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An Estimation Method for number of carrier frequency

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ABSTRACT: This paper proposes a method that utilizes AR model power spectrum estimation based on Burg algorithm to estimate the number of carrier frequency in single pulse. In the modern electronic and information warfare, the pulse signal form of radar is complex and changeable, among which single pulse with multi-carrier frequencies is the most typical one, such as the frequency shift keying (FSK) signal, the frequency shift keying with linear frequency (FSK-LFM) hybrid modulation signal and the frequency shift keying with bi-phase shift keying (FSK-BPSK) hybrid modulation signal. In view of this kind of single pulse which has multi-carrier frequencies, this paper adopts a method which transforms the complex signal into AR model, then takes power spectrum based on Burg algorithm to show the effect. Experimental results show that the estimation method still can determine the number of carrier frequencies accurately even when the signal noise ratio (SNR) is very low.

Keywords: hybrid modulation signal; AR model; Burg algorithm

1 INTRODUCTION

With the rapid development of weapons technology, the ratio of conventional radar signals (such as the single carrier frequency, the fixed repetition frequency and the no modulation in pulse signal) reduced gradually. Meanwhile, complex radar signals (such as the frequency staggering or agile, the pulse Doppler, PSK, FSK, LFM) increase gradually, and even appears a more complicated radar pulse signal—the intra-pulse hybrid modulation signal [1]. A typical characteristic of the intra-pulse hybrid modulation signal is that a single pulse signal contains two to four sub-pulses with coherent frequency. Each sub-pulse has a different carrier frequency. The difference between the hybrid modulation radar signal and the FSK hybrid modulation signal is shown by their narrow-band random modulation within their sub-pulse.

In terms of the current complex radar signal, the modulation forms of the sub-pulse are fundamentally the same. Complex radar intra-pulse modulation forms mainly include as follows: FSK, FSK-LFM, and FSK-BPSK. Modern electronic intelligence system and electronic support system must achieve the fast, real-time processing of various special radar signal in order to meet the requirements of current and future environment electronic warfare. At present, for the intra-pulse hybrid modulation signals of radar, few research work has been reported in the domestic and foreign journals. Therefore, studying and analyzing the complex radar signal is of great significance [2].

According to the general processing procedure of the signal intra-pulse modulation recognition, the primary task is to determine the number of pulse signal's carrier frequencies. In view of the frequency modulation characteristics of the three kinds of the complex radar signals, the authors use the AR model power spectrum estimation algorithm [3] to estimate the number of carrier frequencies in single pulse based on Burg algorithm. Through the AR model power spectrum estimation algorithm, the obtained power spectrum is smoother, and not sensitive to the factors of noise and random phase fluctuation compared with other methods. The algorithm can still determine the number of carrier frequency of sub-pulses accurately even in the low SNR circumstance.

2 SIGNAL MODEL

2.1 *FSK signal*

The time domain expression of FSK signal is shown as follows:

$$s(t) = Arect(t/T)\exp\left(j2\pi f_k t\right) \qquad 0 \le t \le T$$

Where, A is the pulse amplitude, $Arect(t/T)$ is the pulse signal envelope, T is the pulse modulation period, f_k is the FSK function. The characteristic of FSK signal pulse is that each sub-pulse has a different carrier frequency, so as long as we increase the number of f_k, it is capable to generate the MFSK signal. The IF of the signal can be directly expressed as follows:

$$\frac{\partial\varphi(t)}{\partial t}\frac{1}{2\pi} = f_k \qquad 0 \le t \le T$$

Figure 1 is the normalized instantaneous frequency (IF) of an FSK signal that contains 4 sub-pulses when SNR is 15dB.

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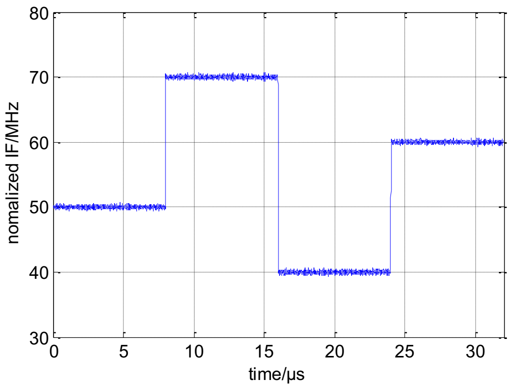


Figure 1. the normalized IF of FSK signal

2.2 *FSK-LFM hybrid modulation signal*

The time domain expression of FSK-LFM hybrid modulation signal is shown as follows:

$$s(t) = Arect(t/T)\exp\left[j2\pi\left(f_k t + Kt^2/2\right)\right] \quad 0 \le t \le T$$

Where, $K = B/T$ is the modulation slope, T is the width of sub-pulse.

