The Problem of Reserved-lane for Hazardous Chemicals Transportation and Heuristic Algorithm

Fuqing Li
School of Electro-Mechanical Engineering, Guangdong University of Technology, Guangzhou, Guangdong, China
Department of Computer Science, Guangdong University of Finance, Guangzhou, Guangdong, China

Chi Long
Department of Computer Science, Guangdong University of Finance, Guangzhou, Guangdong, China

Naiqi Wu
School of Electro-Mechanical Engineering, Guangdong University of Technology, Guangzhou, Guangdong, China

ABSTRACT: Hazardous chemicals transportation requires a high-level of security assurance. It is an effective transport strategy that reserved-lanes are set in a transportation route to avoid accidents such as rear-end collision and crash. However, other vehicles will be inevitably influenced if a lane is set as the reserved-lane. Thus, how to minimize the influence by setting the reserved-lane becomes a problem needed to be studied. The mathematical programming model of the reserved-lane setting for hazardous chemicals transportation is established in this paper. According to the characteristics of the problem, the heuristic algorithm based on Dijkstra algorithm is proposed. This algorithm is a simple and intelligible, satisfactory solution that can be obtained in polynomials.

Keywords: hazardous chemicals transportation; problem of reserved-lane; Dijkstra algorithm; heuristic algorithm

1 INTRODUCTION

With the development of chemical industry in China and the needs of economic construction, the road transportation of hazardous chemicals becomes more and more frequent. Hazardous chemicals transportation is known as “mobile bomb” due to frequent accidents. According to statistics at home and abroad, the hazardous chemicals transportation accidents account for 30-40% of the total number of hazardous chemical accidents. It causes a great loss and forms a potential threat to the public social security [1]. There are many reasons for the frequent accidents of hazardous chemicals transportation such as improper management, insufficient safety awareness of drivers, poor conditions of transport vehicles and so on. Another important factor is the driving road condition. It is extremely easy to cause traffic accidents when special vehicles for hazardous chemicals transportation and normal vehicles are in the same road. Once there is an accident, a huge number of casualties will happen, especially in crowded cities. For example, on June 29, 2012, a serious accident of 20 deaths and 27 injuries was caused by an oil tank truck accident, which was collided by a truck at the rear end at the Nangang section of Guangzhou Yanjiang high-speed road [2]. Similar accidents can be greatly avoided by setting reserved-lanes for hazardous chemicals transportation vehicles on the road network. In fact, many experts have suggested that this kind of lanes should be set. Besides, some places in special circumstances have also taken this measure. Although the traffic safety of hazardous chemicals transportation can be improved by setting reserved-lanes, it will inevitably cause inconvenience to other vehicles. Therefore, it is a problem worthy of studying that which lanes can be selected as reserved-lanes for not only meeting the needs of the hazardous chemical transportation safety but also minimizing the impact on other vehicles.

The problem of hazardous chemical transportation is substantially a path selection problem. A lot of scholars have regarded the hazardous chemical transportation problem as the shortest path problem and carried out extensive researches based on factors such as traffic flow, driving cost, time and security. For example, List and Wijeratne, et al [3-4] carried out a study on the path selection problem of hazardous materials transportation in a random network, proposed the comparison principle of paths based on random conditions, and designed an algorithm for the solution. Chang, et al [5] studied the probability distribution function of transportation time, population coverage rate and accident rate based on random time-varying conditions and put forward an algorithm of relevant problems. Hang Wei, et al [6-7] studied the path selection of hazardous materials transportation...
with time windows limit in a random time-varying network, established a corresponding mathematical model, proved the condition of obtaining the shortest path, and provided the relevant algorithm for solution. The setting of reserved-lanes is essentially a path selection problem. Yingfeng Wu, et al \[6-9\] built up a linear integer programming model and designed a heuristic algorithm based on the practical application of large sports games. Zhou, et al \[10\] studied the problem of reserved-lanes for hazardous materials transportation by combining the two studies mentioned earlier and established a relevant model by taking into account multiple optimization goals. But it is essentially the same as the reserved-lanes setting in large sports games. Feng, et al \[11\] stipulated a time limit for transportation when studying LRPTN problem, tried to prove the complexity by assimilating LRPTN problem to STDG problem, and designed the iteration algorithm based on the cut-and-solve algorithm.

