Numerical Study on Characteristics of 3D Cavitating Hydrofoil

Wei Cao, Hao Xu*, Huaixun Ren & Cong Wang

School of Astronautics, Harbin Institute of Technology, Harbin, Heilongjiang, China

ABSTRACT: The commercial software ANSYS CFX, APDL and Workbench are applied for modeling the hydrodynamic and structural interactions and characteristics of an elastic hydrofoil by means of a two-way FSI method. The SST (Shear Stress Transport) turbulence model and the simplified Rayleigh-Plesset equations are employed for the cavitating flow simulation. Both CFX and APDL solvers are set to be transient. The fluid and solid computational domains are sequentially solved to simulate the interactions between the hydrofoil and the cavitating flow. The results show that the difference in stiffness of common metal materials has trifling effects on hydrofoil performance. But variations in cavitation number and angle of attack will dramatically affect the hydrodynamic and structural interactions and characteristics.

Keywords: hydrofoil; cavitation; fluid-structure-interaction; hydrodynamic; load

1 INTRODUCTION

With the rapid development of high-speed vessels and flexible composite materials in recent years, cavitation problems on some newly developed marine hydrofoils and propellers become striking. Cavitation, especially cloud cavitation, is highly unsteady and may cause serious vibration, deformation, even damage to the nearby structure [1] [2]. Up till now most research on cavitating hydrofoils regard the hydrofoil as fixed rigid, neglecting interactions between the hydrodynamic loading and the hydrofoil structure. These conventional methods that only focus on cavitation and hydrodynamic loads are appropriate for traditional rigid hydrofoils with high stiffness and strength. But these methods are incapable of predicting the distribution and dynamic characteristics of the load on the solid structure that are essential for the utility of more flexible materials on hydrofoil designs. Most previous studies on interactions between hydrofoil and cavity fluid fields are grounded on experiments [3] [4] [5] [6]. Though it is indispensable, advancements in computer hardware and numerical methods open up new ways for cavitation study, which are much more detailed than any affordable experiment in the foreseeable future.

The present paper is a numerical study utilizing two-way fluid-structure interaction (FSI) method on cavitating hydrofoils based on CFX, Mechanical APDL and ANSYS Workbench. The k- ω based SST model is employed as the turbulence model for its high accuracy in simulation of foils at large angle of attack [7]. The Zwart Model based on Rayleigh-Plesset equation is employed as the cavitation model. This model is proved to have a good general validity for cavitating hydrofoils [7]. The discretization at the fluid-structure interface is assumed to be matching. The influences of hydrofoil stiffness, cavitation number,

and angle of attack on hydrodynamic and structural characteristics of a Clark-Y type hydrofoil are investigated.

2 BASIC THEORY

2.1 Fluid dynamics

2.1.1 Governing equations

In order to solve the multiphase flow problem which is involved in cavitation, the homogeneous mixture model is selected. This model assumes all phases share the same flow field, as well as other relevant fields such as temperature and turbulence. The distribution of each individual phase in the flow field is described by the density field and the variation of volume fraction.

The continuity equation for the mixture flow is as follows:

$$\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m \vec{v}_m) = 0 \quad (1)$$

Where $\vec{v}_m$ is the mixture velocity, and $\rho_m = \sum \alpha_k \rho_k$ is the mixture density with α_k as the volume fraction of each phase satisfying $\sum \alpha_k = 1$.

The momentum equation for the mixture flow is as follows:

$$\frac{\partial (\rho_m \vec{v}_m)}{\partial t} + \nabla \cdot (\rho_m \vec{v}_m \vec{v}_m) = -\nabla p + \nabla \cdot [\mu_m (\nabla \vec{v}_m + \nabla \vec{v}_m^T)] + S \quad (2)$$

Where $\vec{v}_m = \sum \alpha_k \rho_k \vec{v}_k / \rho_m$ is the mixture velocity; $\mu_m = \sum \alpha_k \mu_k$ is the mixture dynamic viscosity; p is pressure and S is the source term.