The main characteristic of the FSK-LFM hybrid modulation signal is that it comprises 2~4 sub-pulses, whose duration are basically the same. In each sub-pulse, the frequency will be linear frequency modulated respectively at different fundamental frequency. Meanwhile, the frequency modulation bandwidth and modulation slope of each sub-pulse are mainly the same.

Figure 2 is the normalized IF of an FSK-LFM that contains 4 sub-pulses when SNR is 15dB.

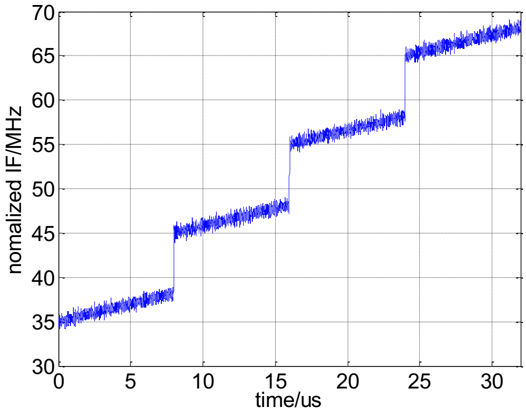


Figure 2. the normalized IF of FSK-LFM hybrid modulation signal

2.3 *FSK-BPSK hybrid modulation signal*

The time domain expression of FSK-BPSK hybrid modulation signal is shown as follows:

$$s(t) = Arect(t/T)\exp\left\{j\left[2\pi f_k t + \pi C_d(k)\right]\right\} \quad 0 \le t \le T$$

Where, $C_d(k)$ is a phase-coded function.

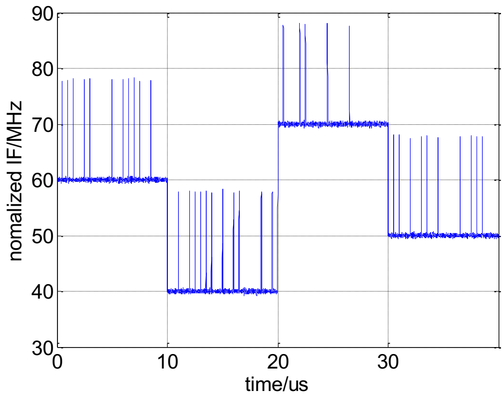


Figure 3. the normalized IF of FSK-BPSK hybrid modulation signal

The main characteristic of FSK-BPSK hybrid modulation signal is that it comprises 2~4 sub-pulses, whose duration are mainly the same. The sub-pulses are provided with coherent frequency. Meanwhile, each of their frequency modulation bandwidth is same. The sub-pulse signal meets the rules of bi-phase coded modulation.

Figure 3 is the normalized IF of an FSK-BPSK hybrid modulation signal when SNR is 15dB.

Figure 1~3 show the normalized IF of three kinds of complex modulated radar signals. We cannot determine whether the long pulse signal contains sub-pulses merely by the figures. Thus, the conventional method of analysing is unable to analyze such complex modulation signal. Therefore, the number detection of unknown pulse signals' carrier frequency is very important.

Generally speaking, in order to detect whether the carrier frequency of long signal is multiple, we always use IF combined with histogram method or Fourier transform. However, the IF combined with histogram method is particularly sensitive to noise, and it cannot determine whether there exists multi-carrier frequencies accurately when SNR is very low. Similarly, Fourier transform is sensitive to noise and phase jump so that the signal spectrum peak will be severely influenced when SNR is very low or the signal is phase-coded modulated.

3 AR MODEL POWER SPECTRUM ESTIMATION BASED ON BURG ALGORITHM

This paper proposes the AR power spectrum estimation based on Burg algorithm [4] to detect the number of unknown pulse signal's carrier frequency. The power spectrum calculated by Burg algorithm is of high resolution, small amount of calculation and high quality. Spectrum lines are much smoother than that obtained by the covariance method and the autocorrelation method. It not only can overcome the influence of background noise, but also can avoid the influence of random phase jump.

