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Vibration mitigation of a bridge cable using a nonlinear energy sink : design and experiment

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Abstract. This work deals with the design and experiment of a cubic nonlinear energy sink (NES) for horizontal vibration mitigation of a bridge cable. Modal analysis of horizontal linear modes of the cable is experimentally performed using accelerometers and displacement sensors. A theoretical simplified 2-dof model of the coupled cable-NES system is used to analytically design the NES by mean of multi-time scale systems behaviours and detection its invariant manifold, equilibrium and singular points which stand for periodic and strongly modulated regimes, respectively. Numerical integration is used to confirm the efficiency of the designed NES for the system under step release excitation. Then, the prototype system is built using geometrical cubic nonlinearity as the potential of the NES. Efficiency of the prototype system for mitigation of horizontal vibrations of the cable under for step release and forced excitations is experimentally demonstrated.

1 Introduction

It has been proved that by endowing nonlinear innate of some special light attachments, namely Nonlinear Energy Sink (NES), it is possible to localize the vibratory energy of important oscillators which are mainly linear [1]. This localization, which is called energy pumping or targeted energy transfer, can be operated for control and/or energy harvesting. This study aims at performing vibration mitigation of the first horizontal linear mode of a bridge cable using an essentially cubic NES prototype. The first part focuses on the experimental modal analysis of the cable, then a theoretical design analysis [2] using complexification of variables and time-multiple scales analysis is performed on a simplified 2-dof model of the first mode of the cable-NES system. The design is numerically tested for a step release excitation and then the prototype system is built and experimentally tested on a cable mounted in a tensioning bench. Experiments under periodic excitation are also performed on the same prototype.

2 Modal analysis of the experimental main system

2.1 Cable physical properties

The study focuses on a real cable tensioned on a bench. Physical properties of the cable are as it follows:

Length $L \approx 21.8$ m, diameter $D = 0.026$ m, area of steel $A = 0.76D^2/4\pi = 4.04\times10^{-4}$ m^2, inertia $I = A^2/4\pi = 2.243\times10^{-8}$ m^4, elasticity modulus $E = 160\,000$ MPa and linear mass $m_l = 3.35$ kg.m^{-1}. The bench has several sensors including accelerometers and a shaker for forced excitation.

2.2 Modal analysis

Modal analysis [3] has been performed with hammer excitation and tension in the cable varying between 50 kN and 200 kN (which means span between ends varying between 21.67 m and 21.72 m). The 9th first horizontal and vertical modes in terms of frequencies have been investigated for each tension. Experimentally identified frequencies have been compared with those obtained by theoretical results from an Irvine cable model [4]. The model has been fitted by reducing linear mass by 0.1 kg. Using modified mass $m_l = 3.25$ kg.m^{-1} the error is under 3% for all modes for a tension in the vicinity of 100 kN. Damping of the first mode has been also evaluated during these experiments.

3 Dynamics of the system

3.1 System equations

The design aims at controlling the first mode of the cable. To simplify the equation and to perform the analytical study we work in the modal domain. To design

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the NES we first reduce the behaviour of the cable to its first mode, i.e. the mode to be controlled. The equation of motion of the first mode of the cable without NES can be written as it follows in Eqns (1):

$$M\frac{d^2X}{dt^2} + \Lambda\frac{dX}{dt} + KX = 0 \qquad (1)$$

where X is the displacement at the mid span of the cable which is reduced to its first mode. The displacement at the mid span is specifically interesting as it is maximum during first mode, which means that if the NES is coupled to the cable at this point it will receive the maximum energy of the 1st mode. M, K, and Λ are classical modal and damping parameters with $M = 35.25$ kg, K = 22759 N.m^{-1} and Λ = 1.25 kg.s^{-1}. These parameters are based on the Irvine model fitted in section 2 for the cable under tension as T = 100 kN, which gives a frequency of 4.04 Hz for the first mode.

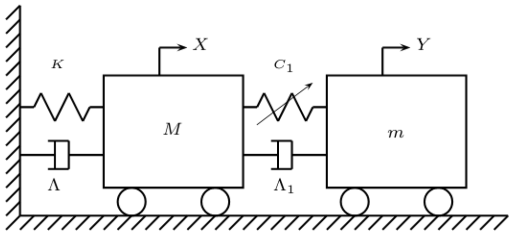

Figure 1. Scheme of the studied system, 1st cable mode coupled with a NES.