2.1.2 Cavitation model

The growth of the gas bubble in cavitating flow is described by the Rayleigh-Plesset equation as follows:

*Corresponding author: xuhaohit@sina.cn

$$R_B \frac{d^2 R_B}{dt^2} + \frac{3}{2} \left(\frac{dR_B}{dt} \right)^2 + \frac{2\sigma}{\rho_f R_B} = \frac{p_v - p}{\rho_f} \quad (3)$$

Where R_b is the bubble radius; σ is the surface tension coefficient between the liquid and vapor; p_v is the vapor pressure; p is the ambient liquid pressure, and ρ_f is the liquid density. With the second order terms and the neglected surface tension, the equation Rayleigh-Plesset reduces to:

$$\frac{dR_B}{dt} = \pm \sqrt{\frac{2}{3} \frac{p_v - p}{\rho_f}} \quad (4)$$

Denoting the bubble per unit volume by N_B and the bubble volume by V_B , the volume fraction γ_g is expressed as follows:

$$\gamma_g = V_B N_B = \frac{4}{3} \pi R_B^3 N_B \quad (5)$$

With the included condensation effect, the inter-phase mass transfer rate per unit volume is expressed as follows:

$$\dot{m}_{fg} = F \frac{3\gamma_g \rho_g}{R_B} \sqrt{\frac{2}{3} \frac{|p_v - p|}{\rho_f}} \text{sgn}(p_v - p) \quad (6)$$

F is the empirical factor. It is 50 for vaporization and 0.01 for condensation.

2.1.3 Turbulence model

The main drawback of Wilcox $k-\omega$ model over cavitating hydrofoil problems is its lack of accuracy in prediction of flow separation under adverse pressure gradients [9]. The main reason for this deficiency is an excessive prediction of the eddy-viscosity. The SST (Shear Stress Transport) model was developed to overcome this deficiency by introducing a limiter to the formulation of the eddy-viscosity:

$$\nu_t = \frac{c_1 k}{\max(c_1 \omega, SF_2)} \quad (7)$$

Where ν_t is the kinetic viscosity; c_1 is the model constant; k is the turbulent kinetic energy; ω is the turbulent frequency; S is an invariant measure of the strain rate, and F_2 is the blending function.

The blending functions are of vital importance to the success of the SST model. The formulation of blending functions is based on the distance to the nearest surface and on the flow variables:

$$F_1 = \tanh \left(\min \left(\max \left(\frac{\sqrt{k}}{\beta' \omega y}, \frac{500\nu}{y^2 \omega} \right), \frac{4\rho k}{CD_{kw} \sigma_{\omega 2} y^2} \right) \right) \quad (8)$$

$$F_2 = \tanh \left(\max \left(\frac{2\sqrt{k}}{\beta' \omega y}, \frac{500\nu}{y^2 w} \right) \right) \quad (9)$$

Where y is the distance to the nearest wall, ν is the kinetic viscosity and:

$$CD_{kw} = \max \left(2\rho \frac{1}{\sigma_{\omega 2} \omega} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_i}, 1.0 \times 10^{-10} \right) \quad (10)$$

2.2 Structural mechanics

As the structural solver is based on the Lagrangian frame of reference, the equilibrium equation of the solid structure becomes:

$$\rho \frac{\partial^2 \vec{u}}{\partial t^2} = \nabla \cdot \vec{\sigma} + \rho \vec{b} \quad (11)$$

Where ρ is the material density; μ is the displacement vector; σ is the Cauchy stress tensor, and b is the body force.

This equation is closed by the constitutive relationship of Hooke's law for an isotropic elastic solid that reads:

$$\vec{\sigma} = 2G\vec{\varepsilon} + \lambda \nabla \cdot \vec{u} \vec{I} \quad (12)$$

Where G and λ are Lamé's constants, ε is the stress tensor and $\vec{I}$ is the second order identity tensor.

2.3 Coupling scheme

2.3.1 Fluid-structure coupling

On the fluid-structure interface, the laws of compatibility and conservation require:

$$\begin{cases} \vec{\sigma}_s \cdot \vec{n} = \vec{\sigma}_f \cdot \vec{n} \\ \vec{v}_m = \frac{d\vec{u}}{dt} \end{cases} \quad \text{on } \Gamma_{S/F} \quad (13)$$

Where σ_f and σ_s are stress tensors for the solid and fluid domain, and v_m is the grid velocity.