3.1 *AR model*

AR model can be expressed by the following difference equation:

$$x(n) = -\sum_{i=1}^{p} a_p(i)x(n-i) + u(n) \tag{1}$$

Where $u(n)$ is the Gauss White Noise sequence; p is the order of AR model; $H(z)$ is the AR model system transfer function:

$$H(z) = \frac{1}{1+\sum_{i=1}^{p} a_i z^{-i}} \tag{2}$$

The calculation formula of AR model power spectrum estimation is obtained by:

$$P_x(k) = \frac{\sigma^2}{\left|1+\sum_{i=1}^{p} a_i W_N^{-ki}\right|^2} \tag{3}$$

The power spectrum diagram describes the distribution characteristics of that signal power changes with frequency. It can be seen from equation (3) that the signal power spectrum diagram will have a spectrum peak in each carrier frequency. Using AR model for power spectrum estimation, we must know model parameters and the variance of white noise sequence. Equation (1) can be transformed as follows:

$$R_x(m) = \begin{cases} -\sum_{i=1}^{p} a_p(i) r_x(m-i), m \ge 1 \\ -\sum_{i=1}^{p} a_p(i) r_x(i) + \sigma^2, m = 0 \end{cases} \tag{4}$$

$$\begin{bmatrix} r_x(0) & r_x(1) & r_x(2) & \cdots & r_x(p) \\ r_x(1) & r_x(0) & r_x(1) & \cdots & r_x(p-1) \\ r_x(2) & r_x(1) & r_x(0) & \cdots & r_x(p-2) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ r_x(p) & r_x(p-1) & r_x(p-2) & \cdots & r_x(0) \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ a_2 \\ \vdots \\ a_p \end{bmatrix} = \begin{bmatrix} \sigma^2 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{5}$$

The matrix form of equation (4) is shown as follows:

Equation (4) and equation (5) are the regular equations of AR model, which are also known as (Yule - Walker) equation.

3.2 *Burg algorithm*

When using the Burg algorithm for power spectrum estimation, we don't add window on the front and back prediction error. Then the Levinson-Durbin recursion method can quickly solve the AR model coefficients. The Levinson-Durbin recursion method begins with low order and increases the order until order p, and gives all parameters of each order. It is helpful to choose the right order of AR model. Burg algorithm is based on the data sequence, which avoids the sequence estimation of the autocorrelation function, and improves the computing speed. Compared with the autocorrelation method, it has better frequency resolution [5].

Burg algorithm is used to make the sum of the front and back prediction error least [6]:

$$P_p^{fb} = \frac{1}{N-P}\sum_{n=P}^{N-1}\left\{\left|e_P^f(n)\right|^2 + \left|e_P^b(n)\right|^2\right\} \tag{6}$$

When order is from 1 to p, $e_P^f(n)$ and $e_P^b(n)$ have the following recursive relations:

$$\begin{cases} e_m^f(n) = e_{m-1}^f(n) + k_m e_{m-1}^b(n-1) \\ e_m^b(n) = e_{m-1}^b(n) + k_m e_{m-1}^f(n-1) \end{cases} \quad m = 1,2,\cdots,p \tag{7}$$

The initial conditions are as follows: $e_0^f(n) = e_0^b(n) = x(n)$

If we take equation (7) into equation (6), P_p^{fb} is only the function of k_m. Make $\partial P_p^{fb} / \partial k_m = 0$, we can obtain as follows:

$$k_m = \frac{-2\sum_{n=m}^{N-1} e_{m-1}^f(n) e_{m-1}^b(n-1)}{\sum_{n=m}^{N-1}[\left|e_{m-1}^f(n)\right|^2 + \left|e_{m-1}^b(n-1)\right|^2]} \quad m = 1,2,\cdots,p \tag{8}$$

Then reuse Levinson - Durbin recursion method can be used to quickly solve the AR coefficient as follows [7]:

$$a_m(m) = k_m$$

$$a_m(k) = a_{m-1}(k) + k_m a_{m-1}(m-k) \quad k = 1,2,\cdots,p$$

$$P_m^{fb} = (1-k_m^2) P_{m-1}^{fb}$$

In general, for the ideal AR model data, Burg algorithm can get accurate power spectrum estimation. But for the white noise and sinusoidal signal, it may sometimes appear line split phenomenon, and the position of spectrum peak and phase is closely related.

