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Study On The Algorithm Of Multifunctional Bone External Fixator

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Abstract. This paper discussed the problem that parameters of bone external fixator are difficult to calculate in the practical application. Positive solution is described in detail. We used MATLAB software to make simulation experiment. The innovation lies the development idea of using inverse position to verify the accuracy of positive solution. Forward displacement analysis was mainly developed using analytical method, which has many advantages, such as small dependence to the inverse solution and the higher precision. With further software development, we will have the algorithms; research model and interface program connected and form high precision smart compliance multifunctional bone external fixator products, which will greatly enhance the overall level of bone external fixation technology and clinical application treatment.

Keywords: bone external fixator, parallel mechanism, positive solution, inverse solution.

1 Introduction

In order to increase the level of clinical application of the bone external fixator, this title discuss the problem of the solution in calculating parameters in the background of related software of bone external fixator which has self-property right.

Bone external fixator uses the Taylor structure in practical application. It has six flexible studdles that connected to the stationary hoop. And it can rotate freely in the joint. When adjusting the length of the studdle, the relative location of the two hoops changes. This Taylor structure can regard as Stewart platform[1-2]. Therefore, this study can regard as the inverse solution and positive solution of the Stewart platform[3-5].Fig.1 shows the application of Stewart platform in femur correction.

Fig 1. The application of Taylor

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2 Modeling

In order to describe Stewart platform which has six axis, we must build its mathematical model. Firstly, build two coordinate systems: the base coordinate system {B} (the origin is in the ob which is the center of the base)and the upper platform coordinate system {P}(the origin is in the op which is the center of the platform),as shown in Fig.2.

In the upper platform coordinate system ,the ZP axis point to the platform's upside , the YP axis and the connecting line of p3 and p4 are vertical. The angle of p4 and p5 is defined as $\theta_p = 15^\circ$, and the other points of junction distribute symmetrically. The angle of p1 and p3 is equal to the angle of p3 and p5,It is 120°,as shown in Fig.2 and Fig.3.In the base coordinate system ,the Zb axis point to the stationary platform's upside , the Yb axis and the connecting line of B3 and B4 are vertical. The angle of B3 and B4 is defined as $\theta_b = 15^\circ$,and the other points of junction distribute symmetrically. The angle of B1 and B3 is equal to the angle of B4 and B6,It is 120°,as shown in Fig.4.

Secondly, define the angle of the Pi axis and the XP axis as λ_i and define the angle of the Bi axis and the Xb axis as Λ_i ,(i=1,2,…,6).we can get some relationship:

$$\Lambda_1=330^\circ-\frac{\theta_b}{2},\quad \Lambda_2=330^\circ+\frac{\theta_b}{2},$$

$$\Lambda_3=90^\circ-\frac{\theta_b}{2},\quad \Lambda_4=90^\circ+\frac{\theta_b}{2},$$

$$\Lambda_5=210^\circ-\frac{\theta_b}{2},\quad \Lambda_6=210^\circ+\frac{\theta_b}{2},$$

$$\lambda_1=270^\circ+\frac{\theta_p}{2},\quad \lambda_2=30^\circ-\frac{\theta_p}{2},$$

$$\lambda_3=30^\circ+\frac{\theta_p}{2},\quad \lambda_4=150^\circ-\frac{\theta_p}{2},$$

$$\lambda_5=150^\circ+\frac{\theta_p}{2},\quad \lambda_6=270^\circ-\frac{\theta_p}{2}.$$

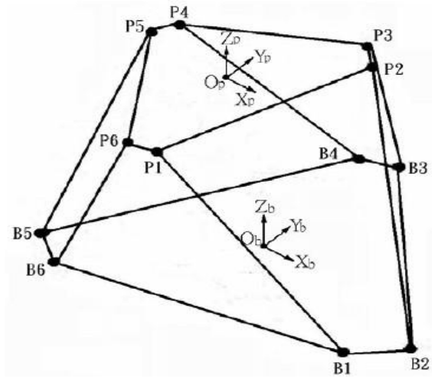

Fig 2. The define of the coordinate.

