

Development of Algorithms for Control of Motor Boat as Multidimensional Nonlinear Object

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Abstract. In this paper authors develop and research system for motor boat control, that allows to move along the stated paths with the given speed. It is assumed, that boat is equipped by the measuring system that provides current coordinates, linear and angular velocities. Control system is based upon the mathematical model, presented earlier (see references). In order to analytically find the necessary controls, all equations were transformed to Jordan controllable form. Besides solution this transformation also allows to handle model nonlinearities and get required quality of movement along the stated paths. Control system includes algorithms for control of longitudinal velocity and boat course. Research of the proposed control system according to boat design limitations for the values of control variables was performed by simulation in MATLAB. Results of two experiments, different in value of the required velocity are discussed.

1 Introduction

One of the major tasks in creation of fast motor boats is development of control system and control algorithms providing boat movement along the stated path with the necessary speed [1, 2, 3]. It is known, that solution of this task requires precise mathematical model describing the boat movement. In this paper, a refined mathematical model of surface boat “Neptune”, introduced in [4] is used. According to this model, motor boat is a multidimensional controllable object. It has two controls and two controlled variables. Thus, boat control system can be synthesized as multidimensional system, based on its model in Jordan controllable form (JCF). This allows both handling model nonlinearities and getting required quality of movement along the stated paths by selection of control laws and algorithms. Developed nonlinear algorithm will be implemented in onboard processing unit and required input will be collected by continuous sensors. Research of closed-loop control system properties was performed by computer simulation.

2 Mathematical model of the motor boat

Considering taken coordinate system (Fig. 1.a), refined mathematical model of the motor boat [2, 4] can be described by the following equations:

$$\begin{aligned} \dot{x}_g &= V_x \cos\varphi + V_z \sin\varphi, \\ \dot{z}_g &= V_z \cos\varphi - V_x \sin\varphi, \dot{\varphi} = \omega_y, \end{aligned} \quad (1)$$

$$J_y \dot{\omega}_y = -c_y \omega_y - k F_t \sin\alpha, \quad (2)$$

$$\begin{aligned} m_z \dot{V}_z &= -c_z |V_z| V_z + F_T \sin\alpha, m_x \dot{V}_x \\ &= -c_x |V_x| V_x + F_T \cos\alpha, \end{aligned} \quad (3)$$

$$y_1 = V_x, y_2 = \varphi,$$

where \dot{x}_g, \dot{z}_g — velocities of changes of boat centre of gravity in fixed coordinate system; V_x, V_z — projections of velocity vector to the axes of body coordinate system XZY ; φ — current boat course, where $\omega_y = \dot{\varphi}$ is an angular velocity of the boat relating to its vertical axis OY . J_y — moment of inertia relating to axis OY ; c_x, c_y, c_z — hydrodynamic coefficients of forces and moments of resistance forces applied to the boat; m_x, m_y — total boat mass including additional water mass relating to OX and OZ axes. F_T — total force of all engines and propellers in the boat (Fig. 1b); α — rotational angle of movable frame with engines and propellers; k — distance from moving force application point to spinning axis of the boat, equal to 0.4 meter; y_1, y_2 — controlled variables of the boat: course and travel speed.

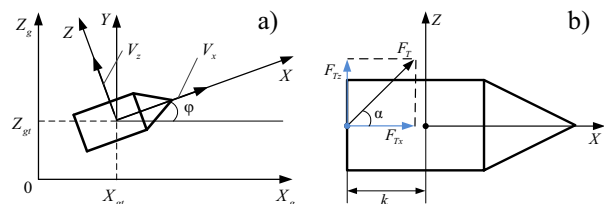


Figure 1. Coordinate system and force projection.

Eq. (1) ~ (3) do not take in to account aerodynamic forces and moments acting on the boat as they are far less then hydrodynamic forces. Flexion of the sea surface,

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pitching, rolling and heaving motions of the boat are not considered. It is assumed that boat moves in still water [3].

Eq. (1) are kinematic ones as they link boat velocities with its coordinates and course. Current coordinates $x_g(t)$, $z_g(t)$ and course $\varphi(t)$ could be defined by integrating Eq. (1) with respect to τ between 0 to t with stated initial conditions $x_g(0)$, $z_g(0)$, $\varphi(0)$ and functions $V_x(\tau)$, $V_z(\tau)$, $\omega_y(t)$, coming from equations (2) and (3) with provision for control actions. Such actions on Eq (2), (3) are force F_T and angle α .

In this case we intend that required boat path is setup by the following laws: change of travel speed $V_x^*(t)$ and change of boat course — angle $\varphi^* = \varphi^*(t)$. Here $V_x^*(t)$, $\varphi^*(t)$ and $\omega_y^* = d\varphi^*/dt$ are given continues functions of time.