4 SIMULATION RESULT AND DISCUSSION

4.1 *FSK signal simulation*

Assume that a FSK signal contains four sub-pulses, and the frequencies of the 4 sub-pulses are 20MHz, 30MHz, 40MHz, and 50MHz. And SNR is 0dB.

Figure 4 (a) is the signal frequency spectrum, and Figure 4 (b) is the power spectrum obtained by AR model power spectrum estimation. Figure 4 shows that the frequency spectrum is seriously influenced by noise in the low SNR condition. It is difficult to judge the time domain signal frequency distribution from the frequency spectrum. However, the power spectrum diagram obtained by AR model power spectrum estimation based on Burg algorithm is extremely excellent. The spectrum lines are much smoother than that in the frequency spectrum, which is not so sensitive to random noise and has significant peaks higher than other "noise peak" at 20MHz, 30MHz, 40MHz, and 50MHz.

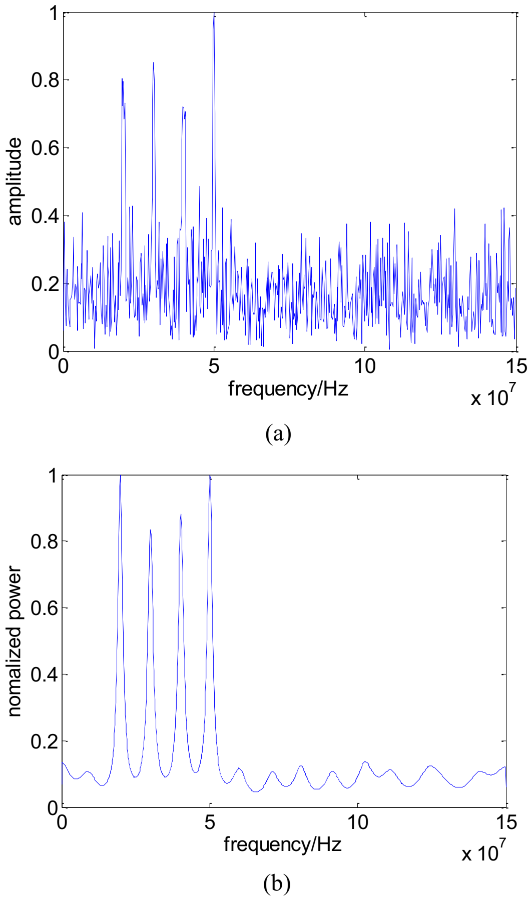


Figure 4. FSK signal frequency spectrum diagram and FSK signal power spectrum diagram

4.2 *FSK-LFM hybrid modulation signal simulation*

A FSK-LFM hybrid modulation signal which contains four sub-pulses is discussed in this subsection when SNR is 0dB. The frequencies of the 4 sub-pulses are respectively 20MHz, 30MHz, 40MHz, and 50MHz.