Then we study a nonlinear 2-dof system including the first linear mode of the cable coupled with the cubic NES. This coupling is represented Figure 1 where Y is the displacement of the NES, C_1 is the cubic coupling associated to the nonlinear function $\alpha \to C_1\alpha^3$ and Λ_1 is the damping of the coupling. The coupling is described by the dimensionless system of Eqns (2):

$$\begin{cases} \ddot{x} + \varepsilon\lambda\dot{x} + \varepsilon\lambda_1(\dot{x} - \dot{y}) + x + \varepsilon c_1(x-y)^3 = 0 \\ \varepsilon\ddot{y} - \varepsilon\lambda_1(\dot{x} - \dot{y}) - \varepsilon c_1(x-y)^3 = 0 \end{cases} \qquad (2)$$

where the derivative $\dot{x}$ is associated to dimensionless time $\tau = \sqrt{\frac{K}{M}}t$ such as $\dot{x} = \frac{dx}{d\tau}$. $\varepsilon = \frac{m}{M} \ll 1$ is a small parameter. Displacement are also dimensionless with the definitions as $x = \frac{X}{X_0}$ and $= \frac{Y}{Y_0}$. Following change of variable are introduced: $\varepsilon\lambda = \frac{\Lambda}{\sqrt{KM}}$, $\varepsilon\lambda_1 = \frac{\Lambda_1}{\sqrt{KM}}$, and $\varepsilon c_1 = \frac{C_1 X_0{}^2}{K}$. These kinds of equations have been studied in [2,5] investigating the behaviour of regular nonlinearities. Other system with non-regular nonlinearities and introduction of perturbation in the main system have been also investigated in [6,7].

3.2 NES design calculations

The system (2) is studied under 1:1 resonance, which means that the cable is the master system, establishing its frequency $\sqrt{\frac{K}{M}}$ to the coupled system. This master frequency is equal to 1 in the dimensionless system such as $\omega_0 = 1$. The following change of variables are introduced:

$$\begin{cases} v = x + \varepsilon y \\ w = x - y \end{cases} \qquad (3)$$

where v is the displacement of the center of mass of the system and w is the relative displacement between the NES and the cable. Then we introduce complex variables of Manevitch [8], separating slow variations of the amplitude envelope from fast variations of oscillations:

$$\begin{cases} \phi_1 e^{i\omega_0\tau} = \dot{v} + i\omega_0 v \\ \phi_2 e^{i\omega_0\tau} = \dot{w} + i\omega_0 w \end{cases} \qquad (4)$$

The system is divided in amplitude and phase such as $\phi_1 = N_1 e^{i\delta_1}$ and $\phi_2 = N_2 e^{i\delta_2}$. For small ε we have $N_1 = \frac{N_1}{\omega_0}$ and $N_2 = \frac{N_2}{\omega_0}$ which are dimensionless displacements respectively close to the amplitude of the cable and the relative displacement between the cable and the NES. Then multiple scales are introduced in derivatives with fast time $\tau_0 = \tau$ and $\tau_j = \varepsilon^j \tau$ for *j=1, 2,..* slow time scales. As exposed in [2,5,6,7] there is a nonlinear relation between N_1 and N_2 if we consider $\tau_0 \to \infty$, i.e. to analyse fixed points of the system. This relation is named as the invariant manifold in slow time τ_0 and is expressed as:

$$N_1 = N_2\sqrt{(1 + \frac{3c_1}{4}N_2{}^2)^2 + \lambda_1{}^2} \qquad (5)$$

The behaviour of the system is attracted on the invariant manifold represented Figure 2.