Thus, movement of the boat with the given travelling speed $V_x^*(t)$ is characterized by corresponding changes of course $\varphi^*(t)$, angular velocity $\omega_y(t)$ and lateral offset $V_z(t)$. Specified boat design includes two independent controls: $u_1 = F_T \cos\alpha$ and $u_2 = F_T \sin\alpha$. Control u_1 allows to change travelling speed V_x . Control u_2 is used for course, angular velocity and lateral shift. Actually this gives and opportunity to control only two variables: angular velocity and lateral shift of the boat having course as non-controlled or course and angular velocity having lateral shift non-controlled. In this paper the last option is taken.

Considering synthesis of the system on the base of Jordan controllable form, we will define boat equations in deviations. Let's introduce state variables — deviations: $x_1 = V_x - V_x^*$, $x_2 = \varphi - \varphi^*$, $x_3 = \omega_y - \omega_y^*$, $x_4 = V_z - V_z^*$. Here $V_x^* = V^* \cos \varphi$, $V_z^* = V^* \sin \varphi$, $\omega_y^* = \dot{\varphi}^*(t)$. Having $V^* = V^*(t)$ and $\varphi^* = \varphi^*(t)$ set by one or another way, functions V_x^* , V_z^* and ω_y^* can be defined straightforward.

To define equations of movement in deviations, it is necessary to differentiate these deviations with respect to time and consider Eq. (2) and (3). The result is:

$$\dot{x}_1 = (-c_x|x_1 + V_x^*|(x_1 + V_x^*) + u_1 - m_x V_x^*)/m_x, \quad (4)$$

$$\dot{x}_2 = x_3, \quad (5)$$

$$\dot{x}_3 = (-c_y(x_3 + \omega_y^*) - k u_2 - J \dot{\omega}_y^*)/J_y, \quad (6)$$

$$\dot{x}_4 = (-c_z|x_4 + V_z^*|(x_4 + V_z^*) + u_2 - m_z V_z^*)/m_z, \quad (7)$$

For the stable system in steady state, it is right to say that $\dot{x}_i = 0, x_i = 0, i = \overline{1, n}$. In this paper, the abovementioned equations mean that $V_x^o = V_x^*, \varphi^o = \varphi^*, \omega_y^o = \omega_y^*, V_z^o = V_z^*$. As in our case, lateral shift is non-controlled, then in steady state Eq. (8) follows from Eq. (4) ~ (7).

$$u_1^0 = c_x|V_x^*|V_x^* + \dot{V}_x^*, u_2^0 = (c_y \omega_y^* + \dot{\omega}_y^*)/k. \quad (8)$$

Consequently, in steady state, control values depend on velocity and acceleration of the boat on the desired path. This conclusion corresponds to physical laws of body movement in viscous fluids one of which is water.

So, kinematics equations (1) and dynamic equations (4) ~ (7) describe movement of the boat according to the desired path. Equations (4) ~ (6) can be used to synthesize the necessary control. Let's suppose that variables $\varphi^* = \varphi^*(t)$, $\omega_y = \omega_y(t)$, $V_z = V_z(t)$ and $V_x = V_x(t)$ are measures be special sensors. As $\varphi^*(t)$ and $V^*(t)$ are given functions of time, then state variables $x_i = 0, i =$

$\overline{1, n}$ in Eq. (4) ~ (7) can be treated as measurable and can be used in synthesis of the system and control algorithms.

3 Development of control for the longitudinal velocity

Control equation $u_1 = F_T \cos\alpha$, defines modification of lateral component of the boat speed. It is included only in one equation (4). Thus, this equation can be considered as equation in JCF and control u_1 can be defined with respect to $x_1 = V_x - V_x^*$ and $\dot{x}_1 = -\lambda_1 x_1$ by the following expression:

$$u_1 = c_x|V_x|V_x + m_x \dot{V}_x^* - \lambda_1 m_x (V_x - V_x^*). \quad (9)$$

But this control equation for u_1 , does not provide astatism of the control system by velocity V_x , and even in steady state and at constant speed it will give some error. To zero the error, let's add integrator to the control device and temporary assign its equation to the model of the considered channel. For this u_1 should be equal to x_5 and $\dot{x}_5 = u_{1,vsp}$, where $u_{1,vsp}$ is an assistant control. Having so, equation of channel, controlling V_x velocity becomes:

$$\begin{aligned} \dot{x}_1 &= \frac{(-c_x|x_1 + V_x^*|(x_1 + V_x^*) + x_5 - m_x V_x^*)}{m_x} \\ &= \phi_1(x_1, x_5), \dot{x}_5 = u_{1,vsp} \end{aligned} \quad (10)$$