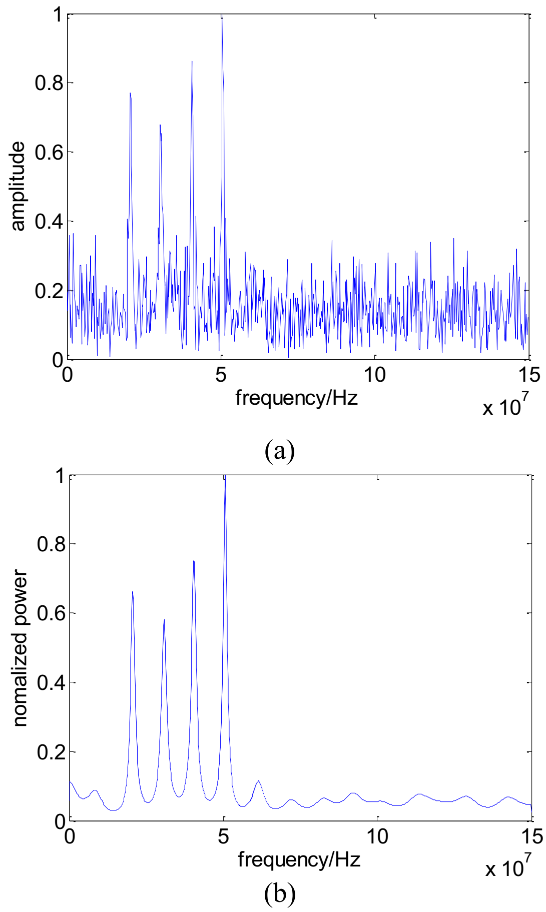


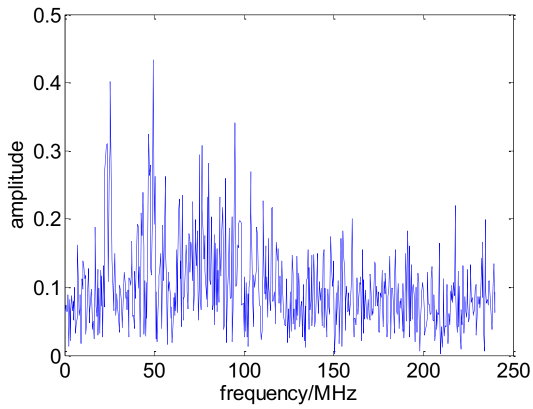
Figure 5. FSK-LFM hybrid modulation signal frequency spectrum diagram and FSK-LFM hybrid modulation signal power spectrum diagram

Figure 5(a) is the signal frequency spectrum, and Figure 5 (b) is the power spectrum estimated through our proposed method. Just like the FSK signal, the frequency spectrum of the FSK-LFM hybrid modulation signal is greatly affected by the noise. However, the power spectrum diagram by AR model power spectrum estimation based on Burg algorithm is extremely excellent, which lines are smooth, and not sensitive to random noise. Figure 5 (b) has a significantly spectrum peak higher than other peak at 20MHz, 30MHz, 40MHz, and 50MHz. Because of intra-pulse chirp, it will have certain broaden spectrum width. However, it has little influence on the number judgment of the carrier frequency for the whole.

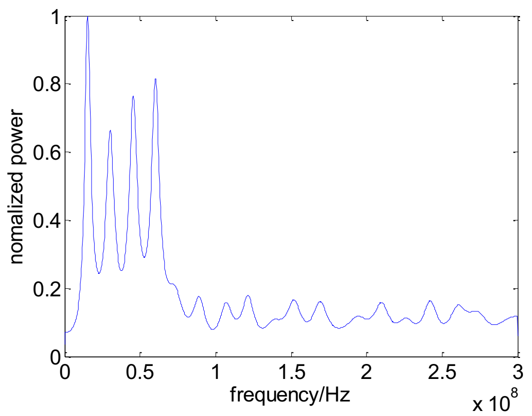
4.3 *FSK-BPSK hybrid modulation signal simulation*

The simulation condition is that a FSK-BPSK hybrid modulation signal contains four sub-pulses when SNR is 0dB, and the frequencies of 4 sub-pulses are 15MHz, 30MHz, 45MHz, and 60MHz.

Figure 6 (a) is the signal frequency spectrum diagram, and Figure 6 (b) is the power spectrum diagram by AR model power spectrum estimation. The characteristic of frequency distribution in the frequency spectrum diagram is not obvious, because of the influence of noise and the fluctuation of the random phase. The bi-phase-coded signal phase change is 0 or π, while frequency is the derivative of phase. Therefore. When phase jumps, frequency will change suddenly, which cause the characteristic of frequency distribution inconspicuous [8]. However, the power spectrum diagram by our proposed algorithm is extremely good, which lines are smooth, and not sensitive to random noise and random phase jump.



(a)



(b)

Figure 6. FSK-BPSK hybrid modulation signal frequency spectrum diagram and FSK-BPSK hybrid modulation signal power spectrum diagram

Figure 6 (b) has significant peaks higher than other peaks at 15MHz, 30MHz, 45MHz, and 60MHz. Because of the existence of phase jump, which does not meet the AR model, when using AR model for power spectrum estimation, influence of noise will be introduced by "model mismatch". Due to the interference of "noise", it can be seen from the Figure 6 (b) that the false spectral peak increased obviously. We can improve the normalization of noise power threshold to eliminate the interference.

