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Modeling and Simulation of Adaptive Control for a Class of Nonlinear System

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Abstract. This paper investigates adaptive control for a class of nonlinear system with uncertain parameters and unknown disturbances. We propose method of improving performance of the controller via modeling and simulation analysis. The method is achieved under different simulation circumstances. It is shown that the global asymptotical stability of closed-loop system can be guaranteed by nonlinear adaptive robust design. The simulation results demonstrate the effectiveness of the presented method.

1 Introduction

A lot of the controlled objects are nonlinear systems with uncertain parameters and unknown disturbances. Linear methods, such as passivity-based control and feedback linearization control, maybe decrease the accuracy of the controller, and even cause deterioration of the system [1, 2]. Because of its significant advantages, adaptive control has become an important tool for study nonlinear systems with uncertain parameters and unknown disturbances. The recent decades has witnessed many achievements in the research of adaptive control. Among them, perhaps the most significant one is the development of a novel adaptive control named backstepping proposed by Kanellakopoulos I., Kokotovic P. V., and Morse A.S. [3]. In the process of controller design, assume that the system is stable in the Lyapunov sense, and then reversely deduce control variable. In recent ten years, backstepping method has been improved and extended constantly [4-6]. Now this method can be used to design the adaptive controller of nonlinear system with multiple uncertain parameters and complex unknown disturbances.

Backstepping based adaptive control requires that the system can be transformed into parametric-strict-feedback form through diffeomorphism transform [7-9]. We will consider the following nonlinear system.

$$\begin{cases} \dot{x}_1 = x_2 + \theta_1 f_1(x_1) \\ \dot{x}_2 = x_3 + \theta_2 f_2(x_1, x_2) \\ \quad \vdots \\ \dot{x}_{n-1} = x_n + \theta_{n-1} f_n(x_1, x_2, \cdots, x_{n-1}) \\ \dot{x}_n = u + \theta_n f_n(x_1, x_2, \cdots, x_n) + \eta \end{cases} \tag{1}$$

where $x := [x_1, x_2, ..., x_n]^T \in \mathbb{R}^n$ is the state, θ_i ($1 \le i \le n$) is the unknown parameter of the i-th derivative equation, u is the control variable of the system and η is a bounded unknown disturbance. The nonlinear functions $f_i(x_1, x_2 \cdots x_i)$, $1 \le i \le n$ are known and smooth and satisfy $f_1(0) = f_2(0,0) = \cdots f_n(0,...,0) = 0$.

2 Adaptive controller design

In the process of controller design, we only consider adding a derivative equation in every step, then deducing parameter update rate and continuing algorithm by defining virtual control.

The design process consists of n steps. At the i-th step, ($1 \le i \le n-1$), we design α_i as the virtual control of x_{i+1}. At the n-th step, the actual control u appears which completes the design.

Step one: Define $\xi_1 = x_1$. Since we know that $f_i(x_1, x_2 \cdots x_i)$ ($1 \le i \le n$) is a function of (x_1 , x_2 , ⋯, x_i) and x_i is a function of ξ_i , so $f_i(x_1, x_2 \cdots x_i)$ can be written as $w_i(\xi_1, \xi_2 \cdots \xi_i)$. And then we design the virtual control of x_2 as follows:

$$\alpha_1 = -C_1 \xi_1 - \hat{\theta}_1 w_1(\xi_1) , \tag{2}$$

where C_1 is an undetermined positive constant, $\hat{\theta}_1$ is the parameter estimation variable of unknown parameter θ_1. Then define the output error

$$\xi_2 = x_2 - \alpha_1 . \tag{3}$$

It can be obtained from (2) and (3) that

$$\dot{\xi}_1 = (\xi_2 + \alpha_1) + \theta_1 w_1(\xi_1) . \tag{4}$$

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Select Lyapunov function of step one

$$V_1 = 0.5\xi_1^2 + 0.5(\theta_1 - \hat{\theta}_1)^2, \quad (5)$$

and its time-derivative, computed with (3) and (4) is given by

$$\begin{aligned}\dot{V}_1 &= \xi_1\dot{\xi}_1 - (\theta_1 - \hat{\theta}_1)\dot{\hat{\theta}}_1 \\ &= \xi_1\xi_2 - C_1\xi_1^2 + (\theta_1 - \hat{\theta}_1)w_1(\xi_1)\xi_1 - (\theta_1 - \hat{\theta}_1)\dot{\hat{\theta}}_1. \end{aligned} \quad (6)$$