In this study, the time limit of transportation mission in Literature \[11\] is eliminated so that the problem can be a general problem of transportation of hazardous chemicals. The integer linear programming model of the problem is established on this basis. The problem of setting reserved-lanes for hazardous chemicals transportation not only has features of the shortest path problem but also has features of reserved-lanes setting. After analyzing these features, this paper designs a heuristic algorithm for the solution of this problem. The idea of this algorithm is to separate the original problem with \( k \) transportation missions into \( k \) sub-problems with a single transportation mission. Each sub-problem can be solved by Dijkstra algorithm of the shortest path problem. A section of the path with the minimum cost of reserved-lanes setting in the solution space of a sub-problem is selected as the reserved-lane at a time. After the reserved-lanes setting cost is set as 0 in the original problem, it will repeat the solution until new lanes no longer need to be set. This heuristic algorithm is simple and intelligible with high solution efficiency. The time complexity is polynomial time, and living examples will suggest that this algorithm is able to obtain satisfactory solutions.

2 PROBLEM DESCRIPTION AND MATHEMATICAL MODEL

The problem of setting reserved-lanes for hazardous chemicals transportation is described as follows: In the traffic network \( G=(V, E), V=\{1,2,\ldots, n\} \) is the set of nodes, and \( E=\{(i, j), j \in V\} \) is the set of road sections. And \( c_{ij} \) is the weight parameter of each road section \((i, j)\) representing the impact factor on other vehicles when the road section \((i, j)\) is set as a reserved-lane. And it is also called the cost of setting. The mission of hazardous chemicals transportation is represented by \( K=\{1,2,\ldots, k|k \leq n\} \), in which \( o_k \) is the source point of task \( k \), and \( d_k \) is the terminal point of task \( k \). It is also stipulated that each task has only one source point and one terminal point. The set of source points of a transportation mission is \( O=\{o_1, o_2, \ldots, o_k\} \), and the set of terminal points is \( D=\{d_1, d_2, \ldots, d_k\} \). \( O \subseteq V, D \subseteq V \). In order to ensure the safety, all vehicles for hazardous chemicals transportation must be on reserved-lanes. The objective is to select the path for vehicles in the traffic network with the minimum cost sum of reserved-lanes setting.

Definitions of variables \( x_{ij}, y_{ij}^k \) are provided as follows:

\[
x_{ij} = \begin{cases} 
1, & \text{Exclusive lanes are set on road section } (i, j) \\
0, & \text{Otherwise}
\end{cases}
\]

\[
y_{ij}^k = \begin{cases} 
1, & \text{Mission } k \text{ has passed through road section } (i, j) \\
0, & \text{Otherwise}
\end{cases}
\]

According to this, the mathematical model of the problem is:

\[
\text{Minimize } \sum_{(i,j) \in A} x_{ij} c_{ij} \tag{1}
\]

Subject to

\[
\sum_{j \in \{o_k, d_k\}} y_{ij}^k = 1, \forall k \in K, o_k \in O \tag{2}
\]

\[
\sum_{i \in \{o_k, d_k\}} y_{ij}^k = 1, \forall k \in K, d_k \in D \tag{3}
\]

\[
\sum_{j \in \{o_k, d_k\}} y_{ij}^k = \sum_{j \in \{o_k, d_k\}} y_{ij}^k, \forall k \in K, \forall i \neq o_k, d_k \tag{4}
\]

\[
y_{ij}^k \leq x_{ij} (i, j) \in A, \forall k \in K \tag{5}
\]

\[
y_{ij}^k \in \{0, 1\}, (i, j) \in A, \forall k \in K \tag{6}
\]

\[
x_{ij} \in \{0, 1\}, (i, j) \in A \tag{7}
\]

