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# Model test on scale effect of the frequency decreases of the reinforce concrete beam due to moment cracks

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**Abstract.** For the experiment on the damage identification of reinforced concrete beams, Scaled model results are often used to investigate the damage identification of in-site structures based on the dynamics. However, the variation of dynamic characteristics with the increasing damage for concrete structures is nonlinear. It is necessary to check the reasonableness that test results with small scales are extended to prototype. In the paper, in order to investigate the relationship between scale and frequency reduction of the reinforced concrete beams due to moment cracks in model test, three different sizes of reinforced concrete beams are designed and made based on elasticity similitude law. Then, step loading method with 5 level is used to gain different damage states of beams, and natural frequencies of the beams in different damage states is measured by hammering test. The experimental results show that with the increase of damage states, the frequencies of the concrete beams are reduced, while the amplitude of reduction changes nonlinearly. The trend of the frequencies reduction of the different sizes beams is the same in the same damage states, but it has nothing to do with reduced-scale. As a consequence, scaled model test can only be used for the qualitative research of the change of dynamic characteristics of the damaged reduced-scale concrete beams.

## 1 INTRODUCTION

In recent years, the construction industry is rapidly developing. Reinforced concrete beam in various forms is the main bearing member of structures. However, structures will be the role of the surrounding environment, such as strong winds and ultra-load, leading structures with varying states of damage and strength decreases, causing casualties and economic loses[1]. Therefore it is necessary to inspect and maintain structures .With damage identification of concrete beams, the extent and location of damage can be diagnosed to timely avoid further damage of structure.

Hammering test, based on the vibration characteristic of reinforced concrete beam, uses force-hammer to hit (incentive) the structure, by means of fast Fourier transform (FFT) with pulse test theory and modal theory to quickly obtain the modal parameters of structures[2].That is easy to operate, especially suitable for field experiments[3]. S.A.Neild[4] collected the acceleration data at mid-span and added the Gabor transform, to obtain the amplitude-frequency curve of beam structure under different damage states.CHENG Xing[5] acquisited the acceleration of reinforced concrete beams through hammering test in different damage states. In combination with the method of time and frequency, the relation curve of the amplitude and frequency of the beam under damage states were got.WANG Li-Heng[6] obtained the relation curve of the amplitude and frequency of the damaged beam, adopted the appropriate window width in time-frequency analysis to summarize the nonlinear dynamic characteristics of the damaged reinforced concrete beams. Jian-Yue WANG [1] also gained the natural frequency of reinforced concrete beams under stochastic subspace identification (SSI) method .

There are some differences in the operation and data processing in literatures above, but hammering test are all used in dynamic measurements. And the results obtained are present in common: with the increases of the damage states of concrete beams, structural natural frequency decreases. However, beams in laboratory are usually designed on the similarity theory and constructed based on the actual bridge according to reduced-scale .The geometry of the model experiment satisfies a proportional relationship, but the influence factors of the frequency conversion which caused by reinforced concrete cracking varies a lot. Frequency change is not only effect by geometry related, but also by the depth, shape and characteristics of the contact surface of cracks and so on. It's worthy of study that the role of frequency variation caused by the damage of reduced scale reinforced concrete beams. In this paper, three reinforced concrete beams are designed based on the similarity theory, and are loaded at 5 levels with step loading method to gain the frequencies of different stages of damage, to investigate the scale effect of frequency change due to the extent of damage caused by concrete beam.

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## 2 Model test design

To study the natural frequency of structure system is equivalent to solve the free vibration equation of multi-degree of freedom system:

$$M\ddot{u} + Ku = 0 \quad (1)$$

Where, $\ddot{u}$ and $u$ is the acceleration and displacement vector of nodes, respectively. $M$ and $K$ is the structure mass matrix and stiffness matrix, respectively Consists of element mass matrix and element stiffness matrix which are available under the calculations below:

$$M_e = \int_v \rho N^T N dv, \quad K_e = \int_v B^T DB dv. \quad (2)$$

Where, $\rho$ is the mass density; $N$ is the shape function; $B$ is the strain matrix; $D$ is the elastic matrix, the function of elastic modulus $E$ and Poisson ratio.
From eq. (1) and eq. (2) it can be learned that to study the vibration characteristics of the structure is mainly to meet the elasticity similitude law that the inertia force of model and prototypes is similar to their elastic restoring force. That is, the proportion relation of the length $l$, the density $\rho$ and elastic modulus $E$ should be coordinated. Based on the dimensional analysis, regarding the length, density and elastic modulus as basic quantity, similar conditions can be established:

$$C_t^2 = C_l^2 C_\rho C_E^{-1} \quad (3)$$

Where, $C_t$, $C_l$, $C_\rho$ and $C_E$ represents the similarity coefficient of time between the prototypes and the models, geometric similarity constant, quality similarity constant and elastic modulus similarity constant, respectively. Among them, $C_l = l_m / l_p$, subscript $m$ and $p$ denote models and prototypes, respectively. Thus, frequency similar constant $C_\omega$ and modal similar constant $C_\phi$ are available as below:

$$C_\omega = \sqrt{C_E C_l^{-2} C_\rho^{-1}}, \quad C_\phi = C_l \quad (4)$$

Accordingly, in this paper the structure model with the same material to structure ,$C_\rho$=1,$C_E$=1. Thus, you can get similar constant frequency as $C_\omega$=1/ $C_l$ between the model and the prototype.

