

# The Dynamic Properties of the Bridge Deck Model Reinforced With FRP Bars

Barbara Markiewicz<sup>1</sup>, Karol Pereta<sup>1</sup>, and Grzegorz Piatkowski<sup>1,a</sup>

<sup>1</sup>Rzeszow University of Technology, Department of Structural Mechanics, Rzeszow, Poland

**Abstract.** Significant application of composites can be observed in numerous fields over the last decades. Composite materials mainly used in the aeronautics industry are gaining more and more application in various areas such as the construction of bridges. The article presents a set of dynamic research carried out on a plate made of lightweight concrete reinforced with composite rods. The tested plate with dimensions 514 x 190 x 18 cm was a model of the actual bridge deck. A multi-channel signal recorder with specialized software was used for performing the measurements and estimating modal parameters of the plate. Dynamic response of the plate on a modal hammer impact was measured with numerous of piezoelectric acceleration sensors. A range of FEM models was created, started with 1-D beam model. Next, 2-D plate and 3-D volume, more complex models were developed. Only in the 3-D model the composite rods were modeled. The basic frequencies and mode shapes obtained for physical model were compared with those for numerical models. There were significant differences. The conducted analysis indicated more complicated than theoretical boundary conditions of the tested plate.

## 1 Introduction

Composites which are relatively new in civil engineering have been used for many years in the aircraft industry. The availability and low price are the biggest advantages of traditional materials in comparison to the price of those new ones, like Fiberglass Reinforced Plastic composites. However, competitive advantages of FRP composites are low weight, high strength and total corrosion resistance which means low maintenance costs. Because of the fact that the exploitation is more expensive, composites are cost effective compared to conventional materials.

FRP composites have been widely used in bridge structures to strengthen the existing bridges since 1997 [1]. However, still no bridge or overpass with the use of composite reinforcement bars or carrying elements has been built in Poland.

The research conducted by consortium Mostostal Warszawa SA and Rzeszow University of Technology has been the first attempt to change the existing situation [2]. This paper presents the research which has been a part of the so-called "Combridge" project. This project is financed by The National Centre for Research and Development in "Demonstrator+" program. The aim of this program is to build the first Polish bridge made of FRP composites [3].

## 2 Description of the tested plate

A double span plate has been studied. A theoretical length of each span has been 240 cm. The discussed plate has

<sup>a</sup>e-mail: gpiatkowski@prz.edu.pl

been a model of the real bridge deck made of a FRP grid-reinforced concrete LC35/38. Bars used in the grids has been diameter of 12 mm.

Static scheme and model dimensions have been presented in Fig. 1.

The plate has been pinned in the centre and supported in the sliding manner near the edges. Support has been implemented in the form of cylindrical roller bearings and steel panels. Plate has been attached to steel panels by means of cement mortar. Theoretical length of each support has been equal 190 cm which is the width of the plate.

Steel profiles and rubber spacers have been used to transmit the load generated by the actuators of the research station on the deck. The load has been transferred to four 40 cm x 40 cm areas. The total additional mass has been 440 kg, Fig. 1.

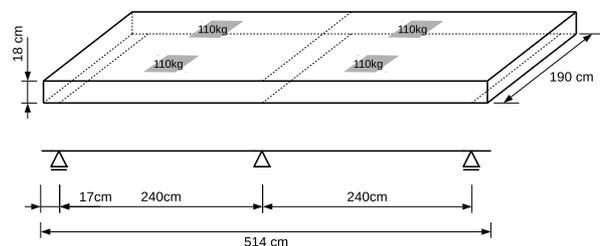


Figure 1. Static scheme and dimensions of the plate

## 3 Measurements

As a result of the measurements, frequencies, vibration modes and damping coefficients have been determined.

The classical modal analysis has been used, which is based on the measurement of the existing force and accelerations at various points of the structure as a response to this force.

The studies have been conducted in several steps, which has been motivated by parallel static research. The first two stages of the test have been as follows:

- stage 1 - uncracked state, mass of a plate,
- stage 2 - uncracked state, mass of a plate + additional mass.

Pulse excitation has been generated with PCB Piezotronics modal hammer weighing 5,5 kg and with a sensitivity of 0.23 mV/N. Soft tip has been used. Pulse excitation has been generated in four points shown in Fig. 2 in the vertical direction.

