Dynamics of bubble rising at small Reynolds numbers

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Abstract. Results of experimental study of a single spherical bubble rising in the non-stationary regime in a viscous liquid (with and without surfactant) at small Reynolds numbers Re<1 have been presented. Improved empirical correlations for drag coefficient of the bubble rising with and without surfactant in the stationary regime have been obtained. Influence of nonstationary effects on the dynamics of bubble rising has been analyzed.

1. Introduction

Objective laws of motion of dispersed-phase particles, in particular gas bubbles in liquid, have an interest for solving the different practical and technological tasks associated with infinite stream flow of a single sphere. We can give, as examples, the processes of sedimentation, barbotage, flotation, etc. Interest in this problem is described by experimental and theoretical difficulties at modeling of two-phase system motion due to the problems of high-speed processes registration and small values of dynamic parameters requiring the use of sensitive elements and correct models production with account of non-stationary components in equation of motion, respectively.

In most works on the mechanics of two-phase flow, the motion of dispersed-phase particles is modeled in a quasistationary formulation with allowance for the smallness of the characteristic times of dynamic relaxation of the particles. However, in solving a number of problem, one must take into account the influence of nonstationary effects on the dynamics of motion of a particle. We can give, as examples, the interaction between particles and a high-power acoustic field [1, 2] and the deceleration of particles in interaction of a heterogeneous jet with an obstacle in the processes of deposition of strengthening coatings on structural materials [3], and also the acceleration of particles in shock waves [4], etc.

In the present work, the results of experimental study of dynamics of a single spherical bubble rising on nonstationary trajectory of its motion at small Reynolds numbers are considered. Bubble motion in pure (without surfactant) and contaminated (with surfactant) liquid is studied.

2. Experimental setup and study technique

The study was carried out on a setup consisting of a vertically installed transparent cell with plane-parallel walls, which was filled with working fluid, a device for bubble generation, and a system for visualizing the process of ascent of a single bubble. A block diagram of the experimental setup is presented in Fig. 1. The cell, of dimension 150 × 150 × 600 mm (of volume 13.5 liters), was

manufactured from optical glass of thickness 5 mm. A solution of castor oil in ethyl alcohol in different percent ratios and glycerin were used as liquids; soft soap was used as surfactant.
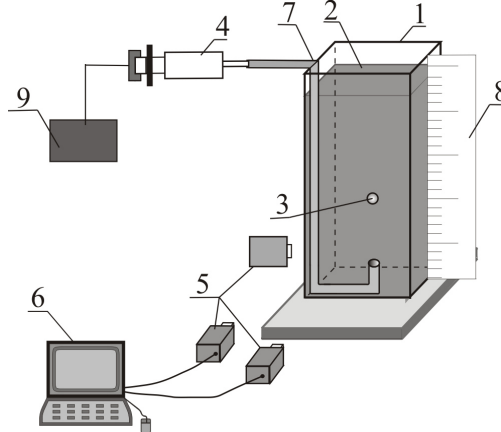

Figure 1. Block diagram of the experimental setup: 1 – cell; 2 – liquid; 3 – bubble; 4 – bubble generator; 5 – video camera; 6 – computer; 7 – tube; 8 – scale rule; 9 – electromotor

Air bubbles were produced with the aid of a mechanism enabling to meter the air volume and hence the initial diameter of a bubble entering the fluid-filled cell. To produce fixed-volume bubbles a d.c. motor which, via the worm gear converting rotational motion to translational motion, set the piston of a syringe in motion was used. To the exit cone of the syringe a hollow aluminum tube of diameter 3 mm was attached, at whose end a changeable injection needle of diameter 0.6–3.0 mm at the bottom of the cell was mounted. The separation of a bubble from the needle occurred in a "natural" manner, when the resultant of all forces was higher than zero, not due to the pressure difference (under the action of an air flow). Spherical bubbles of equal diameter with a zero initial velocity were formed.