We can obtain parameter update rate from (6) that

$$\dot{\hat{\theta}}_1 = w_1(\xi_1)\xi_1. \quad (7)$$

Step two: The time derivative of ξ_2 is given by

$$\begin{aligned}\dot{\xi}_2 &= x_3 + \theta_2 f_2(x_1, x_2) - \frac{\partial\alpha_1}{\partial x_1}\dot{x}_1 \\ &= x_3 + \theta_2 f_2(x_1, x_2) - \dot{x}_1(-C_1 - \frac{\partial(\hat{\theta}_1 w_1)}{\partial x_1}). \end{aligned} \quad (8)$$

Define the virtual control of x_3 as below:

$$\alpha_2 = -C_2\xi_2 - \xi_1 - \hat{\theta}_2 f_2(x_1, x_2) + \dot{x}_1(-C_1 - \frac{\partial(\hat{\theta}_1 w_1)}{\partial x_1}), \quad (9)$$

where C_2 is an undetermined positive constant, $\hat{\theta}_2$ is the parameter estimation variable of unknown parameter θ_2. Select Lyapunov function of step two as follows:

$$V_2 = 0.5\xi_2^2 + 0.5(\theta_2 - \hat{\theta}_2)^2, \quad (10)$$

and its time-derivative, computed with (1), (8) and (9) is given by

$$\begin{aligned}\dot{V}_2 = &-C_2\xi_2^2 - \xi_1\xi_2 + \xi_3\xi_2 + \\ &(\theta_2 - \hat{\theta}_2)w_2(\xi_1, \xi_2)\xi_2 - (\theta_2 - \hat{\theta}_2)\dot{\hat{\theta}}_2. \end{aligned} \quad (11)$$

We can obtain the second parameter update rate from (11) that

$$\dot{\hat{\theta}}_2 = w_2(\xi_1, \xi_2)\xi_2. \quad (12)$$

Step i ($3 \le i \le n-1$): As the same as step two, it can be obtained that

$$\begin{aligned}\dot{\xi}_i &= \dot{x}_i - \dot{\alpha}_{i-1} \\ &= x_{i+1} + \theta_i f_i(x_1, x_2 \cdots x_i) - \sum_{j=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_{j-1}} \cdot \dot{x}_{j-1}. \end{aligned} \quad (13)$$

Define the virtual control

$$\alpha_i = -C_i\xi_i - \xi_{i-1} - \hat{\theta}_i f_i(x_1, x_2, \cdots x_n) + \sum_{j=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_{j-1}}\dot{x}_{j-1}, \quad (14)$$

where C_i is an undetermined positive constant, $\hat{\theta}_i$ is the parameter estimation variable of unknown parameter θ_i. Select Lyapunov function $V_i = 0.5\xi_i^2 + 0.5(\theta_i - \hat{\theta}_i)^2$, and its time-derivative can be written as below:

$$\begin{aligned}\dot{V}_i = &-C_i\xi_i^2 - \xi_{i-1}\xi_i + \xi_i\xi_{i+1} + \\ &(\theta_i - \hat{\theta}_i)w_i(\xi_1, \xi_2, \cdots, \xi_i)\xi_i - (\theta_i - \hat{\theta}_i)\dot{\hat{\theta}}_i \end{aligned}. \quad (15)$$

According to (15), parameter update rate is

$$\dot{\hat{\theta}}_i = w_i(\xi_1, \xi_2, \cdots, \xi_i)\xi_i. \quad (16)$$

Step n: The time derivative of ξ_n can be written as

$$\begin{aligned}\dot{\xi}_i &= \dot{x}_n - \dot{\alpha}_{n-1} \\ &= u + \theta_n f_n(x_1, x_2 \cdots x_n) - \sum_{j=1}^{n}\frac{\partial\alpha_{n-1}}{\partial x_j} \cdot \dot{x}_j. \end{aligned} \quad (17)$$