Dynamic response of the plate has been measured by piezoelectric accelerometers. The vertical accelerations have been measured by 27 accelerometer sensors. All the measurement points have been located on the upper surface of the plate and located as in Fig. 2. The placement of measurement points has corresponded to support scheme of the plate. Sensor offset from the longer edges of the plate has been 5 cm and from the shorter ones 17 cm. A measurement mesh of a total size 480 cm x 180 cm has been received.

Multi-channel signal recorder Scadas Mobile's LMS International has been used in measurements. Signals in the frequency range 0-256 Hz with a resolution of 0.125 Hz have been recorded during acceleration measurements. Recorder software allows for the classical experimental modal analysis including estimation of poles using Poly-max method and visualization of the results.

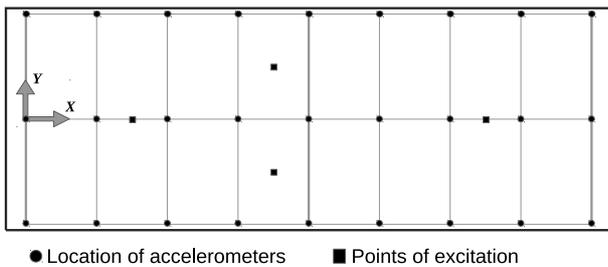


Figure 2. Location of measurement points

### 3.1 Modal test results

The vibration of the plate has been measured in stage 1. In second stage the additional mass of metal and rubber spacers located on the plate has been included.

Dynamic characteristics which are the results of modal analysis for stage 1 have been shown in Table 1. Mode shapes for the first two modes have been presented in Fig. 3 and Fig. 4. The range of the values of the first and second frequencies have been presented in the min-max variability because excitation for modal analysis has been made in different points.

Table 1. Modal test results – stage 1

	Minimum frequency [Hz]	Maximum frequency [Hz]	Minimum damping coefficient [%]	Maximum damping coefficient [%]
f1	36,43	36,66	1,05	1,16
f2	50,36	50,57	1,47	1,76

Table 2. Modal test results – stage 2

	Minimum frequency [Hz]	Maximum frequency [Hz]	Minimum damping coefficient [%]	Maximum damping coefficient [%]
f1	35,70	35,85	1,92	2,26
f2	45,49	45,65	3,09	3,73

Dynamic characteristics which are the results of modal analysis for stage 2 have been shown in Table 2. As expected, additional mass resulted in decreasing of the frequencies and increasing of damping coefficient values. The obtained in both stages mode shapes of the plate have suggested problem with one of the supports.

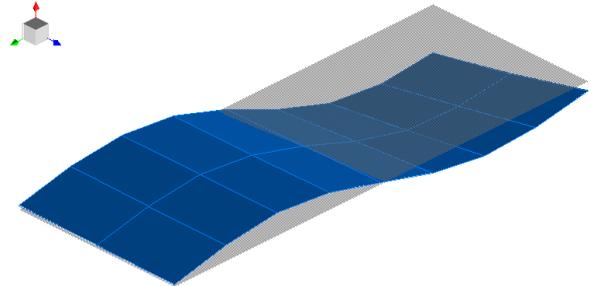


Figure 3. First mode shape – stage 1

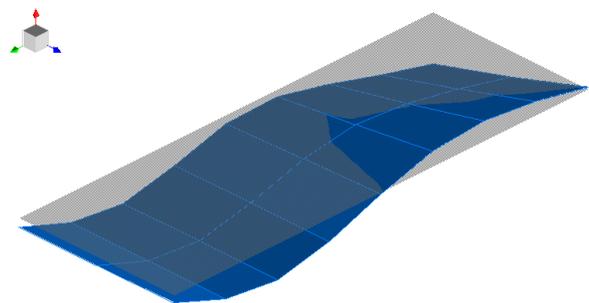


Figure 4. Second mode shape – stage 1

## 4 Analytical model

Even in case of complex structures preliminary estimates of the dynamic properties can be based on the simplest numerical model. The first model of a studied plate has been

a two-spanned beam, for which two first frequencies have been calculated using closed formulas. The formulas (1) and (2) have been determined for half schemes of symmetric beam and appropriate boundary conditions. The first mode shape has been shown in Fig. 5 and frequency for this mode has been described by formula (1).

$$f_1 = \frac{k^2 \pi^2}{l^2} \sqrt{\frac{EI}{\mu}} \quad (1)$$

where:

$l$  - span length,

$\mu$  - linear density,

$EI$  - stiffness,

$k$  - number of particular type of mode.