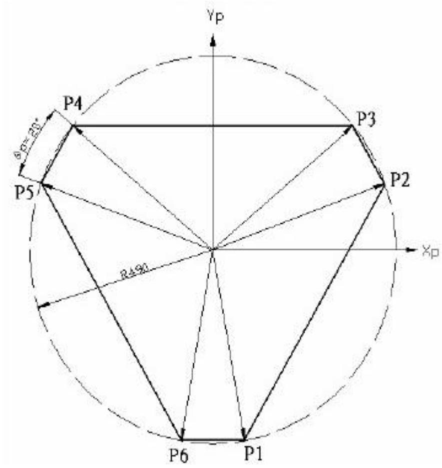

Fig 3. The joint angle of the upper.

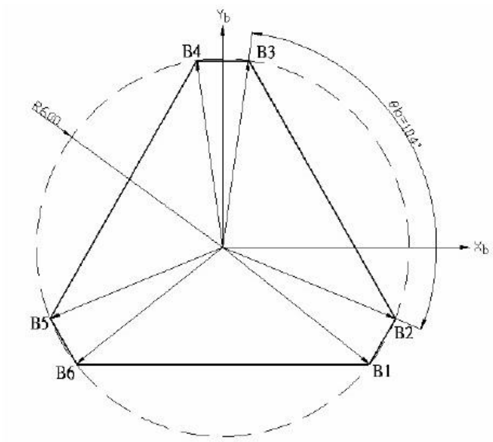

Fig 4. The joint angle of the lower.

Thirdly, define the vector of the point P_i which is in the {P} system as ${}^P p_i=(p_{ix}\ p_{iy}\ p_{iz})^T$ and define the vector of the point B_i which is in the {B} system as ${}^{B}b_i$=$(b_{ix}\ b_{iy}\ b_{iz})^T$.Then:

$${}^{P}p_i=[r_p\cos(\lambda_i)\ \ r_p\sin(\lambda_i)\ \ 0]^T \quad (1)$$

$${}^{B}b_i=[r_b\cos(\Lambda_i)\ \ r_b\sin(\Lambda_i)\ \ 0]^T \quad (2)$$

r_p is the platforms circumradius and r_b is the base's circumradius . The last, we define two working spaces: Cartesian Space: $[x\ y\ z\ \alpha\ \beta\ \gamma]^T$; Joint Space: $[S_1\ S_2\ S_3\ S_4\ S_5\ S_6]^T$.

$[x\ \ y\ \ z]^T$ is the location vector of the upper platform; $[x\ \ y\ \ z]^T$ is the attitude angle vector of the upper platform; $[S_1\ S_2\ S_3\ S_4\ S_5\ S_6]^T$ is the length vector of the six studdles. Till then, the mathematical model of the parallel structure is done.

3 Positive solution algorithm

When knows the six inputs, we just use the analytical method to solve. Here we establish the upper platform's coordinate system: $O-XYZ$ and establish the lower platforms coordinate system: $O-XYZ$ .They have the relationship:

$$X=[T]X'+P \quad (3)$$

Among them:

$$[T]=\begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix},$$

$$P=\{X_P \quad Y_P \quad Z_P\}^T$$

In the eq.(3), $|T|$ is the direction cosine matrix of the upper; P is the position vector of the upper's center in the fixed-coordinate system. With the spatial geometric relationship of the parallel mechanism, when know the sizes, we can solve the coordinate value of each hinge point of the moving platform in the $O-XYZ$ system. Then the vector of the driver's length in the fixed-coordinate system is:

$$l_i=(b_i-B_i)\,,\qquad i=1\,,\,2\,...\quad (4)$$

The equation of the inverse solution of the parallel mechanism is as follows:

$$l_i=\sqrt{l_{ix}^2+l_{iy}^2+l_{iz}^2}\,,\quad i=1,2.......6 \quad (5)$$

Divide the six independent equations in eqn.(5) into three part: $\left(l_1^1\ \ l_6^2\right)$, $\left(l_2^2\ \ l_5^2\right)$ and $\left(l_3^2\ \ l_4^2\right)$,and do the addition and subtraction about them, and simplify ,we get six independent equations in other form. And by using some constraint equations and relationships we can get :