In the mathematical model of the problem of setting reserved-lanes for hazardous chemicals transportation, the objective function (1) requires that the cost of setting reserved-lanes should be the minimum in the whole traffic network. The constraint condition (2) stipulates each mission which starts from their own source points. The constraint condition (3) stipulates each mission which reaches their own destinations. The constraint condition (4) stipulates that the inflow and the outflow of each mission at the intermediate node (except the source point and the terminal point) should be the same, and makes sure that vehicles come in and go out from the same node. Constraint conditions (2)-(4) guarantee that each transportation mission has a feasible path from the source point to
the terminal point. The constraint condition (5) stipulates that only the road section \((i, j)\) passed by vehicles can be set as the reserved-lane. Constraint conditions (6) and (7) stipulate that the value range of variables is \(\{0, 1\}\).

3 ALGORITHM DESIGN

Features of the problem are presented as follows:

1. The objective of the problem is to minimize the cost of setting reserved-lanes for the traffic network. This objective of solution is consistent with the shortest path problem.

2. The setting of reserved-lanes is featured by multiple usages from one setting. Once a certain road section is set as a reserved-lane, all missions can pass through the section. But the cost of setting this road section is calculated only once. This problem is different from the shortest path problem due to this feature. In literatures [3-7], if vehicles repeatedly pass through a certain road section, weights (expense, cost and so on) of the section should be repeatedly calculated.

3. As for transportation problems, many scholars first establish mathematical models and then solve problems with methods of mathematical programming. There are also many scholars who design the solution algorithm from the angle of graph theory. It is more simple and intelligible to analyze the problem studied in this paper from the angle of graph theory.

4. The algorithm complexity of the problem is NP-Hard. Literature [11] assimilates a LRPTN problem as a STDG problem, proving that the algorithm complexity of the LRPTN problem is NP-Hard. If it is stipulated in the problem that sources of all missions are from the same node, then the problem can be assimilated as a STDG problem. Thus, the complexity is also NP-Hard. The computation time of NP-Hard problem is in exponential growth with the problem scale. Accurate algorithms are only applicable to computation cases of small scale. Therefore, this paper designs a heuristic algorithm so as to solve practical transportation problems of medium and large scale.

In the reserved-lanes setting for hazardous chemicals transportation, the problem is a typical shortest path problem if there is only one mission, which can be directly solved with Dijkstra algorithm in the time of \(O(n^2)\). Because the cost only needs to be calculated for one time if a reserved-lane is used repeatedly, the problem is not NP-Hard.

Step2. Let us suppose that \(L = \emptyset(i = 1, 2, L, k)\), and we solve each transportation mission \(k \in K\) with Dijkstra algorithm so as to obtain the path with the minimum cost. If the road section \((i, j)\) is within the path of mission \(k\) and the cost of reserved-lanes setting \(C_{i, j}\) is not 0, then the road section is incorporate in \(L_k\), incorporate the road section with the minimum cost of reserved-lanes setting of \(L\) into the set \(L_{\text{set}}\). Accumulate the cost of reserved-lanes optimal solutions of \(k\) missions do not necessarily form the solution of the whole problem. Only in the worst case, there are no identical sub-paths (road sections) between two mission paths and each optimal solution of \(k\) missions is the solution of the original problem as well as the supremum solution. Thus, it suggests that an optimal solution should be the solution with the most common sub-paths (road sections). That is also to say, the cost of reserved-lanes setting of the whole network should be the minimum and same paths should be reused in each mission as much as possible.