Define

$$\begin{aligned}u = &-C_n\xi_n - \xi_{n-1} - \\ &\hat{\theta}_n f_n(x_1, x_2 \cdots x_n) + \sum_{j=1}^{n}\frac{\partial\alpha_{n-1}}{\partial x_j} \cdot \dot{x}_j - \eta^*, \end{aligned} \quad (18)$$

where C_n is an undetermined positive constant, $\hat{\theta}_n$ is the parameter estimation variable of unknown parameter θ_n. And select Lyapunov function $V_n = 0.5\xi_n^2 + 0.5(\theta_n - \hat{\theta}_n)^2$. The time derivative of V_n can be written as

$$\begin{aligned}\dot{V}_n = &-C_n\xi_n^2 - \xi_{n-1}\xi_n + (\theta_n - \hat{\theta}_n)w_n(\xi_1, \xi_2, \cdots, \xi_n)\xi_n - \\ &(\theta_n - \hat{\theta}_n)\dot{\hat{\theta}}_n + (\eta - \eta^*)\xi_n \end{aligned}, \quad (19)$$

where η^* is an undetermined function. According to (19), parameter update rate is

$$\dot{\hat{\theta}}_n = w_n(\xi_1, \xi_2, \cdots, \xi_n)\xi_n. \quad (20)$$

Select Lyapunov function of the whole system

$$V = \sum_{j=1}^{n} V_i, \quad (21)$$

and its time-derivative, computed with (6), (11), (15) and (19) is given by

$$\dot{V} = -\sum_{i=1}^{n} C_i\xi_i^2 + (\eta - \eta^*)\xi_n. \quad (22)$$

If the system is stable in the Lyapunov sense, the time derivative of V should be less than or equal to zero. The design of our adaptive stabilizer takes as the same method as in [10], that is, we choose

$$\eta^* = |\eta|_{\max} sign\xi_n. \quad (23)$$

Because $V \ge 0$ and $\dot{V} \le 0$, the system is stable in the Lyapunov sense. Further, $\dot{V}$ is a semi-negative definite function and $\ddot{V}$ is a continuous function of ξ_i ($1 \le i \le n$), so it can be proved by Barbalat lemma that: $\xi_i \to 0$, that's to say $x_i \to 0$, as $t \to \infty$. The system is global asymptotically stable [11-13].

3 Simulation illustrations

Consider the following nonlinearly parametrized parametric-strict-feedback system:

$$\begin{cases} \dot{x}_1 = x_2 - \theta x_1^2 \\ \dot{x}_2 = u + 2 + x_1 x_2 + \eta \end{cases}. \tag{24}$$

We will compare dynamic performance of three kinds of nonlinear system by using Matlab/Simulink.

The first one is a nonlinear system without uncertain factors. The controller design and simulation are carried out with the choice: $\theta = 1$, $\eta = 0$. Define $\xi_1 = x_1$ and virtual control

$$\alpha_1 = -C_1 \xi_1 + x_1^2, \tag{25}$$

so the error variable

$$\xi_2 = x_2 + C_1 \xi_1 - x_1^2. \tag{26}$$

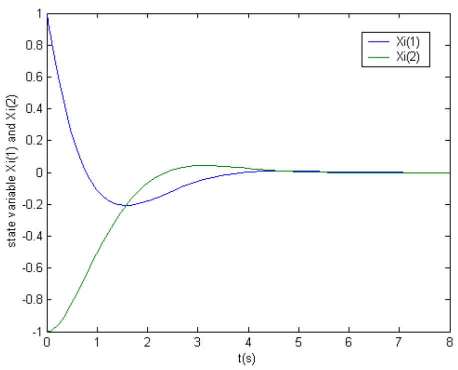
The time derivative of the error variable can be written as follows:

$$\dot{\xi}_2 = u + 2 + x_1 x_2 + (C_1 - 2\xi_1)\dot{\xi}_1. \tag{27}$$

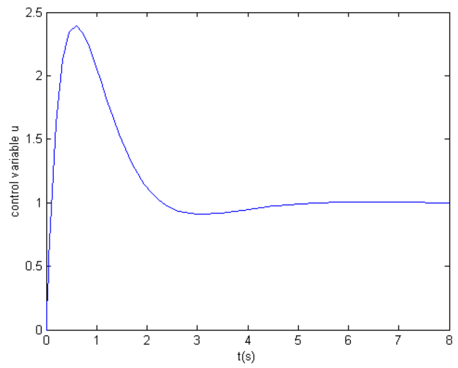
According to the algorithm of section 2, state equations and control equation are given by

$$\begin{cases} \dot{\xi}_1 = \xi_2 - \xi_1 \\ \dot{\xi}_2 = -\xi_2 - \xi \\ u = -2\xi_2 - 2 - \xi_1^3 + \xi_1 \xi_2 - \xi_1^2 \end{cases}. \tag{28}$$

In the process of design, we set undetermined positive constants as $C_1 = 1$. The simulation results are shown in Figure 1. It can be seen from the simulation results that the designed controller is globally asymptotically stable.



