

# Structural Robust Optimal Design based on Orthogonal Experimental and Parametric Finite Element Analysis

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**Abstract.** In this paper, based on orthogonal experiment, piccolo tube dynamic optimization was performed to achieve the purposes of reducing the time of sample, identifying the key parameters and optimizing design. Firstly, parameterized finite element model of the piccolo tube was built. After that, the orthogonal experiment under four design variables covering three levels was performed. After 10 times dynamics simulation analysis (9 times simulation for orthogonal experiment and 1 time simulation for optimal design verification experiment), the optimal design of the piccolo tube was quickly determined. The first order natural frequency of the piccolo tube was improved to 1771HZ from initial design 1496.3HZ. The trend and sensitivity between the design variables and structural performance were obtained. The analyze process shows that the proposed method is simple and efficient. It can provide technical reference for piccolo tube optimization design and other complex structures.

## 1 Introduction

In traditional structural optimization design problems, design variables take as continuous variables of design space [1-2]. But in the practical engineering problems, design variables are often within limited level. To perform optimization design with continuous variables is usually call for large computational expense [3]. And the obtained optimal design result merely is optimal design of the optimal model results, which is not necessarily can be applied to practical engineering problems. In addition, in practical engineering, the structural performance always exhibits some degree of variations due to uncertainties of material properties, loading conditions, geometric dimensions, etc. Hence, a proper design procedure must reasonably account for the inherent uncertain nature of a structural system [4].

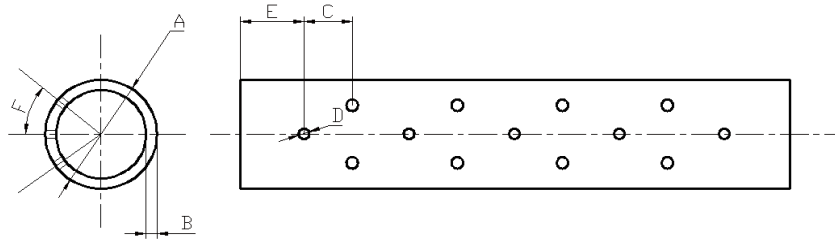
Under the situation of design variables within limited design level, using DOE (Design of Experiments) method can not only find the regularity between structural performance and different designs levels, but also can save

the computational cost of optimization design[5]. The uncertainty analysis can be introduced to ensure the design result is more robust [6]. In this paper, an aircraft piccolo tube is taken as research target. Based on ANSYS software, a parametric finite element model of piccolo tube is build. Then we combine the orthogonal experiment and uncertainty analysis to perform the structural robust optimization design. By the proposed method, the trend relationship between design variables and structural performance is readily obtained, and the optimal combination of design level is determined to ensure a robust optimal design. The analyze process shows that the proposed method is simple and efficient. It can provide technical reference for piccolo tube optimization design and other complex structures.

## 2 Analysis Model

The geometry model of piccolo tube is shown Fig. 1:

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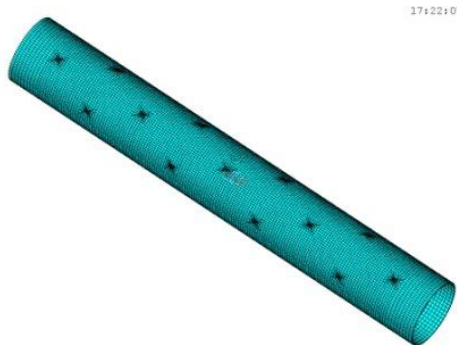
**Fig. 1** The geometry model of piccolo tube

Size parameter and material properties of piccolo tube structure is shown in table 1:

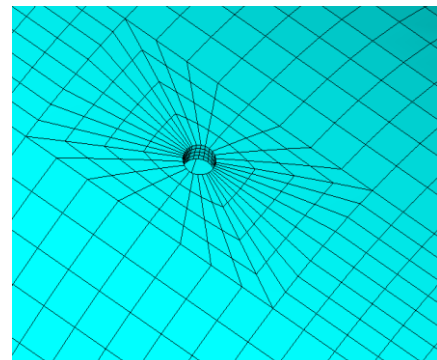
**Table 1** Size parameter and material properties of piccolo tube structure