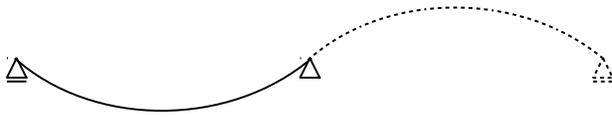


Figure 5. Antisymmetric eigen mode shape of a beam for  $k=1$

The second mode shape has been shown in Fig. 6 and frequency for this mode has been described by formula (2).

$$f_2 = \left(\frac{4k+1}{4}\right)^2 \frac{\pi^2}{l^2} \sqrt{\frac{EI}{\mu}} \quad (2)$$

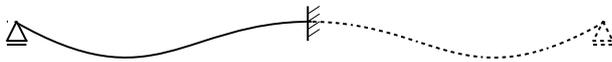


Figure 6. Symmetric eigen mode shape of a beam for  $k=1$

The data used in calculations: the Young's modulus equal to 21,562 GPa, moment of inertia 92340 cm<sup>4</sup> and linear density  $\mu = 673,74$  kg/m. The length of each span has been equal to 2,4 m. The cantilevers have been omitted.

For second stage, homogeneous mean density with mass of additional elements located on the deck have been adopted and the value of linear density has been increased to 765,41 kg/m.

## 5 Numerical model

### 5.1 FEM 1-D and 2-D models

The ADINA program has been used for building of numerical models.

The Young's modulus in numerical models has been the same as in analytical model and volume density has been equal to 1970 kg/m<sup>3</sup>, which is also corresponding with the above mentioned data.

Two numerical BEAM [4] models have been created to compare obtained frequencies with analytical results.

Table 3. Frequencies of two first mode shapes for 1-D model

Freq. [Hz]	Calculation		ADINA	
	Without A.M.	With A.M.	Without A.M.	With A.M.
f1	46,88	43,98	46,70	41,82
f2	73,25	68,72	72,86	66,15

where:

A.M. - additional mass.

Table 4. Frequencies of two first mode shapes for 2-D model

Freq. [Hz]	The support ratio [-]			
	1/1	1/2	1/4	1/8
f1	41,84	40,82	36,45	36,33
f2	65,76	62,37	51,92	51,72

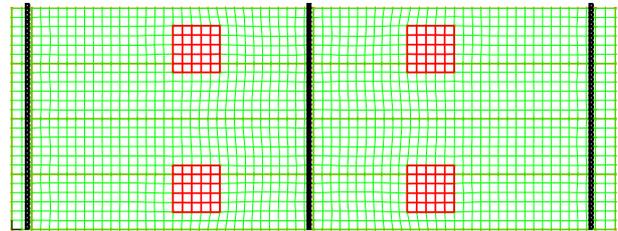


Figure 7. 2-D model of the plate

The cantilevers have been considered in all numerical models. In the first model only the mass of the plate has been considered. In the second model additional mass has been included. The results have been shown in Table 3.

Received in the ADINA results differ from those obtained analytically by less than 0.5% in the case without the additional mass. The difference is due to the fact that the analytical calculation does not include overhangs. Consideration of additional masses on appropriate sections of the beam reduces the numerically calculated vibration frequency compared to the frequency obtained with the analytical model of a mean weight of the whole span length, but the difference in frequency values is less than 5%.

Comparing these results with the results of the measurements given in Tables 1 and 2, the calculated frequencies seem to be too high. The analytical model and numerical model 1-D are limited in terms of defining the actual conditions of the support of the plate. It is impossible therefore to consider the lack of contact between the plate and the support.

In order to consider deviation from the theoretical support the 2-D model has been created. It has been formed with the use of eight-node type SHELL elements [4]. The plate has been analysed as non-cracked, and the stiffness of reinforcement has been omitted. Four models in which the ratio of the length of the left support to width of the plate was as follows: 1/1, 1/2, 1/4, 1/8, in order to test the effect of these boundary conditions on the basic frequen-