(a) State variables.



(b) Control input.

Figure 1. Simulation results without uncertain factors.

The second kind is a nonlinear system with uncertain parameters. Assume that the uncertain parameter $\theta = 1$ and the bounded unknown disturbances $\eta = 0$. Define $\xi_1 = x_1$ and virtual control

$$\alpha_1 = -C_1 \xi_1 + \hat{\theta} x_1^2, \tag{29}$$

where $C_1 = 1$. As the same as previous algorithm, we define error variable to complete the design of the first-order system and continue algorithm. At last, design the control variable:

$$u = -2 - 2\xi_2 + (2\hat{\theta} - 1)\xi_1 \xi_2 + (1 - 3\hat{\theta} + \hat{\hat{\theta}})\xi_1^2 + (2\hat{\theta}^2 - \hat{\theta} - 2\hat{\theta}\hat{\hat{\theta}})\xi_1^3. \tag{30}$$

State equations and observer equations are given by

$$\begin{cases} \dot{\xi}_1 = \xi_2 - \xi_1 - (\theta - \hat{\theta})\xi_1^2 \\ \dot{\xi}_2 = -\xi_2 - \xi_1 - (\theta - \hat{\hat{\theta}})(1 - 2\hat{\theta}\xi_1)\xi_1^2 \\ \dot{\hat{\theta}} = -\xi_1^3 \\ \dot{\hat{\hat{\theta}}} = \xi_1^2 \xi_2 - 2\hat{\theta}\xi_1^3 \xi_2 \end{cases}. \tag{31}$$

The simulation results of state variables and parameter estimation variables are shown in Figure 2. The simulation curves of state variables are substantially unchanged.

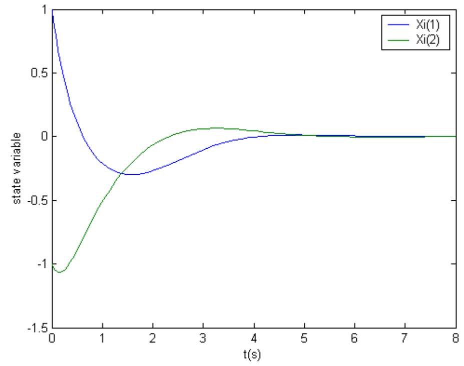
The third one is a nonlinear system with uncertain parameter $\theta = 1$ and unknown bounded disturbances. Equation (30) is modified as

$$u = -2 - 2\xi_2 + (2\hat{\theta} - 1)\xi_1 \xi_2 - \eta^* + (1 - 3\hat{\theta} + \hat{\hat{\theta}})\xi_1^2 + (2\hat{\theta}^2 - \hat{\theta} - 2\hat{\theta}\hat{\hat{\theta}})\xi_1^3. \tag{32}$$

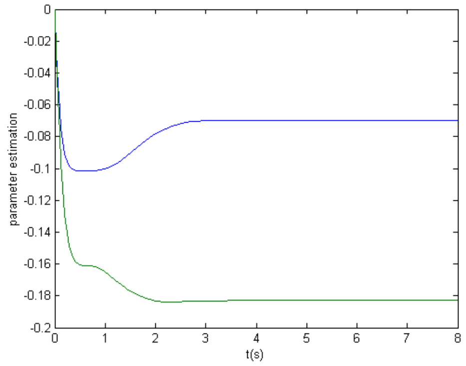
State equations and observer equations are given by

$$\begin{cases} \dot{\xi}_1 = \xi_2 - \xi_1 - (\theta - \hat{\theta})\xi_1^2 \\ \dot{\xi}_2 = -\xi_2 - \xi - (\theta - \hat{\hat{\theta}})(1 - 2\hat{\theta}\xi_1)\xi_1^2 + (\eta - \eta^*) \\ \dot{\hat{\theta}} = -\xi_1^3 \\ \dot{\hat{\hat{\theta}}} = \xi_1^2 \xi_2 - 2\hat{\theta}\xi_1^3 \xi_2 \end{cases}. \tag{33}$$

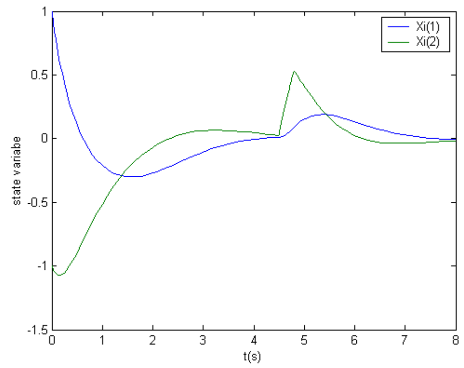
Now we research relationships of system performance and the designed η^*. Assumed that η is a single pulse. Since Lyapunov's second method is just the sufficient condition for stability of nonlinear systems [14], it is not necessary for the system stability of that the time derivative of Lyapunov function is less than or equal to zero. If $\eta^* = 0$, as shown in Figure 3, the unknown disturbance would cause the oscillation of the system, and even lead to the collapse of the system.