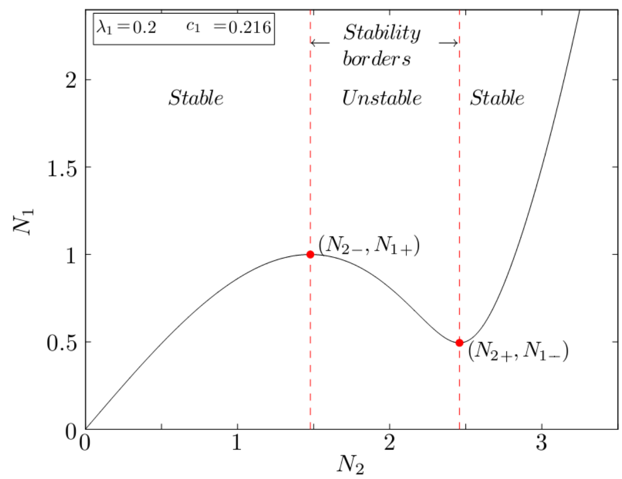

Figure 2. Invariant manifold in fast time for c_1= 0.216 and λ_1 = 0.2

This invariant exhibits two stable branches and one unstable branch, separated by two extrema. These extrema cause two different behaviours in the dynamic of the system. If initial condition on N_1 is high enough, i.e. $N_1 > (N_{2-}, N_{1+})$ the dynamic of the system will follow the invariant manifold until minimum point, i.e. (N_{2+},N_{1-}) and bifurcates through the unstable zone to reach the stable part on the left to finally reach the static equilibrium at point *(0,0)*. If initial condition is such as $N_1 < (N_{2-},N_{1+})$ the system will directly reach the stable branch on the left until static equilibrium. It has been exposed in [5,6,7,9] that the first behaviour when the dynamic of the system bifurcates is more efficient for vibration control. As a result the invariant manifold and its local maximum (N_{2-}, N_{1+}) can be used for design purpose. In our case the theoretical design threshold is X_0, i.e. a displacement equal to 1 in the dimensionless system. Finally design the NES requires to find a couple of values such as c_1 and λ_1 corresponding to an invariant with $N_{1+} = 1$. With an additional mass such as $\varepsilon = 0.01$ and a threshold at $X_0 = 0.015$ m we obtain:

$$\begin{cases} c_1 = \dfrac{C_1 X_0{}^2}{\varepsilon K} = 0.216 \\ \lambda_1 = \dfrac{2\sqrt{KM}}{\zeta_1} = 0.2 \end{cases} \tag{6}$$

The experimental device creates a mainly cubic nonlinearity with two linear springs geometrically assembled, each spring has a stiffness K_r and a free length $L_r = 0.07$ m. The geometrical nonlinearity is defined by relation $K_r = L_r{}^2 C_1$ which gives $K_r = 1070\ N.m^{-1}$. The efficiency of this design is confirmed by numerical integration of equations represented figure 3 comparing analytical prediction and numerical integration of general equations (2).

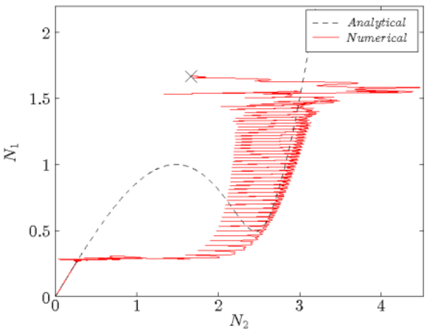

Figure 3. Theoretical invariant manifold of the dimensionless system versus the results obtained from numerical integration of general system equations for a free response with efficient NES design.

4 Experimental results of targeted energy transfer between the cable and the NES

A NES prototype for cable is built and set on the cable. The design of the prototype is close to the theoretical design with a mass $m = 0.370\ kg$ (approximately 1% of the first mode modal mass) and two linear springs of rigidity $K_r = 1000\ N.m^{-1}$. The damping of the NES is not evaluated but reduced as much as possible using lubricant.

4.1 Step release tests

Two step release tests are performed. One test is performed for a step at 0.01 m, which means 0.005 m above the design threshold, the other is performed at 0.025 m, which means 0.01 m above the threshold. Results are represented in figure 4 comparing dynamical behaviour of the cable with and without the coupled NES for the same step release excitation. Both experiments show vibration mitigation, which means that the NES system has a broadband efficiency in the vicinity of the design. Nevertheless the control is much more efficient for the step release above the design threshold (figure 4.b) with a return to equilibrium state after 7 s contrary to the test under the design threshold which shows slow mitigation as traditional damped systems. Figure 4.b underlines the effect of passing through the bifurcation for the control efficiency. Figure 5 compares experimental result and analytical prediction of the invariant manifold. The invariant manifold is plotted for different NES damping values as this variable have not been evaluated. Figure 5.a.b shows good agreement between analytical prediction and experimental results except for damping which has been over estimated in our design. Figure 5.a describes the case where the dynamics of the system decreases without going through the bifurcation whereas figure 5.b underlines clearly the interest of a good design that allows the dynamic to decrease suddenly.