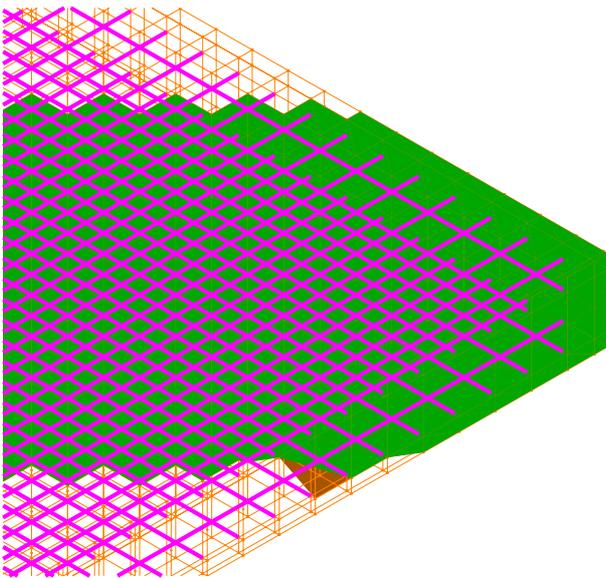
**Table 5.** Frequencies of two first mode shapes for 3-D model

Freq. [Hz]	The support ratio [-]				
	Unreinforced	Reinforced			
	1/1	1/1	1/2	1/4	1/8
f1	41,31	42,40	41,33	36,67	36,04
f2	65,73	67,38	63,76	52,69	52,49

cies. In Fig. 7 the model with entire supported width has been shown.

**5.2 FEM 3-D model**

The 3-D model plate has been made of eight-node SOLID elements [4]. The plate has been analysed as uncracked, again. The composite reinforcement has been included with the use of two-node TRUSS element [4]. The Young’s modulus of this composite rods has been equal 57.8 GPa. A fragment of the model is presented in Fig. 8.



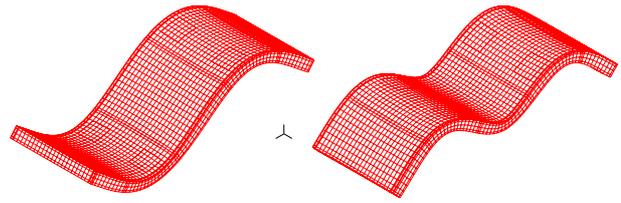
**Figure 8.** Fragment of the 3-D model with reinforcement

Solid model has been made for the same support options as the 2-D model. The results have been listed in Table 5.

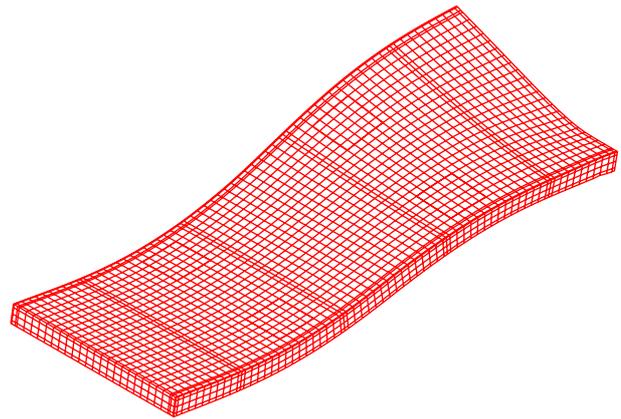
The mode shapes of 3-D model with 1/1 support ratio have been shown in Fig. 9. They have been such like these obtained for 1-D and 2-D models, compare to Fig. 5 and Fig. 6.

The frequency of the plate has increased of 2-3% by included reinforcement. The frequency has decreased in case of reducing the length of the contact between the plate and one of the support of the edges. The difference between the results for the total support and support 1/8 of the width reaches 22% for second mode.

The Modal Assurance Criterion (MAC) was used to compare the test and numerical mode shapes. The correlation between test eigenvector and numerical eigenvectors



**Figure 9.** Two first mode shapes of 3-D model with 1/1 support ratio



**Figure 10.** First mode shape of 3-D model with 1/4 support ratio

for FEM 3-D models with considered ratios of support 1/1, 1/2, 1/4 and 1/8 is accomplished by means of the MAC value:

$$MAC = \frac{(x_T^T x_N)^2}{(x_T^T x_T)(x_N^T x_N)} \tag{3}$$

where:

$x_T$  - test eigenvector

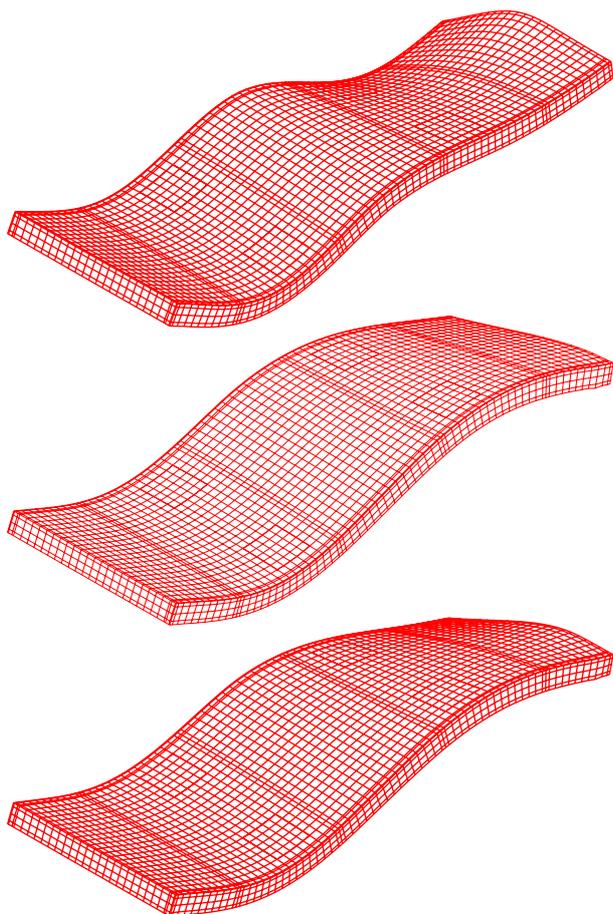
$x_N$  - numerical eigenvector

The first mode shape is anti-symmetrical like in tested plate. Numerical forms differ from each other only with length of stabilized segment and are comparable to these obtained in tests.

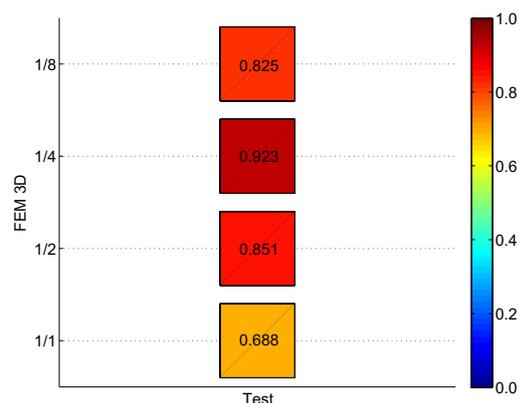
The second mode shape for 1/2 support ratio was still symmetrical like for a beam model. For 1/4 and 1/8 support ratio the nature of the second mode shape was rather anti-symmetrical, what can be observed in Fig. 11. That mode shape is similar to this obtained in test, Fig. 4. Calculated MAC values confirm this observation. The results presented above allow for the conclusion that the actual length of support has been close to 1/4 of the width of the plate.

**6 Conclusions**

The way of support implementation has an significant impact on the gap between measurements and models concerning natural frequencies, because one line support was implemented differently than expected. The most similar to results obtained in modal test are frequencies calculated



**Figure 11.** Second mode shape of 3-D model with 1/2, 1/4 and 1/8 support ratio

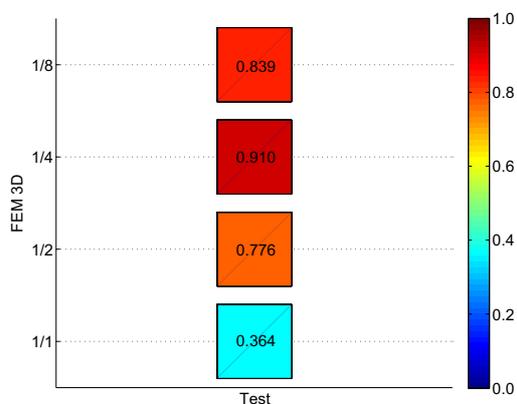


**Figure 12.** 2-D presentation of MAC values of first mode eigenvectors

for both 2-D and 3-D model as well, with length of support equal to 1/4 width of the plate on one of the edges. The similarities refer not only to values (2% difference in the frequencies for the first mode, 15% for the second) but also to mode shapes, what have been presented in Fig. 12 and Fig. 13.

### References

- [1] Siwowski T., Radomski W., *Inżynieria i Budownictwo* **7**, page 382-388 (1998)
- [2] Siwowski T., Kulpa M., Poneta P., *Materiały Budowlane*, **11**, page 58-60 (2014)
- [3] Siwowski T., Kaleta D., Kulpa M., Poneta P., *Polskie Drogi*, **6**, page 84-89 (2014)
- [4] Bathe Klaus Jorgen, *Finite Element Procedures* (Prentice Hall, 1996)



**Figure 13.** 2-D presentation of MAC values of second mode eigenvectors