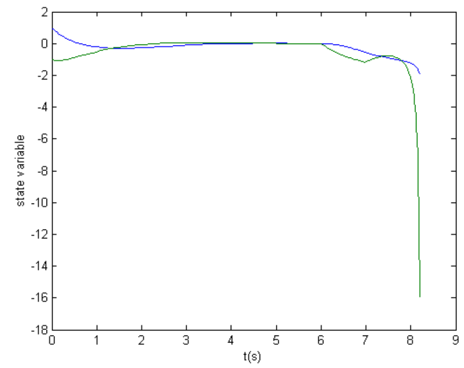
(a) State variables.



(b) Parameter estimation variables.
Figure 2. Simulation results with uncertain parameters.

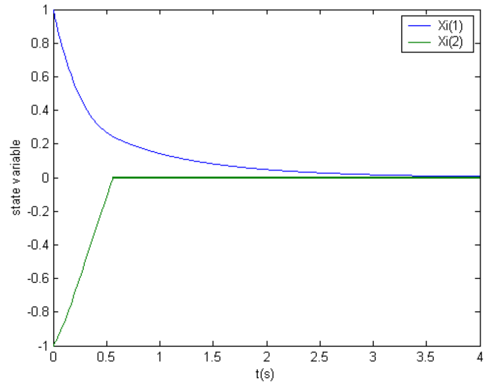


(a) State variables. (positive single pulse appears from 4.2s to 4.5s.)

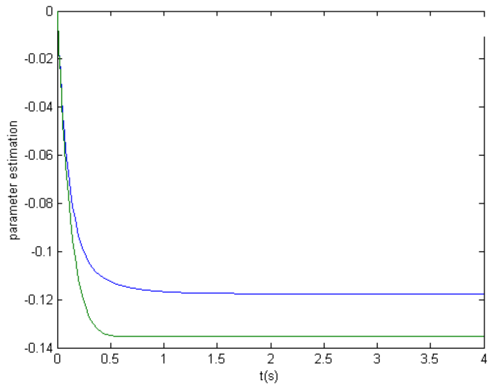


(b) State variables. (negative single pulse appears from 6 to 7s.)
Figure 3. The oscillation or collapse of the system.

In order to solve this problem, we introduce advanced algorithms of robust control. In the process of controller design, select $\eta^* = |\eta|_{\max} sign(x_n)$ to ensure that the system is stable in the Lyapunov sense. Whether the disturbance is a single pulse or continuous harmonics, as shown in Fig.4, global stabilization can be achieved both. It can be seen from Figure 4 that the design controller has strong robustness and the system has a strong anti-jamming capability. Compared Figure 4 with Figure 2, adjustment time of state variables and parameter estimations are significantly reduced.



(a) State variables.



(b) Parameter estimation variables.
Figure 4. Simulation results with uncertain parameters and unknown bounded disturbances.

In view of simulation results of Figure 4, we assume $\eta^* = b(x) sign(x_n)$ and $0 < b(x) \le |\eta|_{\max}$. Function $b(x)$ affects adjustment time and anti-jamming capability of the system. The relationship between $b(x)$ and system performance is shown as follows:

Adjustment time of state variables and parameter estimations will reduce with the increasing of $b(x)$.

The system can return to the original equilibrium state if it is independent of the design of η^*. The amplitude and adjustment time of oscillation caused by disturbances can be decreased when we increase the function $b(x)$.

When $b(x) = |\eta|_{\max}$, the system will not oscillate and can maintain the original equilibrium state immediately.

4 Conclusions

In this paper, we research the design of adaptive controller for nonlinear system and complete the following work: We design adaptive controller of N-order nonlinear system with parameter-strict-feedback form, considering unknown parameters and unknown bounded disturbances. We complete the simulation experiment of different circumstances and analyze the simulation results. We present a method of improving the performance of the system.

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