As $\partial \phi_1(x_1, x_5)/\partial x_5 = m_x^{-1}$, then Eq. (12) are in JCF when $\phi_1(x_1, x_5) = 1$. This allows to find the necessary equation using method of analytical synthesis for nonlinear control systems using JCF. Method is described in details in [5]. According to it, let's introduce new variables $w_1 = x_1, w_2 = \dot{w}_1 + \lambda_1 w_1$. In this case:

$$\begin{aligned} w_2 &= \dot{w}_1 + \lambda_1 w_1 = \\ &= -m_x^{-1}[c_x|x_1 + V_x^*|(x_1 + V_x^*) - x_5 + m_x \dot{V}_x^*] + \lambda_1 x_1 \end{aligned} \quad (10)$$

or

$$w_2 = \phi_1(x_1, x_5) + \lambda_1 x_1, \quad (11)$$

where

$$\phi_1(x_1, x_5) = -m_x^{-1}[c_x|x_1 + V_x^*|(x_1 + V_x^*) - x_5 + m_x \dot{V}_x^*]. \quad (12)$$

According to [5], in order to define control, it is required to get derivative with respect to time for variable w_2 . As this function directly depends on set of variables that are functions of time, then preliminary it is necessary to get partial derivative for these variables. In particular, w_2 depends on function $\overline{\phi_1}(x_1, x_2)$ (Eq. (12)). And the latter depends on variable $x_1 = x_1(t)$ and $V_x^* = V_x^*(t)$. Function $\overline{\phi_1}(x_1, x_2)$ has multiplication $|\zeta|\zeta$ that includes function modulus. Taking this into account, development of formula for derivative of this multiplication with ζ in explicit form will be:

$$\frac{d|\zeta|\zeta}{d\zeta} = \begin{cases} d\zeta^2/d\zeta, & \zeta > 0 \\ -d\zeta^2/d\zeta, & \zeta < 0 \end{cases} = \begin{cases} 2\zeta, & \zeta > 0 \\ -2\zeta, & \zeta < 0 \end{cases} = 2|\zeta| \quad (13)$$

Besides, left and right parts of the Eq. (10) depend not only upon state variables, x_1, x_2 , but also V_x^* and \dot{V}_x^* . If $V_x^* \neq const$, then in general case $V_x^* = V_x^*(t)$ and $\dot{V}_x^* = \dot{V}_x^*(t)$. So, following [5] control $u_{1,vsp}$ can be found using condition $\dot{w}_2 = -\lambda_2 w_2$. Taking derivative

with respect to time for Eq. (11) with respect to Eq. (12), (13) and (10) we get $\dot{w}_2 = -m_x^{-1}[2c_x|x_1 + V_x^*|(\varphi_1(x_1, x_5) + \dot{V}_x^*) - u_{1,vsp} + m_x \dot{V}_x^*] + \lambda_1 \varphi_1(x_1, x_5) = -\lambda_2 w_2$. And with regard to Eq. (11):

$$u_{1,vsp} = 2c_x|x_1 + V_x^*|(\varphi_1(x_1, x_5) + \dot{V}_x^*) - m_x[(\lambda_1 + \lambda_2)\varphi_1(x_1, x_5) + \lambda_1 \lambda_2 x_1] \quad (14)$$

As $u_1 = x_5$ and $x_1 = \dot{V}_x - V_x^*$, then from Eq. (10) and (14) the following formulas for the astatic boat control device and be produced:

$$\dot{x}_5 = 2c_x|x_1 + V_x^*|(\varphi_1(x_1, x_5) + \dot{V}_x^*) - m_x[(\lambda_1 + \lambda_2)\varphi_1(x_1, x_5) + \lambda_1 \lambda_2 x_1 - \dot{V}_x^*], u_1 = x_5. \quad (15)$$

In such a way, boat velocity control device is described by Eq. (15). Its output values gives F_T and α as $u_1 = F_T \cos \alpha$. Positive constants λ_1 and λ_2 in Eq. (15) define length and character of the transient process for boat velocity. When choosing these constants it is necessary to account the limitation for control modulus u_1 . Important to note that increase of constants λ_1 and λ_2 leads to shortening of the transition process but also increase the necessary $|u_1|$.