4.4 *Algorithm adaptability at different SNR*

Without loss of generality, we assume that the normalized power spectrum of the noise is not more than 0.2. We execute 2000 Monte Carlo simulations to discuss the adaptability of the proposed method for the FSK signal, the FSK - LFM hybrid modulation signal, and the FSK-BPSK hybrid modulation signal at different SNR.

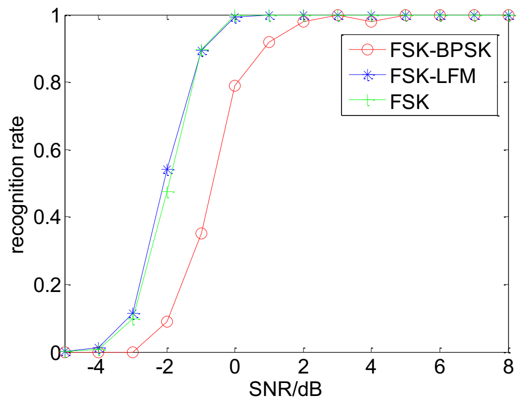


Figure 7. Recognition rate at different SNR

Figure 7 shows the adaptability of the algorithm adaptability where the three kinds of signals select suitable order of AR model. We assume that each signal has four sub-pulses.

Figure 7 shows that algorithm performance on FSK signal is the most optimal, because the FSK signal is completely consistent with the AR model. While the FSK-LFM hybrid modulation signal intra-pulse is joined with the narrowband linear frequency modulation, it will broaden the power spectrum peak. Through improving the selected order of AR model which is higher than that of the FSK, the power spectrum can get a good resolution. As the AR model order selection is very changeable, we did a large number of experiments to obtain the most suitable one. We found that the order between 30 and 40 can obtain the best effect. If the AR model order is too low, it will smooth the spectrum peak, and reduce the resolution of spectrum. Therefore, the power spectrum diagram is unable to show the structure and details of the spectrum peak and valley; if the AR model order is too high, it will increase the sham frequency components. But for the FSK-BPSK hybrid modulation signal, due to the existence of phase jump which does not meet the AR model, the introduction of the interference terms cannot be eliminated by the high order AR model. In a word, the algorithm recognition rate on FSK-BPSK hybrid modulation signal is slightly lower compared with the FSK signal and the FSK-LFM hybrid modulation signal under low SNR condition. Experiments show that the recognition rate for the aforementioned three kinds of signal can reach 98% when SNR is 2dB.

4.5 *Algorithm adaptability for different number of sub-pulses*

In this subsection, we discuss how the algorithm performance changes with the number of sub-pulses. We focus on the FSK signal, which contains 2 to 9sub-pulses, and 2000 Monte Carlo simulations are performed.

Figure 8 shows that the signal recognition rate of 9 sub-pulses can reach 95% at 0 dB SNR if the normalized noise power threshold is not more than 0.25. If the normalized noise power threshold is not more than 0.2, the recognition rate can only reach 80%.

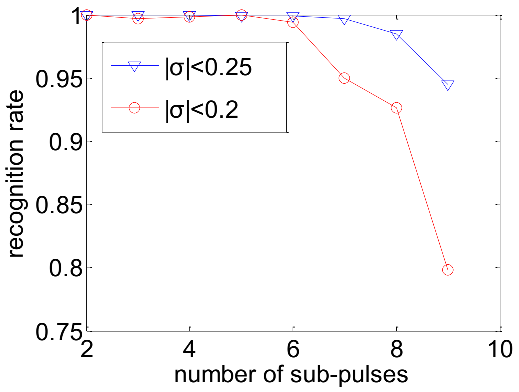


Figure 8. Recognition rate of different number of FSK signal sub-pulses

5 CONCLUSION

An algorithm for estimating the number of sup-pulses' carrier frequency has been proposed in this paper. The method solves the problem to judge whether a single pulse signal has many sub-pulses that can't be determined by traditional method. The power spectrum diagram, which is obtained by AR model spectrum estimation method based on Burg algorithm is excellent. Its lines are much smoother than that estimated by other method. The algorithm not only can overcome the influence of background noise, but also can avoid the influence of random phase jump. Therefore, it still can determine the number of sub-pulses' carrier frequency more accurately under the condition of low SNR. The experiment results show that this method can achieve the high recognition rate when the SNR is 0 dB and has strong practicability and feasibility.

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