The visualization system incorporated light sources (two 18 W luminescent lamps) installed on the back panel throughout the cell height, a Panasonic HDC-SD60 digital video camera, and two Citius C100 high-speed video cameras. Multiangle video recording of the process under study enabled to improve precision and to ensure monitoring of the change in the parameters of ascent of the bubble in different stages of its motion. The first camera was used for determining the bubble size; the recording field was 5 × 5 cm with a twofold magnification. The second camera recorded the bubble's dynamics in the initial nonstationary step of motion, whereas the third recorded it on the entire mechanical trajectory of the bubble. Video recording was carried out with a spatial resolution of 1280 × 670 pixels with a rate of no less than 700 frames per second and an exposure time of 0.5–2.0 ms. A scale rule with a scale factor of 1 mm calibrated with allowance for the optical distortion by cell walls was used to monitor the distance covered by the bubble.

3. Experimental results

The study of bubble rising dynamics was performed at following parameters: D=(0.82–5.5) mm, μ_l=(10^{-3}–2.4) Pa·s, ρ_l=(935–1260) kg/m^3, Re=3.5·10^{-3}–0.6; surfactant content z=(0.01–0.13) %.

A series of experiments was performed to improve precision of drag coefficient in a stationary regime. Analysis of experimental data on drag coefficient of a single spherical bubble shows that for bubble rising in liquid with surfactant drag coefficient at Re<1 nearly coincide with Stocks' law C_D=24/Re (full line in Fig. 2), and for bubble moving without surfactant drag coefficient at Re<1 is almost equal to Hadamard-Rybczynski' law C_D=16/Re (dash-doted line in Fig. 2). However, this effect (change mode of bubble motion) is observed only for certain time of presence t of bubble in

liquid. At surfactant content less than 0.091% and $t<5$ s the effect is not observed. It means that particles of surfactant did not manage to subside on the bubble surface. In Fig. 2 experimental data are shown by dots.
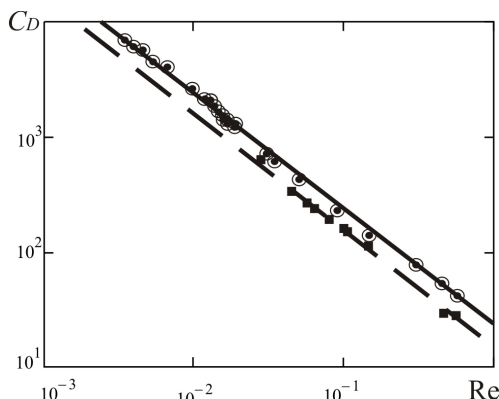

Figure 2. Dependence of drag coefficient of spherical bubble on Reynolds number: ● – with surfactant; ■ – without surfactant

The value of the experimental drag coefficient C_D was determined from the equation of motion of a bubble in a stationary regime from the formula

$$C_D = \frac{4}{3} g(\rho_l - \rho) \frac{D}{\rho_l u^2} ,$$

where g is the gravity acceleration; ρ_l, ρ is the density of liquid and gas phase, respectively; D is the bubble diameter; u is the bubble rising velocity.

The relative error in determining C_D was calculated from the formula

$$\delta C_D = \sqrt{(\delta D)^2 + 4(\delta u)^2} ,$$

where $\delta D = 3$ % and $\delta u = 0.1$ %. In calculating δC_D, the errors of measurement of the density ρ and ρ_l were not taken into consideration by virtue of their smallness. The error of experimental determination of drag coefficient did not exceed $\delta C_D \sim 3$ % for the conditions of the conducted experiments. Approximations $C_D(\mathrm{Re})$ for a bubble rising with and without surfactant have been obtained, respectively,

$$C_D = \frac{24.06}{\mathrm{Re}} ,$$

$$C_D = \frac{15.5}{\mathrm{Re}} .$$

Mean square deviation for the correlations obtained is approximately the same and equal to $\delta \approx 1.9\%$.

To evaluate the influence of the force related to the additional force F_{ad}, we consider equations of ascent of the bubble in the form

$$\frac{du}{dt} = g\left(\frac{\rho_l}{\rho} - 1\right) - \frac{3}{4} \frac{A\mu_l}{\rho D^2} u , \qquad (1)$$

$$\frac{du}{dt} = 2g - \frac{3}{2}\frac{A\mu_l}{\rho_l D^2}u , \qquad (2)$$

where $A=24.06$ – for the bubble rising with surfactant; $A=15.5$ – for the bubble rising without surfactant.