$$\left.\begin{aligned} F_1 &= (l_3^2+l_4^2)/2-(R^2+R_0^2)-W \\ &\quad -2Y_pB_{3Y}+2(d_{11}b_{3X}'B_{3X}+d_{22}b_{3Y}'B_{3Y}) \\ &\quad -2b_{3Y}'(d_{12}X_P+d_{22}Y_P+d_{32}Z_P)=0 \\ F_2 &= (l_3^2-l_4^2)/4+X_PB_{2X}+d_{21}b_{3X}'B_{3Y}+ \\ &\quad d_{12}b_{3X}'B_{3Y}-b_{3X}'(d_{11}X_P+d_{21}Y_P+d_{31}Z_P) \\ &\quad =0 \\ F_3 &= d_{11}d_{12}+d_{21}d_{22}+d_{31}d_{32}=0 \end{aligned}\right\} \quad (6)$$

Solve the equations with the optimization theory. Its objective function is composed by least square principle.

It is:

$$S(XYZ)=\sum_{j=1}^{3}F_i^2(X,Y,Z) \quad (7)$$

4 Simulation result

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As shown in Table 1, the relative error is small. It can reach the level of 10^{-2}. The iterative calculation lead to the relative error. It is within the acceptable level.

Table 1. Experimental data

No	Inputs (length)	Positive Outputs (pose)	Relative error
1	$\begin{bmatrix}150\\167\\155\\165\\157\\160\end{bmatrix}$	$\begin{bmatrix}9.11\\4.63\\105.68\\-0.0978\\-0.0049\\-0.0534\end{bmatrix}$	$\begin{bmatrix}0.01\%\\-0.20\%\\-0.21\%\\0.14\%\\0.20\%\\0.06\%\end{bmatrix}$
2	$\begin{bmatrix}124\\112\\103\\126\\123\\137\end{bmatrix}$	$\begin{bmatrix}2.36\\0.3\\105.06\\0.1096\\0.1124\\-0.1396\end{bmatrix}$	$\begin{bmatrix}0.18\%\\2.10\%\\2.69\%\\-1.44\%\\-2.17\%\\-0.64\%\end{bmatrix}$
3	$\begin{bmatrix}127\\116\\120\\124\\127\\113\end{bmatrix}$	$\begin{bmatrix}-8.78\\-0.2\\105.21\\-0.0989\\-0.0989\\-0.0209\end{bmatrix}$	$\begin{bmatrix}3.14\%\\6.38\%\\4.23\%\\-7.32\%\\-7.11\%\\1.72\%\end{bmatrix}$
4	$\begin{bmatrix}117\\116\\122\\130\\116\\129\end{bmatrix}$	$\begin{bmatrix}5.24\\-13.14\\105.23\\0.0983\\0.0408\\-0.0291\end{bmatrix}$	$\begin{bmatrix}0.69\%\\4.40\%\\-4.68\%\\-4.26\%\\4.15\%\\0.69\%\end{bmatrix}$
5	$\begin{bmatrix}129\\144\\125\\75\\116\\117\end{bmatrix}$	$\begin{bmatrix}4.9389\\30.7393\\96.9103\\-0.0738\\-0.3654\\-0.1884\end{bmatrix}$	$\begin{bmatrix}-0.91\%\\0.38\%\\0.23\%\\0.97\%\\-0.02\%\\-0.50\%\end{bmatrix}$

5 Summary

At the present stage, the technology of bone fixing in the external has been the important part in the field of bone's cure. This study is working on solving the problem that the parameters of bone external fixator are difficult to calculate in the practical application.

In the study of the Stewart platform, the analysis of positive solution is the basement of its application. The paper study mainly study the forward displacement analysis used analytical method that with many advantages, such as programming faster, calculate easier, higher precision, especially the initial value calculatio-n is more easier to get. We can get the pose of the upper platform through inputting the length of six bars. By this way we can make the bones rest orate gradually. And this method supply a sufficient way about the application of positive solution.

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