2.3.2 Software coupling

The coupling between Mechanical APDL and CFX is carried out by the Multi-Field External (MFX) solver. A two-way coupled analysis is executed by solving each computational domain sequentially and transferring the data between solvers in turn. The hydrodynamic load is firstly calculated by CFX and then transferred to Mechanical APDL as the boundary condition on the fluid-solid interface. Mechanical APDL then returns the deformation of the solid structure thus forming up a new geometry. Adaptations are then made to the mesh accordingly. Both CFX and Mechanical APDL solvers are set to be transient. The MFX solver time step governs the elapsed time of the simulation. The time step in CFX, as well as in Mechanical APDL, is identical to the MFX time step at all times.

3 GRIDS AND BOUNDARY CONDITIONS

The foil is a uniform cross-section Clark-Y type hydrofoil with a 70mm chord length(C) and 68mm wingspan. Since CFD and FEA solver require different meshes, the fluid field and the solid structure are separately meshed.

As for the fluid field, the domain inlet is $5C$ the upstream trailing edge and the outlet is $8C$ downstream as illustrated in Figure 1.

Figure 1. Geometric dimension of the flow field

Figure 2. Three differently fined grids

Figure 3. Lift coefficient of different grids.

Figure 4. Meshing of the solid structure

A C-H type grid is generated for the domain. The inlet boundary is set to be velocity inlet and the outlet is set to be pressure outlet. The slip condition is assumed on all boundary walls and the foil surface is specified as no-slip wall. The grid independence was investigated through three similar grids with different cell numbers. As shown in Figure 2, the coarser grid SN1 consists of 391000 cells, the medium SN2 700000, and the finer SN3 1009000. The investigation was carried out with 8° angle of attack (AoA) at Reynolds number $Re=7.0 \times 10^5$ and a cavitation number $\sigma=0.8$. As illustrated in Figure 3, it was found that the total lift coefficient by SN2 and SN3 varies within 3%, while the results by SN1 differed more 10% from the others. In consideration of both accuracy and efficiency requirements, the simulation was afterward carried out on the 700000-cell grid SN2.

The solid structure of the hydrofoil is discretized by 20-node hexahedron element SOLID186 as revealed in Figure 4. It is set to be fixed on one end while the other four faces interact with the fluid field.

4 RESULTS AND DISCUSSIONS

4.1 Effects of hydrofoil stiffness

The effects of hydrofoil stiffness on hydrodynamic and structural characteristics were investigated by means of different materials. Three commonly-used engineering materials, namely magnesium alloy, aluminum alloy and structural steel are applied. Their properties are listed in Table 1. The two-way FSI simulation mentioned above was implemented with $\sigma=1.2$, at $Re=7.0 \times 10^5$ and 8° AoA.

Table 1. Material properties

Material	Density	Young's Modulus E	Poisson's ratio ν
	kg/m ³	GPa	
Magnesium alloy	1800	45	0.35
Aluminum alloy	2770	71	0.33
Structural steel	7850	200	0.30

Though operated under the steady inlet flow, all three foils exhibit similar periodic shedding phenomena as illustrated in Figure 5. The cavitation sheet approximately accumulates during the first half of the shedding cycle and followed by the formation of a U-shaped bubble^[10]. It then sheds micro bubble clusters from the trailing part until the next shedding cycle begins. However, the cycle for these three materials slightly differs from each other with 59.2ms for magnesium alloy, 59.8ms for aluminum alloy, and 60.2ms for structural steel. As illustrated in Figure 6 and Figure 7, the lift and drag coefficients also slightly differ under the given condition.

The hydrofoil stiffness also has trifling influence on stress magnitude and distribution. The maximum von

Mises stress for magnesium alloy foil, which is 21.1MPa in magnitude and the largest in all three, differs by only 1% from that of the structural steel. The von Mises contour in a cycle for the magnesium alloy foil is demonstrated in Figure 8.