Equation (1) describes the process of ascent of the bubble without allowance for F_{ad}, whereas equation (2), with allowance for F_{ad}. The solution of these equations has the identical form

$$u(t) = u_0\left[1 - \exp\left(-\frac{t}{t_0}\right)\right], \qquad (3)$$

where u_0 is the stationary velocity of the bubble; t_0 is the characteristic time of dynamic relaxation of the bubble.

The formula for the distance covered by the bubble has the form

$$h(t) = h_0\left[\frac{u(t)}{u_0} + \ln\left(1 - \frac{u(t)}{u_0}\right)\right], \qquad (4)$$

where $h_0 = u_0 t_0$ is the characteristic distance covered by the bubble.

The time and the length of the portion of establishment of the stationary regime $u(t_*) = 0.99u_0$ are $t_* = 5t_0$ and $h_* = 3.6h_0$.

The velocity of stationary ascent is the same for equations (1) and (2) and is determined by the formula

$$u_0 = \frac{4}{3}\frac{\rho_l D^2 g}{A\mu_l} .$$

The time of dynamic relaxation for equation (1) is written in the form

$$t_0 = \frac{\rho}{\rho_l g}u_0 = \frac{4}{3}\frac{\rho D^2}{A\mu_l} ,$$

and for equation (2), in the form

$$t_0 = \frac{u_0}{2g} = \frac{2}{3}\frac{\rho_l D^2}{A\mu_l} .$$

The values of t_* and h_* increase with Reynolds number; therefore, to analyze nonstationary effect we consider results of a series of experiments for the following parameters: $D=3.2$ mm, $\rho_l =935$ kg/m^3, $\mu_l =0.23$ Pa·s, Re=0.47. Fig. 3 gives results of calculations of the bubble velocity from formulas (3) and (4) at $A=15.5$ for a nonstationary regime of ascent. Experimental values of the velocity are shown as dots in Fig. 3; the time t_* and the length h_* of the portion of establishment of stationary regime are shown as dashed lines. These dependences for bubble velocity rising $u(t)$ and $u(h)$ in pure liquid qualitatively agree with ones for bubble moving in contaminated liquid.
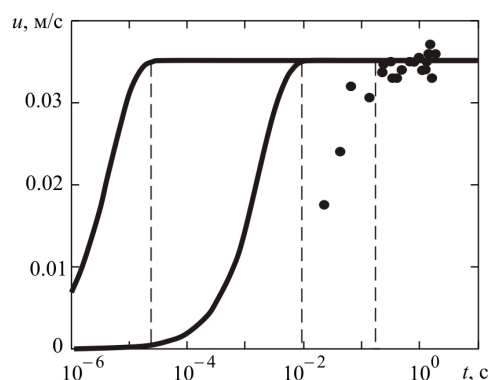
Figure 3. Ascending velocity of the bubble vs. time without allowance (1) and with allowance (2) for the force related to the additional mass

The reason for the substantional disagreement between the calculated and experimental dependences $u(t)$ and $u(h)$ and between the values of t_* and h_* for the nonstationary regime of ascent of the bubble is that the additional component of the drag force, i.e., the «hereditary» Basset force, is disregarded in equations (1) and (2).

4. Conclusion

According to the experimental results, the following conclusions can be made.
- New experimental data on the ascent bubble velocity on the stationary region of its trajectory with and without surfactant in the range of Reynolds numbers Re=$3.5 \cdot 10^{-3}$–0.6.
- Improved empirical dependences for drag coefficient of the bubble moving in pure and contaminated liquid in the range of Reynolds numbers Re=$3.5 \cdot 10^{-3}$–0.6 have been obtained.
- It is shown that viscous drag force and «hereditary» Basset force make a main contribution to drag force on the nonstationary region of its trajectory. For the nonstationary regime of motion it is necessary to take into account the force related to the additional mass and Basset force.

References

1. E. Visitski, A. Petrov, M. Shunderyuk, Applied mathematics and mechanics, **5**, 73 (2009).
2. E. Visitski, A. Petrov, Reports of RAS, **6**, 408 (2006).
3. D. Mikhatulin, Yu. Polegaev, D. Reviznikov, *Heat transfer and body destruction in supersonic heterogeneous flow* (Moscow, Yanus, 2007).
4. A. Fedorov, D. Tropin, Combustion, Explosion, and Shock Waves, **2**, 49 (2013).