Structural parameter	Originally designed values	Optional design specifications			standard deviation
Tube diameter A	60mm	58	60	62	0.3mm
Wall thickness B	2mm	1.8	2	2.2	0.2mm
End distance C	40mm	38	40	42	0.3mm
Pore diameter D	2mm	1.8	2	2.2	0.2mm
Pitch E	45				0.2mm
Hole angle F	45				0.3mm
Modulus of elasticity E	2.1E8Mpa				21000 Mpa
Poisson's ratio $\lambda$	0.3				0.02
Density $\rho$ (kg/m <sup>3</sup> )	7.8E3 kg/m <sup>3</sup>				78 kg/m <sup>3</sup>

Finite element model is shown in Fig. 2, and the local mesh is shown in Fig. 3:



**Fig. 2** Mesh model of piccolo tube



**Fig.3** Local mesh of piccolo tube

The first four natural frequency of piccolo tube is shown in table 2 as below:

**Table 2** The first four natural frequency of piccolo tube

Modal order	1	2	3	4
Natural frequency	1538.4	1613.8	1819.0	1827.5

The first four vibration of piccolo tube is obtained mode analysis as shown in Fig. 4.

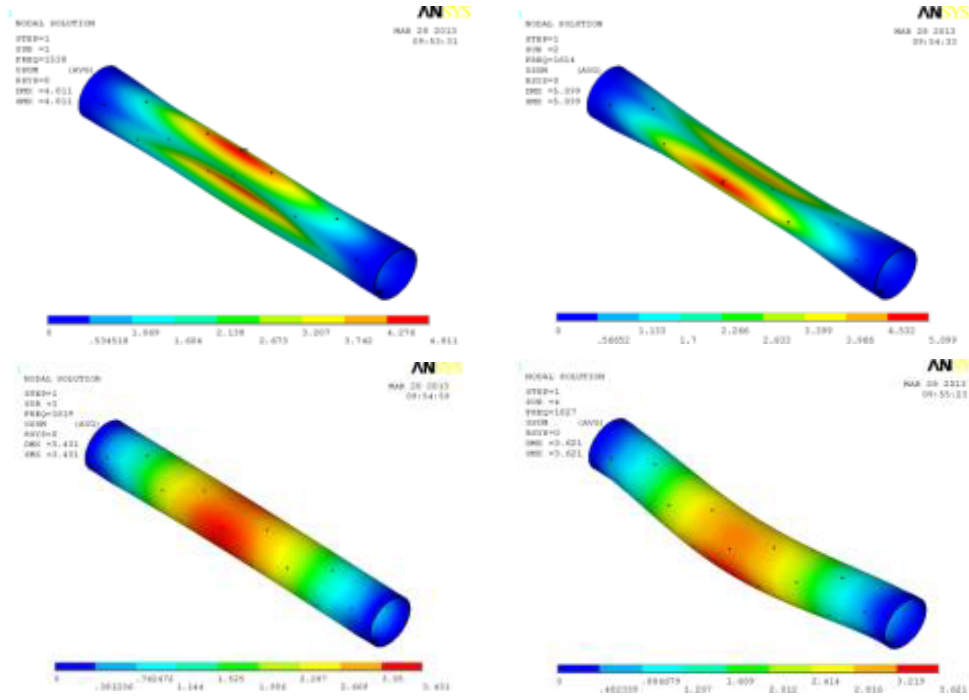


Fig. 4 The first four modal analysis of piccolo tube

### 3 Analysis Method

Orthogonal experiment design is based on the principle of balance of experiment design to arrange the test (test point in the preferred area balanced distribute, called orthogonal in mathematics), which is only part of the all combinations of tests. But it is representative, and can reflect the influence of various factors at different combinations to structural performance. The method was first proposed by statistician RA Fisher. Based on this method, Dr. Taguchi proposed a three-stage product design ideas in the 1970s, which is also named as Taguchi robust design method. To illustrate the characteristics of orthogonal experiment, comparative analysis of full factorial experiments and orthogonal experiments is performed by an example with three factors on three levels. If all possible combinations of three factors on three levels is considered, the times of combination experiments is 27, its schematic diagram is shown in Fig. 5. If according to the orthogonal experiment method, only 9 times combination experiments is needed, its schematic diagram is shown in Fig. 6.

