MATEC Web of Conferences 22, 01030 (2015)
DOI: 10.1051/matecconf/20152201030

Model for Conflict Resolution with Preference Represented as Interval Numbers

Haojie Yuan & Yong Qu
College of Science, Naval University of Engineering, Wuhan, Hubei, China

ABSTRACT: This paper takes the preference information of players to the outcomes in a form of interval numbers, and uses the possibility degree formula for the comparison between two interval numbers. In order to obtain the credible actions of players in a conflict, the concept of risk tolerance is introduced. After the stability of all feasible outcomes analyzed, the equilibrium solution of the conflict will be obtained. A military conflict scenario is given to explain the rationality and feasibility of the new model finally.

Keywords: interval number; possibility degree; risk tolerance; conflict resolution; stability analysis

1 INTRODUCTION

Since differences of opinion seem to arise whenever and wherever human beings interact with one another, conflict is present everywhere, such as, a couple trying to decide where to take their honeymoon, the negotiations over free trade among nations and the war strategy of a country in military attacks[1].

After determining the players, strategies, and preference information for a conflict model, each outcome is analyzed for stability for each player. In most situations, preference information is always represented in the form of preference vector, which is just a sequence of all outcomes from the most preferred to the least preferred to the player.

This method has two deficiencies. First, because of the uncertainty of the objective things and the fuzziness of human thought, sometimes it is difficult for players to clear sequence of different outcomes. Second, in the preference vector, if an outcome is better than another, it is not much better than the second one.

In this paper, the preference of players to outcomes is represented as interval numbers. For different two outcomes, the corresponding two interval numbers can be compared by a possibility degree formula. After introducing the concept of risk tolerance of players, the stability of all feasible outcomes can be analyzed. Finally, the method is utilized to study a military conflict scenario.

2 POSSIBILITY DEGREE METHOD FOR RANKING INTERVAL NUMBERS

Definition 1: For any a^-, $a^+ \in R$, if $a^- \le a^+$, closed interval $a=\left[a^-,a^+\right]$ is called a "interval number". Specially, a degrades into a real number when $a^-=a^+$.

Since the preference to the outcomes is represented as interval numbers, it is more truly to reflect the fuzziness of players, and could be able to describe how much the preferred one outcome is better than another. When a player makes decision in a conflict, he must compare all outcomes and then choose the best one.

Assume that $a=\left[a^-,a^+\right]$ and $b=\left[b^-,b^+\right]$ are interval numbers which are the player's preference to two outcomes,, and denote $l_a=a^+-a^-$, $l_b=b^+-b^-$. There are three different formulas to compare two interval numbers as follows.

Definition 2[2]:

$$P(a \ge b) = \min\left\{\max\left(\frac{a^+ - b^-}{l_a + l_b},0\right),1\right\}$$ is called

the possibility degree of $a \ge b$.

Definition 3[3]:

$$P(a \ge b) = \frac{\max\left\{0, l_a + l_b - \max\left(b^+ - a^-\right)\right\}}{l_a + l_b}$$ is

called the possibility degree of $a \ge b$.

Definition 4[4]:

$$P(a \ge b) = \frac{\min\left\{l_a + l_b, \max\left(a^+ - b^-,0\right)\right\}}{l_a + l_b}$$ is

called the possibility degree of $a \ge b$.

Xu[4] and Gao[5] have proved the equivalence of the three definitions. The relationship between a and b can be denoted as $a \underset{P}{\ge} b$. This definition method has intuitive sense in probability. Suppose that a,b are random variables which obey uniform distribution on $\left[a^-,a^+\right]$ and $\left[b^-,b^+\right]$ respectively, the possibility degree of $a \ge b$ is the probability of random variable a, no less than random variable b.

It is easy to prove that the possibility degree defined as above satisfies the following properties [6].

Theorem 1 Assume $a=\left[a^-,a^+\right]$, $b=\left[b^-,b^+\right]$, $c=\left[c^-,c^+\right]$, then

(1) $0\le P(a\ge b)\le 1$.