Figure 5. Cavitation bubble variation pattern in a cycle

Figure 6. Lift coefficient for different materials

Figure 7. Drag coefficient for different materials

Figure 8. Von Mises stress contour for the magnesium alloy foil

4.2 Effects of cavitation number

The investigation on effects of cavitation number was executed on an aluminum alloy foil under five different cavitation numbers $\sigma=0.8, 1.0, 1.2, 1.4$, and 1.6 , at 8° AoA and $Re=7.0 \times 10^5$. Under $\sigma=1.4$ and 1.6 , the cavitation on foil surface presented itself as a steadily attached sheet cavitation with some few bubbles shedding from the trailing part^[11]. A cloud cavitation came into being under $\sigma=0.8, 1.0$, and 1.2 . As illustrated in Figure 9, A U-shaped breakdown and periodic shedding of the cavitation bubble appeared at this stage of bubble development.

Figure 10 and Figure 11 show the effect of cavitation number on lift (C_L) and drag (C_D) coefficients of the hydrofoil. Both C_L and C_D curves tend to be horizontal straight lines under $\sigma=1.4$ and 1.6 in the above two figures. This suggests a stable cavitation bubble and thus the hydrodynamic loads on the hydrofoil can be considered to be stable. Under $\sigma=0.8, 1.0$, and 1.2 , both C_L and C_D fluctuates periodically, showing strong unsteadiness. And this fluctuation increases along with the decrease of σ .

As for the structural characteristics of the hydrofoil, the von Mises stress reaches a maximum value of 21.4MPa under $\sigma=1$ when the flow field fluctuates and the inlet flow speed is relatively high. Table 2 shows this maximum stress under each σ .

Though variations for different cavitation number, the absolute value for this variation of the maximum von Mises stress is no more than 20% of that under $\sigma=1$. The cavitation number mostly affects the loading fluctuation pattern. Under $\sigma=0.8, 1.0$, and 1.2 , the von Mises stress fluctuates by the same period of that of the cloud cavitation. But the time at which the maximum stress appears differently. This fluctuation is relatively small under $\sigma=1.4$, and 1.6 , the loadings under these circumstances can be considered to be stable. The structural loading fluctuation patterns are illustrated in Figure 12.

Figure 9. Cavitation form based on different σ Figure 10. Lift coefficient based on different σ Figure 11. Drag coefficient based on different σ Figure 12. Loading patterns based on different σ

4.3 Effects of attack angle

To study the effects of angle of attack, simulations with $Re=7.0 \times 10^5$ and $\sigma=1.2$ at 4° , 6° , 8° , 10° and 12° AoA were executed.

Table 2. Maximum von Mises stress based on different σ

σ	0.8	1.0	1.2	1.4	1.6
Maximum stress(MPa)	17.61	21.44	20.92	19.16	19.96

Figure 13. Lift coefficient at different AoA

At 4° AoA, no cavitation arose on the foil surface. A steadily attached sheet cavitation came about as the angle of attack was increased to 6° . This form of cavitation showed strong steadiness with only a few bubbles shedding from the trailing part. Based on 8° , 10° , and 12° AoA, the cavitation on the hydrofoil turned into cloud cavitation which breaks and sheds periodically from the foil surface. And this unsteadiness became greater under a larger angle of attack.

Figure 13 and Figure 14 shows the effect of angle

Figure 14. Drag coefficient at different AoA

Figure 15. Von Mises contour at AoA= 8° , 10° , 12°

of attack on lift (Cl) and drag (Cd) coefficients of the hydrofoil. At 4° (no cavitation) and 6° (sheet cavitation) AoA, both Cl and Cd tend to be straight, showing little fluctuation. At 8°, 10°, 12° AoA, both curves fluctuate periodically, showing strong unsteadiness. This fluctuation becomes more violent as the angle of attack increases. In the meantime, both average and maximum values for Cd also increase with the growth of angle of attack. However, the maximum value for Cl at 12° AoA is slightly smaller than that at 10° due to stall^[11].