Let us suppose that \(X\) is the solution of the problem. It includes two parts: \(X_1\) and \(X_2\). \(X = X_1 + X_2\). \(X_1\) stands for the sum of reserved-lanes setting cost of repeatedly used road sections. \(X_2\) stands for the sum of reserved-lanes setting cost of a road section only for the use of a certain mission. \(X_1\) should be as small as possible in order to make \(X\) the minimum. Therefore, after the solution of \(k\) shortest path problems decomposed from the original problem with Dijkstra algorithm, paths with less cost of reserved-lanes setting should be selected for repeated use. The obtained heuristic algorithm is presented as follows.

Let us suppose that \(L_{(i=1,2,\ldots,k)}\) is the set of road sections (for mission \(k\)) then, \(L = \bigcup_{i=1,2,\ldots,k} L_{(i)}\). \(L_{\text{set}}\) is the set of road sections for reserved-lanes and \(C\) is the sum of reserved-lanes setting cost.

Algorithm:
1. Initialization: \(L_{\text{set}} = \emptyset, C = 0\)
2. Do
   a) \(L = \emptyset(i = 1, 2, L, k)\)
   b) Compute the shortest cost path for all \(k \in K\) by Dijkstra Algorithm
   c) if \((i, j)\) in the path of task \(k\) and \(C_{ij} \neq 0\), then \(L = L \cup \{(i, j)\}\)
   d) \(L = L \cup \{(i, j, k)\}\)
   e) if \(C_{uv}\) of \((u, v)\) is the least in \(L\), then \(L_{\text{set}} = L_{\text{set}} \cup \{(u, v)\}\)
   f) \(C_{\text{set}} = 0\)
   g) \(C_{\text{set}} = 0\)
3. While \(L \neq \emptyset\)
4. Output \(C, L_{\text{set}}\)

The solution process of the algorithm is:

Step1. The set of reserved-lanes \(L_{\text{set}}\) is initialized as an empty set and the cost of reserved-lanes setting \(C\) is initialized as 0.

Step2. Let us suppose that \(L = \emptyset(i = 1, 2, L, k)\), and we solve each transportation mission \(k \in K\) with Dijkstra algorithm so as to obtain the path with the minimum cost. If the road section \((i, j)\) is within the path of mission \(k\) and the cost of reserved-lanes setting \(C_{ij}\) is not 0, then the road section is incorporated in \(L_k\), incorporate the road section with the minimum cost of reserved-lanes setting of \(L\) into the set \(L_{\text{set}}\). Accumulate the cost of reserved-lanes setting of this road section to \(C\) and set the cost of reserved-lanes setting of this road section as 0 (Reserved-lanes only need to be set for one time, so the cost of reserved-lanes setting needs to be calculated once).

Step4. Go back to Step 2 and repeat the process until \(L\) is an empty set (namely no new road sections need to be considered in all missions).

Step5. Output the final results \(C\) and \(L_{\text{set}}\).

The process is shown in Figure 1.
4 EXAMPLES AND EXPERIMENTS

The solution process can be illustrated with a simple example. The traffic network and the cost of reserved-lanes setting of each road section are shown in Figure 2. Let us suppose that there are two transportation missions. Mission 1 is to transport from node 1 to node 4 and mission 2 is to transport from node 1 to node 5. The solution process is: (1) In initialization, \( L_{init} = \emptyset, C = 0 \); (2) \( L_1 = \emptyset, L_2 = \emptyset \); (3) Respectively calculate the paths with the smallest setting cost of mission 1 and mission 2 through Dijkstra algorithm, \( L_1 = \{(1,3), (3,5)\}, L_2 = \{(1,3), (3,5)\} \); (4) Set the road section \( (3,5) \) as a reserved-lane, \( C_{init} = 0 \). Respectively calculate the paths with the smallest setting cost of mission 1 and mission 2 through Dijkstra algorithm, \( L_1 = \emptyset, L_2 = \emptyset \); (5) Set the road section \( (3,5) \) as a reserved-lane, \( C_{init} = 0 \). Respectively calculate the paths with the smallest setting cost of mission 1 and mission 2 through Dijkstra algorithm, \( L_1 = \emptyset, L_2 = \emptyset \); (6) Set the road section \( (5, 4) \) as a reserved-lane, \( C_{init} = 0 \). Respectively calculate the paths with the smallest setting cost of mission 1 and mission 2 through Dijkstra algorithm, \( L_1 = \emptyset, L_2 = \emptyset \); (7) Set \( C_{init} = 0 \). Respectively calculate the paths with the smallest setting cost of mission 1 and mission 2 through Dijkstra algorithm, \( L_1 = \emptyset, L_2 = \emptyset \); (8) Set the road section \( (3, 5) \) as a reserved-lane, \( C = 4 \). Respectively calculate the paths with the smallest setting cost of mission 1 and mission 2 through Dijkstra algorithm, \( L_1 = \emptyset, L_2 = \emptyset \); (9) Set \( C_{init} = 0 \). Respectively calculate the paths with the smallest setting cost of mission 1 and mission 2 through Dijkstra algorithm, \( L_1 = \emptyset, L_2 = \emptyset \). Output the result.