## 3 Testing process

This paper presents three concrete beams, using the 42.5-strength cement, particle size less than 1cm of gravel, medium sand. The size, reinforcement and mass of each beam are shown in Table 1.

Four dividing point loading method that the static loading schematic is shown in Fig.1,is used to load on the beams with size 3.12m, 4.16m, 5.20m ,whose multi-stage loading data is shown in Table 2, Table 3 and Table 4 respectively. The test order is shown as below:

(1) Hammering tests to determine the natural frequencies of the beam.

(2) Static loading with four-point step-loading way. Loading conditions of each concrete beams will be shown subsequently. In the experiment, each static loading level requires 30 minutes in order to ensure full performance of applied load. Dynamic test is conducted after unloading which needs 30 minutes to make sure the phenomenon of unloading manifested.

**Table 1** Size and reinforcement of concrete beams

| Number of the beams / properties | 1 | 2 | 3 |
|---|---|---|---|
| length [m] | 3.12 | 4.16 | 5.20 |
| width [mm] | 120 | 160 | 200 |
| height [mm] | 240 | 320 | 400 |
| tensile reinforcements | 2B12 | 2B16 | 2B20 |
| compression reinforcements | 2B12 | 2B16 | 2B20 |
| stirrups | A6@120 | A6@160 | A6@200 |
| mass[kg] | 250 | 560 | 1000 |

**Table 2** Beam's multi-stage loading data (3.12m beam)

| Load states | Jack load value[kN] | Load of loading point[kN] | Maximum bending moment of beam[kN.m] | Status description after loading |
|---|---|---|---|---|
| 1 | 6 | 3 | 3.12 | Cracks appear in mid-span pure bending segment |
| 2 | 12 | 6 | 6.24 | Crack expand |
| 3 | 18 | 9 | 9.36 | Crack expand |
| 4 | 24 | 12 | 12.48 | Crack coalesce |
| 5 | 27 | 13.5 | 14.04 | Crack coalesce, loading can't increase |

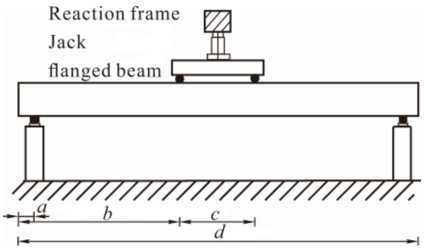
**Table 3** Beam's multi-stage loading data (4.16m beam)

| Load states | Jack load value[kN] | Load of loading point[kN] | Maximum bending moment of beam[kN.m] | Status description after loading |
|---|---|---|---|---|
| 1 | 18.6 | 9.3 | 12.90 | cracks appear in mid-span pure bending segment |
| 2 | 27.9 | 13.95 | 19.34 | Crack expand |
| 3 | 37.2 | 18.6 | 25.79 | Crack expand |
| 4 | 46.5 | 23.25 | 32.24 | Crack coalesce |
| 5 | 47.5 | 23.75 | 32.93 | Crack coalesce, loading can't increase |

**Table 4** Beam's multi-stage loading data (5.20m beam)

| Load states | Jack load value[kN] | Load of loading point[kN] | Maximum bending moment of beam[kN.m] | Status Description After loading |
|---|---|---|---|---|
| 1 | 14.2 | 7.1 | 11.83 | cracks appear in mid-span pure bending segment |
| 2 | 28.4 | 14.2 | 23.67 | Crack expand |
| 3 | 42.6 | 21.3 | 35.50 | Crack expand |
| 4 | 56.8 | 28.4 | 47.33 | Crack coalesce |
| 5 | 71 | 35.5 | 59.17 | Crack coalesce, loading can't increase |

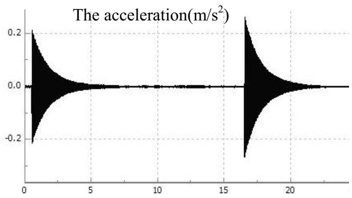
Using the built-in spectrum analysis function of the software called DASP, you can easily get the structure's amplitude-frequency characteristic curve. The natural frequencies can be obtained as the peak point of the curve, damping values can also be obtained, as shown in Fig.2.