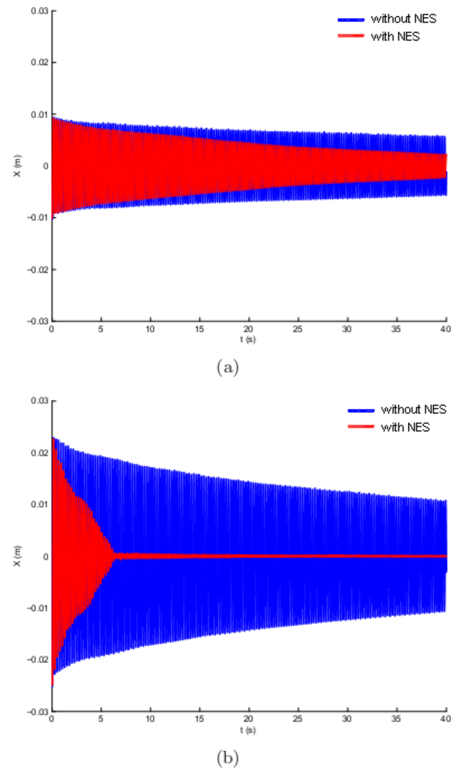

Figure 4. Experimental results comparing test with and without NES for a step release test (a) under the threshold and (b) above the threshold

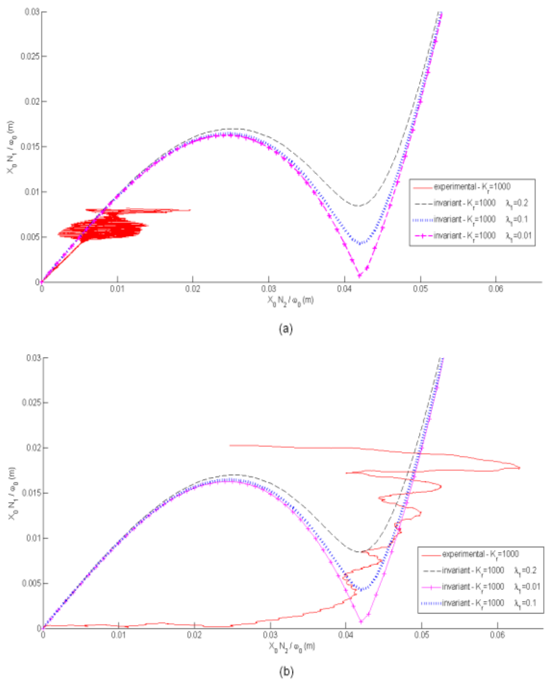

Figure 5. Experimental results compared to theoretical invariant manifolds for several damping values.

These results show the validity of the design procedure and the efficiency of the NES to control step release excitation.

4.2 Frequency response under forced excitation

Without considering design for forced excitation the same NES is tested using a shaker on the cable. The shaker delivers sinusoidal excitation for frequencies in the vicinity of the first frequency of the cable at 3.96 Hz (frequency has decreased because of thermic effect). The signal is such as we obtain a 0.025 m displacement in the mid span of the cable. Using analytical developments (they are not presented in this paper and are out the scope of the current paper) we can show that the design of the NES is supposed to limit displacement of the cable at 0.015 m.

Figure 5 represents results obtained in the frequency domain for forced tests with and without a NES. The dotted lines represent the linear response of the cable without NES, plain line represents the response with a NES and horizontal dotted line represents the theoretical threshold. These results have been obtained by doing different experiments at each frequency step, waiting for stationary regime at each step. Figure 5 underlines the efficiency of the NES for forced excitations and the “cutting” effect for the main system displacement at the threshold with a non periodic behaviour. Several experiments in the vicinity of the first have also shown good robustness of the device.

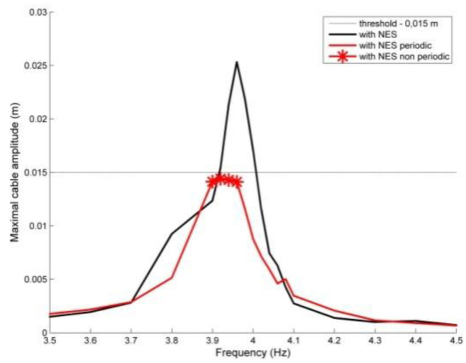

Figure 6. Result Frequency response of the cable with and without coupled NES. The cable is excited by external periodic force with the frequency in the vicinity if the first mode and the NES threshold.

5 Conclusion

This work presents an analytical procedure to design a nonlinear energy sink in order to control horizontal vibration of a cable. The procedure starts from analytical expansions leading to a design value. The design is confirmed numerically and then tested experimentally on a real cable tensioned in a bench for step release excitation. Experiments show good agreement with theoretical results and underline the efficiency of the nonlinear energy sink. The efficiency is especially high regarding the added mass, 1 % of first modal mass of the cable and only ~ 0.5 % of the full cable mass. Experiments show good result for step release but also for forced excitations which show the efficiency of the properly designed device for transient and stationary regimes.

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