A more general case can be also considered. The diagram in literature [12] is used as the traffic network for experiments. The weight in each side (road section) is randomly produced between \([0.5, 1] \). Start points and terminal points of missions are randomly selected in the diagram of traffic network. \( |V| \) is the number of nodes, \(|A|\) is the number of sides (road sections), \(|K|\) is the number of missions, \( N_{av} = \frac{|A|}{|A|+|K|} \) is the network density, \( C_{opt} \) is the result calculated with the mathematical programming software Cplex 12.3, \( C_a \) is the result calculated with the heuristic algorithm, and \( \frac{C_a - C_{opt}}{C_{opt}} \times 100\% \) stands for the difference. Take the average of each calculation example after 30 times of tests and experimental results are shown in Table 1.

Figure 1. Flow chart of the algorithm

Figure 2. Instance graph
It can be seen from the earlier results that all solutions of the heuristic algorithm are inferior to those of the mathematical programming software Cplex. But the difference diminishes with the increase of network density. The more missions there are, the smaller the difference is. The reason is that there are more accessible paths and road sections of common missions among nodes if the network density is stronger. Likewise, the more missions there are, the larger the possibility of repeated road sections between missions is. Therefore, the heuristic algorithm is more likely to obtain the optimal path in a reserved-lanes selection.

However, the advantage of the heuristic algorithm in time computation emerges with the increase of the problem scale. The time of solving the shortest path problem with Dijkstra algorithm is \( n^2 \). There are altogether \( k \) missions and the time of completing one circulation is \( kn^2 \). Only one road section is set in one circulation. The number of road sections of the whole traffic network is \( A \) and the time of circulations is no more than \( \frac{A}{n} \). So, the computation time of the heuristic algorithm is \( kA n^2 \), which belongs to polynomial time. However, Cplex adopts the iterative algorithm of branch and bound, the computation time of which is in exponential growth with the increase of problem scale. In practical problems, the number of nodes in the traffic network is relatively large and the problem scale is large as well. So, mathematical programming software like Cplex is helpless for this kind of problems. Now the heuristic algorithm proposed in this paper can be used to solve the problem by sacrificing the quality of solutions.

### 5 CONCLUSION

In order to avoid greater harms brought by traffic accidents like rear-end collision and crash in hazardous chemicals transportation, this paper proposes that a lane of a certain road section should be set as the reserved-lane for vehicles of hazardous chemicals transportation on the basis of allowing other vehicles to keep on passing. This is a new problem about how to minimize the influence on other vehicles in a reserved-lanes setting. The time complexity of the problem is NP-Hard. Thus, the heuristic algorithm that can be solved in polynomial time is designed according to actual transportation situations such as numerous nodes, numerous road sections and large problem scale. However, the algorithm is simple and intelligible with imperfect solution quality, so carrying out studies on algorithms with better quality is still our job for the next step.

### REFERENCES


