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HIGH-POWER WIND TURBINE: PERFORMANCE CALCULATION

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Abstract. The paper is devoted to high-power wind turbine performance calculation using Pearson's chi-squared test the statistical hypothesis on distribution of general totality of air velocities by Weibull-Gnedenko. The distribution parameters are found by numerical solution of transcendental equation with the definition of the gamma function interpolation formula. Values of the operating characteristic of the incomplete gamma function are defined by numerical integration using Weddle's rule. The comparison of the calculated results using the proposed methodology with those obtained by other authors found significant differences in the values of the sample variance and empirical Pearson. The analysis of the initial and maximum wind speed influence on performance of the high-power wind turbine is done

1. INTRODUCTION

Increased prices for fossil fuels, the development of the northern and Arctic regions, intensification of agricultural production, the development of individual building encourage the use of heat and power systems based on wind turbines [1, 2]. The successful development of wind energy depends on the solution of three main problems:

- identification of the energy value of wind in this region and the most favorable places for the wind turbines installation;
- creating high-performance wind turbine;
- determining their functions in the energy sector in the region [2-4].

The influence of noted factors should be analyzed during the design stage by applying various methods predicting the characteristics of wind turbines.

To approximate the experimental data on the distribution of wind speeds various functions are used, but a two-parameter Weibull exponential function is the most frequently involved [1,2].

In [5] a method of wind turbine selection and evaluation of the average performance taking into account local climatic conditions based on the Weibull distribution function is proposed.

Reference [6] describes an automated method of rational choice for low-power wind turbine systems of autonomous power satisfying the consumer's price / performance ratio. It doesn't have the block testing statistical hypothesis on the applicability of the Weibull distribution.

Based on an improved version of the method [5] the analysis of the performance of high power wind turbine is done.

2. METHODOLOGY

Depending on the wind speed, electrical power generated by wind turbines with rated power equal to:

$$N(U) = N_{nm}\varphi(U), \tag{1}$$

where $\varphi(U)$ is the operating characteristic of wind turbines. For systems with constant speed wind wheel in the nominal mode approximation is applicable:

$$\phi(U) = \begin{cases} 0, \ U < U_0 \\ (U/U_0)^3, U_0 < U \le U_{nm} \\ 1, U_{nm} < U \le U_{mx} \\ 0, U > U_{mx}. \end{cases} . \tag{2}$$

Here, U_0 – the initial speed starting wind wheel's rotation; U_{nm} – the nominal speed of the wind turbine; U_{mx} - the maximum speed deriving the wind turbine from the operating mode. For example, in [7] wind turbine Wincon-200 was

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used. The average wind speed at the site at hub height of wind turbines was about 6.0 m / s. Upon reaching U_0 values of 4.5 m/s and maintaining that speed for more than a minute the manager controller signaled on start-up the wind turbine. The maximum installed power is achieved with U = 14 m/s. When U_{mx}>25 m/s wind turbine was automatically stopped to avoid damage. If U<20 m/s wind turbine was started again.

The experimental data on wind speed are summarized using the Weibull distribution function [5] proposed to describe the failures of mechanical systems in the initial period of operation, the calculation of their longevity [8]:

$$F(U) = 1 - \exp\left[-(U/c)^k\right], \tag{3}$$

where k – parameter of the form, and c - the parameter, close to the average wind speed founding by the formula:

$$U_c = c\Gamma(1 + 1/k). \tag{4}$$

The average power density of the wind flow (power per unit area of the wind wheel) is:

$$P_{0c} = 0{,}5\rho c^3 \Gamma(1 + 3/k), \tag{5}$$

where ρ – air density (1.2 kg/m^3); $\Gamma(x)$ – the gamma function.

The interpolation expression is selected for the gamma function calculation [9]. This approach was implemented in the automation of calculating the objects reliability obeying the Weibull distribution.

The nominal power of the selected wind turbine should be approximately equal to the value:

$$P_{nm} \approx \eta_p \cdot \eta_g \cdot C_p \cdot S \cdot P_{0c}, \tag{6}$$

where S – the area of the wind wheel; η_p – the efficiency of the wind wheel's rotor (0.9); η_g – the efficiency of the electric generator (0.95); C_p – the power factor taking into account the share of wind flow's power received by wind turbine (0.45).

Meteorological services record the parameters of wind on a standard height of weather vane h_f=10 m. Modern wind turbines' windwheels axles are at a height of 10 to 100 m [1, 2]. The formula is used to determine the average wind speed at these altitudes [1]:

$$U_{hc} = U_{fc}\left(\frac{h}{h_f}\right)^b, \tag{7}$$

where U_{fc} – the average wind speed at a weather vane; parameter b is equal to 0,143 for open spaces.

The average performance of wind turbines over a period of time T is:

$$W_c = \varphi_c N_{nm} T, \tag{8}$$

where φ_c – the average operating characteristic (the coefficient of wind turbine available capacity). Using Weibull distribution function [5]:

$$\phi_c = (c/U_{nm})^3\left[\gamma(a,\, U_{nm}) - \gamma(a,\, U_0)\right] + \exp\left[-\left(\frac{U_{nm}}{c}\right)^3\right] - \exp\left[-\left(\frac{U_{mx}}{c}\right)^3\right]. \tag{9}$$

Here $a = 1 + 3/k$; $\gamma(a, x)$ – the incomplete gamma function [5].

To find the values of $\gamma(a, x)$ in this paper providing high accuracy a numerical integration by Weddle's rule were used [10].