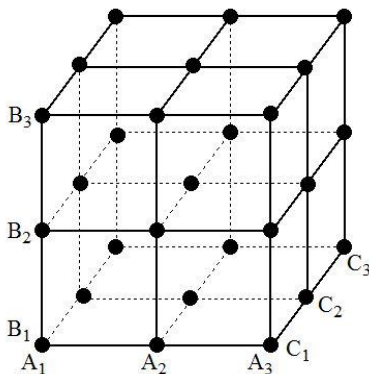


Fig. 5 Schematic diagram of full factorial experiment with three factors on three levels

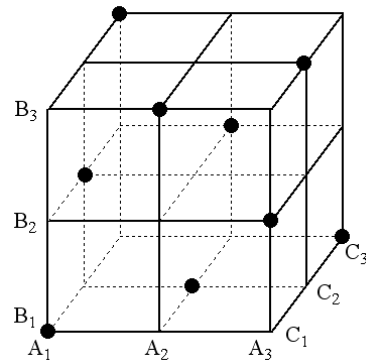


Fig. 6 Schematic diagram of orthogonal experiment with three factors on three levels

Taking into account the presence of uncertainties resulting from machining error and measurement error, size parameters are trade as normal distribution. Then based on  $3\sigma$  theory, the standard deviation of size parameters can be calculated as follows:

$$3\sigma = \Delta \tag{1}$$

Where  $\Delta$  denotes dimensional tolerance. Then we introduced SNR (Signal Noise Ratio) to reflect the influence of uncertainties to structural performance. SNR can be used to express the system response deviations from the ideal response. The higher the SNR is, the higher system robustness has. As the aircraft engine frequency is about 1500Hz, while the first natural frequency of piccolo tube is 1538.4HZ. The two of them is very close. In order to avoid the occurrence of resonance, the goal of optimization design is to improve the first natural frequency of piccolo tube. Therefore in this paper, a look great characteristic SNR is established as follows:

$$SN = -10 \lg \left( \frac{1}{N} \sum_{i=1}^N \frac{1}{y_i^2} \right) \quad (2)$$

Where  $SN$  denotes SNR (Signal Noise Ratio),  $y_i$  denotes the first natural frequency of piccolo tube at the  $i$  times sampling,  $N$  denotes the sampling times. In this

paper, we use the ANSYS to perform uncertainty analyze based on Mento Carlo method by 500 times random sampling.

The analysis results of orthogonal experiment design are shown in Table 3:

**Table 3** Analysis results of orthogonal experiment design

Test	Factors				Test result Y(SN)
	A	B	C	D	
1	$A_1=0.0558$	$B_1=0.0018$	$C_1=0.0405$	$D_1=0.0009$	$y_1=64.6994$
2	$A_1=0.0558$	$B_2=0.002$	$C_2=0.045$	$D_2=0.001$	$y_2=64.5312$
3	$A_1=0.0558$	$B_3=0.0022$	$C_3=0.0495$	$D_3=0.0011$	$y_3=63.3501$
4	$A_2=0.062$	$B_1=0.0018$	$C_2=0.045$	$D_3=0.0011$	$y_4=64.0619$
5	$A_2=0.062$	$B_2=0.002$	$C_3=0.0495$	$D_1=0.0009$	$y_5=63.2342$
6	$A_2=0.062$	$B_3=0.0022$	$C_1=0.0405$	$D_2=0.001$	$y_6=64.1389$
7	$A_3=0.0682$	$B_1=0.0018$	$C_3=0.0495$	$D_2=0.001$	$y_7=62.7821$
8	$A_3=0.0682$	$B_2=0.002$	$C_1=0.0405$	$D_3=0.0011$	$y_8=63.6594$
9	$A_3=0.0682$	$B_3=0.0022$	$C_2=0.045$	$D_1=0.0009$	$y_9=62.8729$
$I_1$	$I_{1A}=y_1+y_2+y_3$ =192.5807	$I_{1B}=y_1+y_4+y_7$ =191.5433	$I_{1C}=y_1+y_6+y_8$ = 192.4977	$I_{1D}=y_1+y_5+y_9$ = 190.8065	$\bar{Y} = \frac{1}{9} \sum_{i=1}^9 y_i \bar{Y}$ $= 63.7033$
$I_2$	$I_{2A}=y_4+y_5+y_6$ =191.4350	$I_{2B}=y_2+y_5+y_8$ = 191.4248	$I_{2C}=y_2+y_4+y_9$ = 191.4660	$I_{2D}=y_2+y_6+y_7$ = 191.4521	
$I_3$	$I_{3C}=y_7+y_8+y_9$ =189.3144	$I_{3B}=y_3+y_6+y_9$ =190.3619	$I_{3C}=y_3+y_5+y_7$ = 189.3665	$I_{3D}=y_3+y_4+y_8$ =191.0715	
$\bar{I}_1$	$\bar{I}_{1A}=(y_1+y_2+y_3)/3$ =64.1936	$\bar{I}_{1B}=(y_1+y_4+y_7)/3$ =63.8478	$\bar{I}_{1C}=(y_1+y_6+y_8)/3$ =64.1659	$\bar{I}_{1D}=(y_1+y_5+y_9)/3$ =63.6022	
$\bar{I}_2$	$\bar{I}_{2A}=(y_4+y_5+y_6)/3$ =63.8117	$\bar{I}_{2B}=(y_2+y_5+y_8)/3$ = 63.8083	$\bar{I}_{2C}=(y_2+y_4+y_9)/3$ =63.8220	$\bar{I}_{2D}=(y_2+y_6+y_7)/3$ = 63.8174	
$\bar{I}_3$	$\bar{I}_{3A}=(y_7+y_8+y_9)/3$ = 63.1048	$\bar{I}_{3B}=(y_3+y_6+y_9)/3$ =63.4540	$\bar{I}_{3C}=(y_3+y_5+y_7)/3$ =63.1222	$\bar{I}_{3D}=y_3+y_4+y_8/3$ =63.6905	
$T$	$T_A = \bar{I}_{iAmax} - \bar{I}_{iAmin}$ =1.0887	$T_B = \bar{I}_{iBmax} - \bar{I}_{iBmin}$ = 0.7256	$T_C = \bar{I}_{iCmax} - \bar{I}_{iCmin}$ = 1.0437	$T_D = \bar{I}_{iDmax} - \bar{I}_{iDmin}$ = 0.2152	
Normal-ization (%)	$\frac{T_A}{T_A+T_B+T_C+T_D}$ = 35.43	$\frac{T_B}{T_A+T_B+T_C+T_D}$ = 23.61	$\frac{T_C}{T_A+T_B+T_C+T_D}$ = 33.96	$\frac{T_D}{T_A+T_B+T_C+T_D}$ = 7	