4 Development of course control

As it was shown above, changes in boat course are described by Eq. (5) and (6).

$$\dot{x}_2 = x_3 = \tilde{\varphi}_1(x_3), \quad (16)$$

$$\dot{x}_3 = \varphi_2(x_3) - kJ_y^{-1}u_2, \quad (17)$$

where $x_2 = \varphi - \varphi^*$, $x_3 = \omega_y - \omega_y^*$,

$$\varphi_2(x_3) = -J_y^{-1}c_y(x_3 + \omega_y^*) - \dot{\omega}_y^*. \quad (18)$$

In this case $d\tilde{\varphi}_1/dx_3 = 1 \neq 0$, so Eq. (16) and (17) are in JCF allowing to find control u_2 . In order to provide astaticism by the course, let's introduce integrator into the control device. If $u_2 = x_6$, then union of integrator equation $\dot{x}_6 = u_{2,vsp}$ with equations (16) and (17) would give the following course control equation (with respect to (17)):

$$\dot{x}_2 = x_3 = \tilde{\varphi}_1(x_3), \quad (19)$$

$$\dot{x}_3 = \varphi_2(x_3) - kJ_y^{-1}x_6 = \tilde{\varphi}_2(x_3, x_6), \quad (20)$$

$$\dot{x}_6 = u_{2,vsp} \quad (21)$$

Eq. (19) ~ (21) are in JCF and in order to get control equations the following additional variables are required:

$$\begin{aligned} \tilde{w}_1 = x_2, \tilde{w}_2 = \tilde{w}_1 + \lambda_5 \tilde{w}_1 = x_3 + \lambda_2 x_2, \tilde{w}_3 \\ = \tilde{w}_2 + \lambda_6 \tilde{w}_2 \\ = \tilde{\varphi}_2(x_3, x_6)(\lambda_5 + \lambda_6)x_3 \\ + \lambda_5 \lambda_6 x_2. \end{aligned} \quad (22)$$

Derivation of variable \tilde{w}_3 with respect to time and substitution of $(\dot{\tilde{w}}_3) = -\lambda_7 \tilde{w}_3$ with the account to Eq. (17) ~ (21) will produce:

$$k^{-1}J_y[(\lambda_5 + \lambda_6 + \lambda_7 - J_y^{-1}c_y)\tilde{\varphi}_2(x_3, x_6) - J_y^{-1}c_y\dot{\omega}_y^* - \ddot{\omega}_y^* + (\lambda_5\lambda_6 + \lambda_5\lambda_7 + \lambda_6\lambda_7)x_3 + \lambda_5\lambda_6\lambda_7x_2]. \quad (23)$$

Character of the transition processes by variables ω_y and φ depends on values of λ_5 , λ_6 , and λ_7 constants.

So, with account to Eq. (21) and Eq. (23), controlling device while calculating course and angular speed can be described by the following equation:

$$\begin{aligned} \dot{x}_6 = k^{-1}J[(\lambda_5 + \lambda_6 + \lambda_7 - J_y^{-1}c_y)\tilde{\varphi}_2(x_3, x_6) \\ - J_y^{-1}c_y\dot{\omega}_y^* - \ddot{\omega}_y^* \\ + (\lambda_5\lambda_6 + \lambda_5\lambda_7 + \lambda_6\lambda_7)x_3 \\ + \lambda_5\lambda_6\lambda_7x_2]. \end{aligned} \quad (24)$$

Equations (15) and (24) allows to find values u_1 and u_2 . Necessary values of the real control actions – total force F_T and rotational angle α can be found by the following equations:

$$\begin{aligned} F_T = \sqrt{u_1^2 + u_2^2}, \\ \alpha = \arctg\left(\frac{u_2}{u_1}\right). \end{aligned} \quad (25)$$

Details of the implementation of F_T and α with provision for design constraints are discussed in paper [4]. Presence of these limitations results in limitations in boat movement stability area. Analytical definition of this area is challenging, so in this paper authors research it by simulation in MATLAB.

5 Simulation of developed algorithms

The following boat values were used for the experiment: $m_x = 50$; $m_z = 50$; $J_y = 15.1$; $c_x = 15$; $c_z = 50$; $c_y = 30$; $k = 0.4$. Regulator parameters: $\lambda_1 = 1$; $\lambda_2 = 1$; $\lambda_5 = 1$; $\lambda_6 = 1$; $\lambda_7 = 1$.

For the first experiment, regulator equation (15) was set by the following required velocity parameters: $V_x^* = 1 \frac{m}{s}$, $\dot{V}_x^* = 0$, $\ddot{V}_x^* = 0$. Necessary angle in Eq. (19) ~ (24) $\varphi^* = 1 \frac{m}{s}$. No requirements were set for angular speed. Simulation results are presented at Fig. 2.

As you can see, boat velocity and orientation angle met the required values.

