

## Adaptive active vibration isolation – A control perspective

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**Abstract.** In many classes of applications like active vibration control and active noise control, the disturbances can be characterized by their frequencies content and their location in a specific region in the frequency domain. The disturbances can be of narrow band type (simple or multiple) or of broad band type. A model can be associated to these disturbances. The knowledge of the disturbance model as well as of the compensator system is necessary for the design of an appropriate control system in order to attenuate (or to reject) their effect upon the system to be controlled. The attenuation of disturbances by feedback is limited by the Bode Integral and the “water bed” effect upon the output sensitivity function. In such situations, the feedback approach has to be complemented by a “feedforward disturbance compensation” requiring an additional transducer for getting information upon the disturbance. Unfortunately in most of the situations the disturbances are unknown and time-varying and therefore an adaptive approach should be considered. The generic term for adaptive attenuation of unknown and time-varying disturbances is “adaptive regulation” (known plant model, unknown and time-varying disturbance model).

The paper will review a number of recent developments for adaptive feedback compensation of multiple unknown and time-varying narrow band disturbances and for adaptive feedforward compensation of broad band disturbances in the presence of the inherent internal positive feedback caused by the coupling between the compensator system and the measurement of the image of the disturbance. Some experimental results obtained on a relevant active vibration control system will illustrate the performance of the various algorithms presented.

### 1. Introduction

#### 1.1. The problem

Attenuation of vibrations and noise is a current challenge in many areas. Often the term “isolation” is used to characterize the process of attenuation of disturbances or noise. One can distinguish several types of isolation:

- Passive: use materials with noise vibration attenuation properties.
- Semi active: change the attenuation properties of the materials used for attenuation.
- Active: use compensation force noise to counteract vibrations or noise.
- Adaptive: since the characteristics of vibration/noise are unknown and/or time varying, adaptation should be added to ANC and AVC.

The active isolation systems are also called Active vibration Control (AVC) and Active Noise Control (ANC). The principles of active vibration control and active noise control are similar. Of course the range of frequencies and the type of instrumentation are different but same control techniques can be used. However the present paper will focus on adaptive active vibration control.

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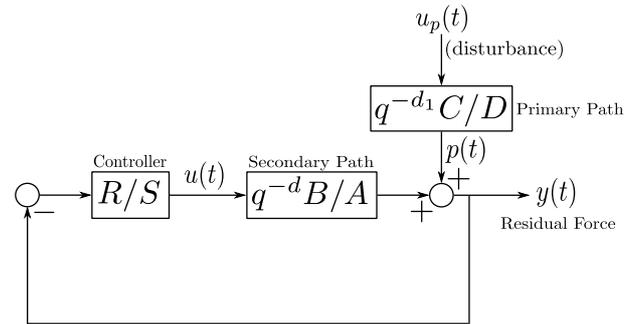
The book [1] gives a compact and clear presentation of the origin and evolution of active vibration control techniques. It should be mentioned that these techniques often have been invented by researchers in the area of vibration isolation and signal processing. The interest of the automatic control community on these subjects is much more recent (started essentially in the nineties). The objective of the present paper is to look at the problem of active vibration control from the perspective provided by the automatic control methodology. From this perspective the vibrations which we would like to strongly attenuate (or eliminate) are called generically “disturbances”. One of the major objective of automatic control is exact rejection or attenuation of disturbances by feedback and feedforward actions.

In many classes of applications like active vibration control (active suspension, control of disk drives) and active noise control, the disturbances acting upon a system and which have to be compensated (rejected, attenuated) can be characterized by their frequencies content and their location in a specific region in the frequency domain. The disturbances can be of *narrow band* type (simple or multiple) or of *broad band* type. Of course a combination of both is possible and what we call broad band may be in certain cases finite band disturbances over a small region in the frequency domain. However the distinction between these two types of disturbances is convenient in order to examine the techniques used for their compensation.

Fundamentally in active control a compensator system is introduced which will generate a “secondary” source.

This compensator (acting through the “secondary path”) conveniently driven will interfere destructively with the disturbance coming from the “original” primary source (in general non accessible) through which is called the “primary path”. In the control terminology the “secondary path” is the *plant to be controlled* in order to reduce as much as possible the effect of the disturbance on the controlled output which in the case of active vibration control is the measured residual acceleration or force. To achieve this generically a feedback controller will be used. An important concept which allows to assess the disturbance attenuation properties, stability of the feedback control loop and robustness is the so called “output sensitivity function” (the transfer function between the disturbance and its effect on the measured output). There are some fundamental issues when approaching the problem of attenuating the disturbances by feedback. The first fundamental issue is related to the properties of the famous “Bode integral” on the modulus of the output sensitivity function expressed in  $dB^1$  which has value zero if the system is open loop stable. Since the objective is to strongly attenuate (even reject totally asymptotically) the disturbance, this may require significant holes (low values) in the magnitude of the sensitivity function which in turn (even with a very careful design) may lead to unacceptable “water bed” effect both in terms of performance (one amplifies at certain frequencies where some disturbance can still be present) as well as in terms of robustness (the modulus margin may become unacceptable<sup>2</sup>). Therefore there are inherent limitations in using feedback for vibration isolation. Basically narrow band disturbances can be rejected by feedback up to a certain number (at least 3 – see [3]). Finite band disturbances sufficiently “narrow” can also be handled by feedback only. However broad band disturbance attenuation will require to use adaptive feedforward compensation in addition. But before moving to the case of feedforward compensation let's mention another fundamental result in feedback control which is of great interest for the problem of vibration attenuation. This is the “internal model” principle which says that the disturbance will be asymptotically cancelled if and only if the controller contains the “model of the disturbance”.