In table 3, the subscript 1, 2, 3 of variables A, B, C, D are each variable corresponding first, second, and third levels. Where  $y_i$  denotes the SNR result,  $I_i$  denotes the sum of signal noise ratio under the  $i$  level,  $\bar{I}_i$  denotes the average value of signal noise ratio under the  $i$  level. In this paper, the design requires the SNR result is the smaller

the better. Hence if the average value of signal noise ratio under the  $i$  level is the minimum among the three levels, the  $i$  level will be the optimal level. According to the results of nine tests, we can select the best combination of the levels from Table 2, which is shown in Fig. 7 marked in the red line.

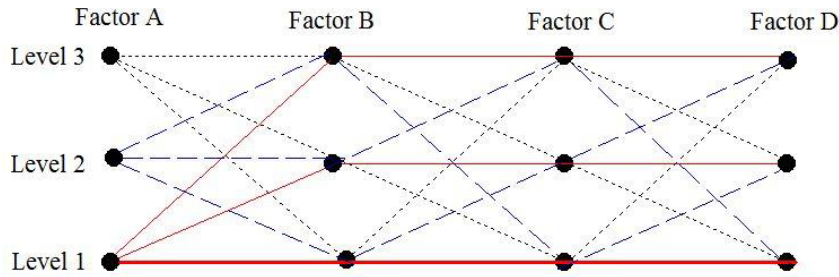


Fig. 7 Network map of orthogonal experiment design

However, the orthogonal experiment is only ninth of the full factorial experiment, there are more trials did not do, optimal combination of nine tests is not necessarily the optimal combination of global. Therefore, in this paper

mathematics we use visual analysis method to obtain the trend of various factor at different levels, which is shown in Fig. 8.