For the second experiment, required velocity was increased to 2 m/s. Limitation for the total force stayed the same $F_T^{max} = 50 N$. Simulation results are presented at Fig. 3.

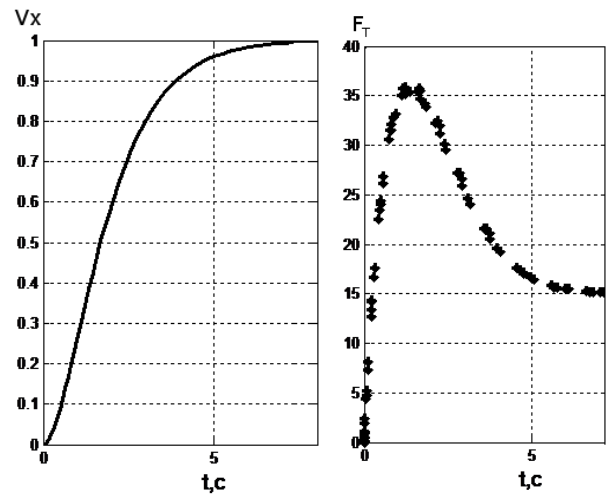


Figure 2. Results of the experiment 1: a) modification of boat velocity; b) rotational angle c) total force; d) modification rotational angle of movable frame.

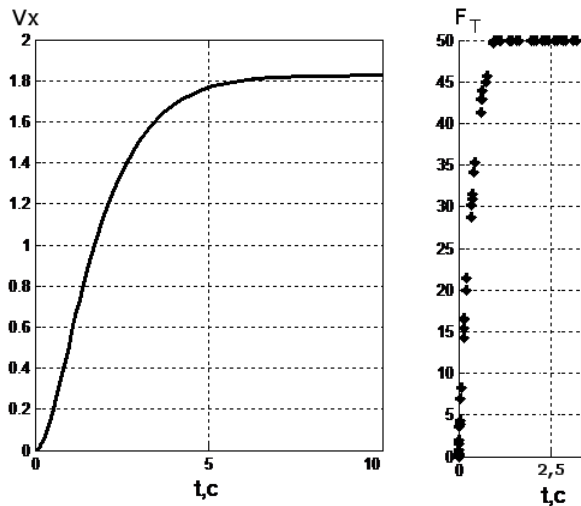


Figure 3. Results of the experiment 2: a) modification of boat velocity; b) rotational angle c) total force; d) modification rotational angle of movable frame

Charts at Fig. 3 show that boat did not meet the required velocity because of limitation for F_T . Requirement for the angle α were met correctly. Results for these two experiments prove efficiency of the proposed algorithms.

6 Conclusions

This paper was devoted to the development of the automatic control system for the motor boat. It is assumed, that boat is equipped by the measuring system that provides current coordinates, linear and angular velocities. Control system was created upon the boat mathematical model, described in paper [4]. To solve the problem, equation of control channels were transformed to Jordan manageable form that allowed to analytically find necessary controls. Research of the proposed control system according to boat design limitations for the values of control variables was performed by simulation in MATLAB. Now the next task is to perform the

experiment with a real boat. It will be necessary to develop hardware of the control system, specify measuring system and setup the equipment. The early prototype is planned for the spring 2016.

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References

1. S. Jacobsen, M. Jadud. *Mobile Robot Control*. 222p. (2006).
2. V. Pshikhopov, Y. Chernukhin, A Fedotov, V. Guzik, M. Medvedev, B. Gurenko & oth. *4th International Workshop on Computer Science and Engineering-Winter, WCSE 2014*, (2014).
3. V. Pshikhopov, M. Medvedev, B. Gurenko, *AMM*. **E**. 490-491, pp. 700-707, (2014), doi:10.4028/www.scientific.net/AMM.490-491.700.
4. B. Gurenko, Proc. of the Second Intl. Conf. on Advances In Mechanical and Robotics Engineering - AMRE 2014. pp. 84-87, (2014), doi:10.15224/978-1-63248-031-6-156.
5. A. Gaiduk, *Facta Universitatis-Series: Automatic Control and Robotics*, pp. 81-90, (2012).
6. A. Inzartsev, L. Kiseljev, M. Medvedev, A. Pavin, *Investigation, Motion Control. Vienna: InTech*, pp. 207–228, (2010).
7. V. Pshikhopov, Y. Chernukhin, V. Guzik, M. Medvedev, B. Gurenko, A. Piavchenko, R. Saprikin, V. Pereversev, V. Krukhmalev, *AMM*, **E** 701 - 702, pp. 704-710, (2015), doi: 10.4028/www.scientific.net/AMM.701-702.704