Being similar to Cl and Cd coefficients, the von Mises stress on the foil at 4° and 6° AoA also showed little variation over time. The structural loading under these two circumstances can therefore be stable. The stress contour at 8°, 10°, and 12° AoA showed the fluctuation cycles as Cl and Cd. As illustrated in Figure 15, all maximum stresses under these three conditions occurred at the same spot on the foil surface at different time. Table 3 shows this maximum stress at each degrees of angle of attack.

Table 3. Maximum von Mises stress based on different AoA

AoA	4°	6°	8°	10°	12°
Maximum stress (MPa)	14.07	16.96	12.92	26.46	25.48

5 CONCLUSIONS

The cavitation problem on hydrofoil was numerically investigated through a two-way FSI method. The development and breakup cycle of the cloud cavitation, along with the periodic fluctuations in lift and drag coefficients and loading distribution, was revealed on an aluminum hydrofoil. The comparison between different foil stiffness led to the conclusion that the differences in commonly-used metal material stiffness have little effects on hydrodynamic and structural characteristics of a cavitating hydrofoil. The variations in cavitation number and angle of attack however dramatically affected the hydrodynamic characteristics and the interaction between fluid and structure, including the form of cavitation and the hydrodynamic loadings on the hydrofoil.

ACKNOWLEDGEMENT

This work is sponsored by NSFC (51149003) and the

Fundamental Research Funds for the Central Universities (Grant No. HIT. NSRIF. 2013033).

REFERENCES

- [1] Arndt R E A, Song C C S. & Kjeldsen M, et al. 2000. Instability of partial cavitation: a numerical/experimental approach.
- [2] Leroux J B, Coutier-Delgosha O. & Astolfi J A. 2005. A joint experimental and numerical study of mechanisms associated to instability of partial cavitation on two-dimensional hydrofoil. *Physics of Fluids* (1994-present), 17(5): 052101.
- [3] Song C. & Almo J. 1967. *An Experimental Study of The Hydroelastic Instability of Supercavitating Hydrofoils*.
- [4] Ducoin A, Astolfi J A. & Sigrist J F. 2012. An experimental analysis of fluid structure interaction on a flexible hydrofoil in various flow regimes including cavitating flow. *European Journal of Mechanics-B/Fluids*, 36: 63-74.
- [5] Akcabay D T. & Young Y L. 2014. Influence of cavitation on the hydroelastic stability of hydrofoils. *Journal of Fluids and Structures*, 49: 170-185.
- [6] Akcabay D T, Chae E J. & Young Y L, et al. 2014. Cavity induced vibration of flexible hydrofoils. *Journal of Fluids and Structures*, 49: 463-484.
- [7] Eleni D C, Athanasios T I. & Dionissios M P. 2012. Evaluation of the turbulence models for the simulation of the flow over a National Advisory Committee for Aeronautics (NACA) 0012 airfoil. *Journal of Mechanical Engineering Research*, 4(3): 100-111.
- [8] Morgut M, Nobile E. & Biluš I. 2011. Comparison of mass transfer models for the numerical prediction of sheet cavitation around a hydrofoil. *International Journal of Multiphase Flow*, 37(6): 620-626.
- [9] Grotjans H. & Menter F R. 1998. Wall functions for general application CFD codes. *ECCOMAS 98 Proceedings of the Fourth European Computational Fluid Dynamics Conference*, pp. 1112-1117. John Wiley & Sons.
- [10] Kubota A, Kato H. & Yamaguchi H, et al. 1989. Unsteady structure measurement of cloud cavitation on a foil section using conditional sampling technique. *Journal of Fluids Engineering*, 111(2): 204-210.
- [11] Hutton S P. 1986. Studies of cavitation erosion and its relation to cavitating flow patterns[C]//*Proc. Int. Symp. Cavitation, Sendai, Japan*. 1: 21-29.
- [12] Ogata Hiroyuki, Ito Yukio. & Oba Risaburo, et al. 1991. Unsteady Performance of a Cavitating Hydrofoil in Stall Conditions. *Transactions of the Japan Society of Mechanical Engineers Series C*, 57(543): 3788-3793.