(2) (Complementary) $P(a\ge b)+P(b\ge a)=1$.

(3) (Transitivity) If $P(a\ge b)\ge 0.5$ and $P(b\ge c)\ge 0.5$, then $P(a\ge c)\ge 0.5$.

(4) $P(a\ge b)\ge 0.5$, if and only if $a^+ + a^- \ge b^+ + b^-$. Specially, $P(a\ge b)=0.5$, if and only if $a^+ + a^- = b^+ + b^-$.

3 STABILITY ANALYSIS

Definition 5: Assume r is a real number, and a,b are two interval numbers which respectively denote a player's preference to outcome 1 and outcome 2, if the player deems outcome 1 is more preferred than outcome 2 when $P(a\ge b)>1-r$, and then r is called the "risk tolerance" of this player.

Definition 6: If a player could reach a more preferred outcome just changing his strategy to while the strategies of the other players remain unchanged, the player will take unilateral improvements (UI's) from the present outcome. This action of the player is called "credible action".

There are four types of stability that can be determined for the outcomes of each player. A particular outcome for a given player can be[7]:

1). ***Rational***. In this situation, the player has no UI he can make from the outcome. The player's strategy is the best he can take given the strategy selection of the other players. A rational outcome is stable, and is denoted by ***r***.

2). ***Sanctioned***. For all UI's availability to the player for a particular outcome, credible actions can be taken by the other players which result in a less preferred outcome for this player. The possibility that a worse outcome could result from changing his strategy deters the player and induces a type of stability that is labeled "s".

3). ***Unstable***. The player has at least one UI from which the other players can take no credible action that would result in a less preferred outcome than the outcome from which the player is improving. A credible action by a player is one which results in a more preferred outcome for himself. An unstable outcome is denoted by "u".

4). ***Stable by Simultaneity***. After the points aforementioned, three types of stability have been determined for all the outcomes for each player, if an outcome is unstable for at least two players, simultaneous action by more than one player could cause a less preferred outcome to occur and thereby induce stability for an outcome which is previously thought to be unstable for a given player. This outcome is also a type of stable, denoted by $\bar{u}$.

Definition 7: If an outcome is stable for all players, this outcome is called "equilibrium solution" of the conflict, which is most likely to happen.

4 CASE STUDY: A MILITARY CONFLICT SCENARIO

Suppose there are two countries, A and B. Country A intends to take military action against country B, which forms a conflict. In Table 1, each country involved in the conflict is listed above the set of options available for that country. Short forms for the countries and options which are given in parentheses are utilized in the text and also subsequent tables. As can be seen in Table 1, country A has the options of attack from the air, attack from the sea and negotiates peace. Country B's options are defense from the air, defense from the sea and negotiating peace.

Suppose two countries correctly know each other's options and strategies. Thirteen feasible outcomes are displayed in Table 1. Each outcome can be written as a column of 1's and 0's, where 1 means the option is selected by the country and 0 indicates that the option is not taken. All the outcomes are numbered at the bottom of Table 1.

The preference to outcome $i(1\le i\le 13)$ for country A is denoted by $a_i=[a_i^-,a_i^+]$, and denoted by $b_i=[b_i^-,b_i^+]$ for country B, which is all listed in Table 2.

Table 1. Feasible Outcomes in the conflict

Country A													
Attack from the air	0	1	1	1	1	0	0	0	0	1	1	1	1
Attack from the sea	0	0	0	0	0	1	1	1	1	1	1	1	1
Negotiate peace	1	0	0	0	0	0	0	0	0	0	0	0	0
Country B													
Defense from the air	0	1	0	0	1	1	0	0	1	1	0	0	1
Defense from the sea	0	0	1	0	1	0	1	0	1	0	1	0	1
Negotiate peace	1	0	0	1	0	0	0	1	0	0	0	1	0
Outcome number	1	2	3	4	5	6	7	8	9	10	11	12	13

