} = C_{21}$, $C_{12} = C_{22}$. Putting the data from Appendix A table into (10), (15) and (18), look-ahead distance is 10meter, and then the transfer function matrix (13) can be gotten.

### 3 Design of $H_{\infty}$ loop shaping controller

From McFarlane and Glover [3,4,5] use the steps of the designing of $H_o$ loop-shaping controller, we can obtain results of the loop-shaping and the controller calculates.
Choosing \( W_i = 0.5 \frac{s + 80}{s + 2.8} \), \( W_j = 1 \), through MATLAB robust and algorithm toolbox obtaining \( \gamma = 2.7812 \) \((1<\gamma<10)\), making \( \varepsilon_{\max} <1 \), satisfy the goal of robust performance.\(^{[4,5,6]}\)

Figure 4 is each singular value curve after shaping, we can see that after shaping the singular value curves of \( W_2G_0W_1 \) and open loop function \( G_0K_s \) (red solid lines and red dotted line) are higher than \( G_0 \) (Black dotted line) in low frequency values, it shows that tracking capability and anti-low-frequency interference have improved with the use of controller. As the same time, \( W_2G_0W_1 \) singular value’s slope of the curve is littler than \( G_0 \) (Curve is dropping fast) in high frequency, it means that after shaping the power to control noise is strengthen, slope of the \( G_0K_s \) curve is same with nominal controlled object \( G_0 \), there is no weaken in control noise after shaping. The result of \( \gamma \) is in a reasonable range, loop shaping is good to satisfy the robust stability’s index.

\[ K = \left[ \begin{array}{c} \psi_r \\ -1.9 \psi_r + 1.8 K_s \\ -0.9 \end{array} \right] \]

\[(19)\]

4 Simulation of tractor-semitrailer route tracking control

4.1 Routes designing

Thinking that widths of road and tractor-semitrailer, when tractor-semitrailer driving, lateral offset is better not more than0.2 to 0.3 meters\(^{[3]}\). So, tractor-semitrailer’s control goal is to let vehicle system drive in the road, if it appears lateral deviation, controller should ensure that lateral offset is not beyond the required range. According to control objectives, it structures a driving route that it is continuous multi-curves, the designing route shows in Figure 6. Assuming that tractor-semitrailer drives in 500 meters road with constant advance speed, then goes to curvature radius of 600 meters’ circular route which has three sections respectively, then, driving out along 500 meters straight. In the total route, steering wheel keeps zero degree firstly, and then changes three times, at last zero again.
5 Conclusions

Tractor-semitrailer model which include DOF of body roll is described in road reference coordinate system as generalized controlled object. $H_\infty$ loop-shaping robust controller is designed and the order of the controller is reduced which could make the tractor-semitrailer stable under perturbed conditions and could guarantee the look-ahead lateral offset keeping in the satisfied range.

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References