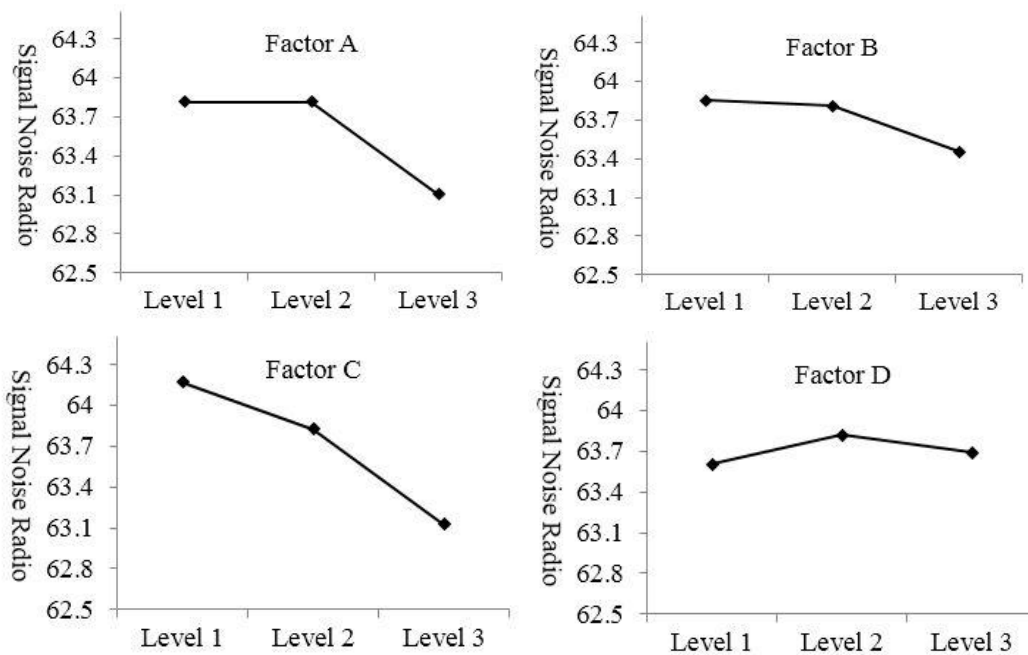


Fig. 8 The trend of various factor at different levels

As can be seen from Fig. 8, in order to get the maximum SNR, we should choose the optimal level combination as  $A_1B_1C_1D_2$ . This test is not included in the nine test, it is the optimal combination of global. Order to verify the effectiveness of the predicted optimal solution, we perform mode analysis under the level combination  $A_1B_1C_1D_2$ . The obtained first natural frequency is 1771.60Hz, and its signal noise ratio is 64.7852, which is better than  $A_1B_1C_1D_1$ .

You can also obtain the importance degree based on information range analysis. From table 2 we can see that  $T_A > T_B > T_C > T_D$ , which means among the four design variables, variable A has greatest impact on the SNR, while variable D has weakest impact. After normalization, we can obtain the sensitivity of each factor on the SNR is shown in Fig. 9.

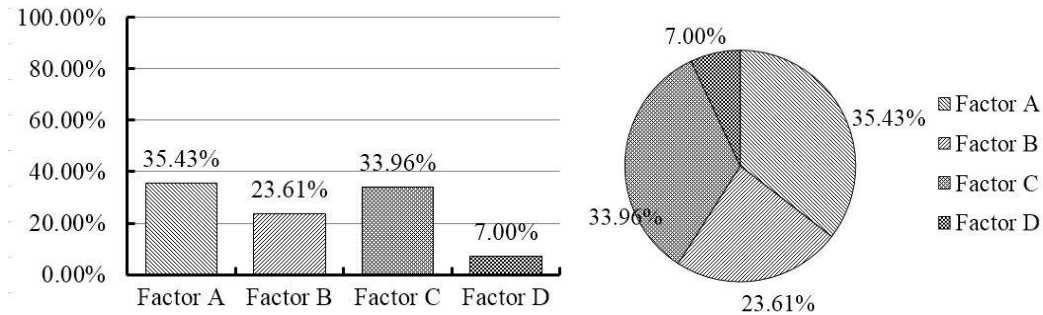


Fig. 9 the sensitivity histogram of each factor on the SNR

## 4 Summary

In practical engineering, the structural performance always exhibits some degree of variations due to uncertainties of material properties, loading conditions, geometric dimensions, etc. Hence, a proper design procedure must reasonably account for the inherent uncertain nature of a structural system. In this paper, based on orthogonal experiment, dynamic optimization for piccolo tube was performed to achieve the purposes of reducing the time of sample, identifying the key parameters and optimizing robust design. By the proposed method, the trend relationship between design variables and structural performance is readily obtained, and the optimal combination of design level is determined to ensure a robust optimal design. The analyze process shows that the proposed method is simple and efficient. It can provide technical reference for piccolo tube optimization design and other complex structures.

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