Table 2. The stability analysis of the conflict

Country A													
Individual stability	u	s	r	r	r	r	u	r	s	s	s	s	u
UI's	4 6 12	6 10					3 11		5	6	3	4 8	5 9
Country B													
Individual stability	r	r	u	u	u	s	r	u	s	r	u	u	r
UI's			2 5	2 3 5	2	7		6 7 9	7		10 13	10 11 13	
Outcome number	1	2	3	4	5	6	7	8	9	10	11	12	13
Overall stability	X	E	X	X	X	E	X	X	E	E	X	X	X

Suppose the risk tolerance of country A and country B are respectively $r_A=0.2$ and $r_B=0.1$. The stability analysis of the conflict is shown in Table 3.

Table 3. Preference in the conflict

Outcome	Country A	Country B
1	$[0.40, 0.50]$	$[0.90, 0.95]$
2	$[0.45, 0.55]$	$[0.80, 0.90]$
3	$[0.75, 0.85]$	$[0.50, 0.60]$
4	$[0.90, 0.95]$	$[0.35, 0.55]$
5	$[0.55, 0.65]$	$[0.65, 0.75]$
6	$[0.70, 0.80]$	$[0.70, 0.80]$
7	$[0.35, 0.45]$	$[0.85, 0.95]$
8	$[0.85, 0.95]$	$[0.40, 0.50]$
9	$[0.50, 0.60]$	$[0.75, 0.85]$
10	$[0.60, 0.70]$	$[0.60, 0.70]$
11	$[0.65, 0.75]$	$[0.45, 0.55]$
12	$[0.80, 0.90]$	$[0.30, 0.40]$
13	$[0.25, 0.35]$	$[0.55, 0.65]$

The outcomes including outcomes 2, 3, 4, 5, 6, 8, 9, 10, 11, 12 are stable for country A. For country B, outcomes 1, 2, 6, 7, 9, 10, 13 are stable. Outcomes 2, 6, 9, 10 are stable for both countries, which are the equilibrium solution of the conflict.

If the risk tolerance of country A and country B both changed into 0, the analysis result is the same as the first situation, that is, outcomes 2,6,9,10 are the equilibrium solution of the conflict. However, if the risk tolerance of country A and country B both change into 0.5, the equilibrium solution changes, which is only including outcomes 6,9,10.

Adjusting the risk tolerance of two players for more situations, it could be find that with the increase of the risk tolerance, there may be more UI's from an outcome, which possibly causes the decrease of the number of equilibrium solution.

5 CONCLUSIONS

In this paper, the preference information about outcomes is represented in the form of interval numbers, which reflects the uncertainty of the preference. In this case, a player compares two outcomes by a possibility degree formula and the risk tolerance he could undertake. After analyzing the stability of all outcomes for each player, the equilibrium solution of the conflict will be obtained. A military conflict scenario is employed to demonstrate how this approach may be applied, and the result shows that equilibrium solution may change with different risk tolerance of players.

REFERENCES

[1] Yakov Ben-Haim. & Keith W.Hipel. 2002. The graph model for conflict resolution with information-gap uncertainty in preferences. *Applied Mathematics and Computation*, 126: 319-340.

[2] Facchinetti G, Ricci R G. & Muzzioli S. 1998 Note on ranking fuzzy triangular numbers. *International Journal of Intelligent Systems*, 13: 613-622.

[3] Da Q L. & Liu X W. 1999. Interval number linear programming and its satisfactory solution. *Systems Engineering -Theory & Practice*, 19(4): 3-7.

[4] Xu Z S. & Da Q L. 2003. Possibility degree method for ranking interval numbers and its application. *Journal of Systems Engineering*, 18(1): 67-70.

[5] Gao F J. 2013. Possibility degree and comprehensive priority of interval numbers. *Systems Engineering – Theory &Practice*, 33(8): 2033-2040.

[6] Qiu D S, He C. & Zhu X M. 2012. Ranking method research of interval numbers based on probability reliability distribution. *Control and Decision*, 27(12): 1894-1898.

[7] Keith W. Hipel, Niall M. Fraser. Using game theory to model political uncertainty.