The calculation of the wind turbine performance is reduced to determining φ_c, i.e. a statistical evaluation of Weibull function parameters k and c at predetermined rates of U_0, U_{nm} and U_{mx}.

The results of long-term measurements of U in the area for each month and for the whole year are in the reference book in tabular form giving the relative frequency of entering rate in the interval expressed in percentage.

In processing of statistical data selective average wind speed and variance are calculated by known formulas [2], [5]:

$$U_{bc} = \sum_{i=1}^{m} w_i U_i, \quad D_b = \sum_{i=1}^{m} w_i (U_i - U_{bc})^2, \tag{10}$$

were m – the number of wind speed's intervals; n – the number of wind speed measurements over a given period equals to 100; n_i – the frequency of wind speed occurrence in the i-th interval; $w_i = n_i / n$ – the relative frequency.

Equating the U_{bc}, D_b from (9) and mathematical expectation and variance of the Weibull-Gnedenko distribution a transcendental equation respecting to the parameter k was found [5]:

$$f(k) = \Gamma\left(1 + \frac{2}{k}\right) / \Gamma\left(1 + \frac{1}{k}\right) - \frac{D_b}{U_{bc}^2} - 1 = 0. \tag{11}$$

The decision was done by the bisection method. And then the parameter c was calculated:

$$c = U_{bc} / \Gamma\left(1 + \frac{1}{k}\right). \tag{12}$$

Further, by means of Pearson's chi-squared test the hypothesis was carried out testing the results of wind speed measurements are consistent with the function of Weibull distribution.

The observed value of Pearson's chi-squared test was calculated by the formula [5], [11]:

$$\chi_n^2 = n\sum_{i=1}^{m}\frac{\left(w_i - p_i\right)^2}{p_i} \ , \tag{13}$$

where p_i – the probability of wind speed hitting in the i-th interval [5], [11]:

$$p_i = \begin{cases} F(U_{2i}) - F(U_{1i}), & i = 1,2,...,m-1 \\ 1 - F(U_{1i}), & i = m. \end{cases} \tag{14}$$

Using Pearson the number of degrees freedom is calculated by the formula: $k_c = s - 1 - r$, where s – the number of bits; r – the number of parameters estimated in the sample [5].

The number of degrees of freedom r of χ^2-distribution equal to the number of bits k minus the number of superimposed connections: $1)\sum_{i=1}^{8} p_i = 1;\ 2)\, m = m_x$.

To provide the automation testing the hypothesis in [12] instead of the tabulated values of the critical points of the distribution a simplified approximation of the Cornish-Fisher has been used having sufficient accuracy and fair value for an arbitrary number of degrees of freedom [13]:

$$\chi_p^2(k) = k + u_p\sqrt{2k} + \frac{2u_p\left(u_p - 1\right)}{3} + \frac{u_p\left(u_p - 7\right)}{9\sqrt{2k}}. \tag{15}$$

The quantile of standard normal distributions level u_p was count by the following formula [13]:

$$u_p = 4{,}91\left[(1-p)^{0{,}14} - p^{0{,}14}\right], \tag{16}$$

with a relative error less than 0.03%.

3. Results and discussion

Table 1 shows the results of calculations on the developed program (the second line) and presented in [5] (the third line).

Table 1. The results of calculations

U_{bc}	D_b	k	c	χ_n^2	χ_p^2
10.452	59.09	1.38	11.442	7.73	26.09
10.453	43.63	1.62	11.672	2.57	26.22

The difference in the values of the sample variance is probably due to the negligence of the calculations made in the paper [5]. Insignificant difference in the values of χ_p^2 confirms the applicability of the approximation (15) and (16).

Since the inequality holds $(\chi_n^2 < \chi_p^2)$, therefore, Weibull function can be used to describe the the considered wind characteristics.

The calculation for the height h equal 40 m showed the following: U_c = 12.74 m/s; c = 14.23 m/s; U_c = 23.35 m/s. Then the conditions of effective wind turbine took the form [5]:

$$\begin{cases} U_0 < 12{,}74\ m/s;\ 12{,}74\ m/s < U_n \le 23{,}35 \text{ m/s}; \\ U_{mx} > U_{nm};\ P_{nm} \approx 0{,}18 d^2 U_{nm}^3. \end{cases}$$

For assessing parameters were selected for modern wind turbines: U_0 = 3 m/s; U_{nm} = 16 m/s; U_{mx} = 25 m/s; d = 50 m [5]. Then the wind turbine rated power P_{nm} = 1.84 MW, and the utilization coefficient of available capacity φ_c = 0.505, i.e. using this wind turbine 50.5% of its installed capacity can be received for a year [5]. The calculation using the developed program showed φ_c is equal to 0.270, hence, the average performance of wind turbines over a period of time T is about two times less than in [5]. In this paper there is no information about methods calculating total and incomplete gamma functions.

The results of the parametric analysis showed the following: varying the wind speed in the range of possible rates (1…5 m/s) does not affect φ_c и P_c values. Conversely, increasing U_{mx} from 21 to 29 m/s resulted in increased φ_s from 0.232 to 0.296 and P_c from 0.43 to 0.55 MW.

4. Conclusion

Thus, based on the improved methodology calculating the performance of high power wind turbines previously used Pearson for statistical hypothesis testing the distribution of the general totality of air velocities by Weibull, clarified the average value operating characteristic and defined the maximum wind speed increasing the nominal capacity of wind turbines.

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