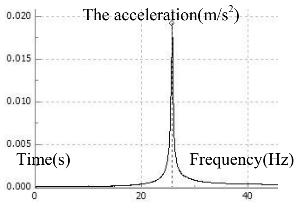



(a)static loading schematic

(b)Static loading experimental figure

**Fig.1** Beam's static loading




(a)The time domain analysis result of acceleration time history

(b)The frequency domain analysis result of acceleration time history

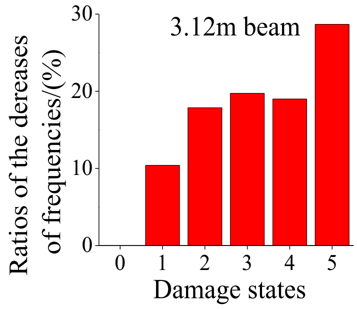
**Fig.2** The time and the frequency domain analysis result of acceleration time history

## 4 Scale Effect Analysis

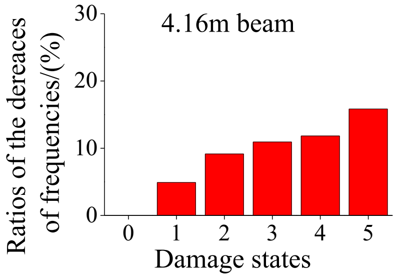
Table 5 shows the frequency variation obtained by the hammering test under different states of damage .It can be seen from the table that with the damage level increases, the trend of structure's natural frequency resulting from hammering test overall is reduced.

**Table 5** natural frequencies under different Damage levels (hammering test)

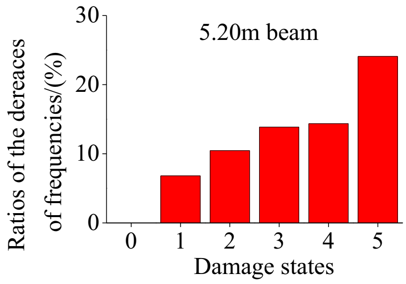
| damage states | 3.12m beam | 4.16m beam | 5.20m beam |
|---|---|---|---|
| 0 | 37.5 | 28 | 25.6875 |
| 1 | 36.8 | 27.125 | 23.9375 |
| 2 | 33.6 | 26.625 | 23 |
| 3 | 30.8 | 25.4375 | 22.125 |
| 4 | 30.1 | 24.9375 | 22 |
| 5 | 30.375 | 24.6875 | 19.5 |
| 6 | 26.75 | 23.5625 | — |




(a) 3.12m beam




(b) 4.16m beam




(c) 5.20m beam

**Fig.3** The reduction ratios of natural frequencies along with different damage states

The reduced proportion of the frequency is defined as follows:

$$D = (F_0 - F_x) / F_0 \times 100\% . \qquad (5)$$

Where, $D$ is the reducing proportion of frequencies, $F_0$ is the natural frequency obtained by hammering test under no damage, $F_x$ is the natural frequency of a damage level. Figure 4 illustrate the reduced ratio of natural frequencies of three beams that with the damage level increases.

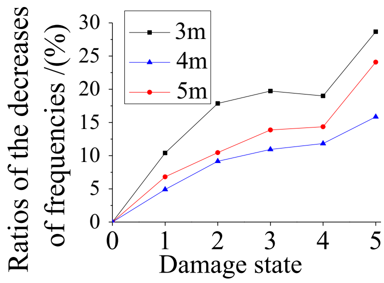



**Fig.4** Comparison of the ratios of the decreases of frequencies in different damage states

From the Fig.3 it can be told that as the damage level increases, the frequency change trend overall is reduced, while in the experiments there are some special points, such as compared with the fourth level damage level of 3.12m,the natural frequency in the fifth is not reduced.

The relationship between the reduced ratio of the natural frequency of 3 reinforced concrete beams and the damage states can be tell in the Fig.4, defining the fifth damage level is destroyed completely. According to the figure, the ratio of the reduced frequency increases with the damage level increases. By comparing the relative relationship of the 3 beams' frequencies, when the damage states of the beam in different lengths increases, the ratio of the frequency decreased has nothing to do with length. It is indicating that the frequency change of concrete beams with different lengths in different damage states has no scale effect.

## 5 Conclusion

Through static loading and hammering test on three different lengths and sizes of reinforced concrete beams carried out alternately, the natural frequencies of reinforced concrete beams under different damage state are obtained to comparative analyze the ratio of the decreases of frequency with the damage state of reinforced concrete beams deepening. Experimental results show that:

(1) At the same state of damage, the trend of change on frequency reducing of different sizes beams are the same, but it has no connection with reduced-scale.

(2) The change of dynamic characteristics of the reduced-scale reinforced concrete beams caused by the damage has no significant scale effect, and it can be used for qualitative research.

## 6 Acknowledgements

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