This brings in view the concepts of “plant model” and “disturbance model”. In order to properly design the feedback controller the knowledge of the “plant model” and “disturbance model” are necessary. The control methodology uses “model based design” known as MBC (model based control). One should distinguish between “knowledge plant model” and “dynamic plant model”. The “knowledge plant model” is obtained from the law of physics and mechanics describing the operation of the compensator system. Unfortunately these models are often not precise enough since on one hand their precision depends on the perfect knowledge of some physical parameters (hard to get) and on the other hand



**Figure 1.** Block diagram of the active vibration compensation by feedback.

it is difficult to model all the elements constituting the compensator system. For this reason one uses which is called the “control dynamic plant model” i.e. a kind of filter which describes the dynamical relationship between the variations of the control input and the output of the system. This kind of model necessary for design can be obtained directly from an experimental test using the techniques of “System Identification”. In most of the applications the characteristics of the compensator systems remain almost unchanged during operation, which means that the associated dynamic control model remains almost unchanged and therefore the parameters of the identified model remain almost unchanged. However for controller design we need in addition the “model of the disturbance”. A common framework is the assumption that the disturbance is the result of a white noise or a Dirac impulse passed through the model of the disturbance. The knowledge of this model together with the knowledge of the model of the secondary path (compensator) allows the design of an appropriate control strategy. In practice in most of the cases the characteristics of these disturbances are unknown or time-varying. While in some particular cases a robust design can be considered [4,5], in most of the situations, as a consequence of the high level of attenuation required, an adaptive approach is necessary for obtaining a good tuning with respect to the disturbance characteristics.

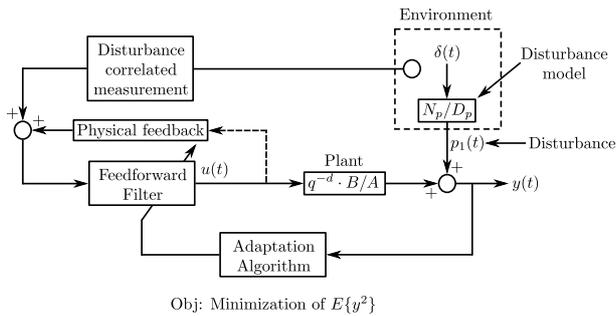
When considering the model of a disturbance, one has to address two issues: (i) its structure (complexity, order of the parametric model) and (ii) the values of the parameters of the model. In general, one can assess from data the structure for such model of disturbance (using spectral analysis or order estimation techniques) and assume that the structure does not change. Therefore adaptation will have to deal with the change in the parameters of the model of the disturbance.

However, when the limitations induced by the Bode integral do not allow to achieve the desired performances by feedback (in particular for the case of broad band disturbances) one has to consider to add a feedforward compensation which requires a “source” correlated with the disturbance to be attenuated.

In a number of applications areas including active noise control (ANC) and active vibration control (AVC), an image (a correlated measurement) of the disturbances acting upon the system can be made available. This

<sup>1</sup> The output sensitivity function is the transfer function between the disturbance and the measured controlled output.

<sup>2</sup> The modulus margin is the minimum distance between the open loop transfer function hodograph and the Nyquist point [2].



**Figure 2.** Adaptive feedforward compensation of unknown disturbances.

information is very useful for attenuating the disturbances using a feedforward compensation scheme. However, the feedforward compensator filter will depend not only upon the dynamics of the plant (the compensator system) but also upon the characteristics of the disturbances. Since the characteristics (the model) of the disturbances are generally unknown and may be time-varying, adaptive feedforward compensation has to be used. Fig. 2 illustrates the adaptive rejection of unknown disturbances by feedforward compensation. A “well located” transducer can provide a measurement, highly correlated with the unknown disturbance (a good image of the disturbance). This information is applied to the control input of the plant through an adaptive filter whose parameters are adapted such that the effect of the disturbance upon the output is minimized.

Adaptive feedforward vibration (or noise) compensation is currently used in ANC and AVC when an image of the disturbance is available. However, at the end of the nineties it was pointed out that in most of these systems there is a physical “positive” feedback coupling between the compensator system and the measurement of the image of the disturbance (vibration or noise) [6–9]. The internal inherent physical positive feedback may cause the instability of the AVC or ANC systems. This will be illustrated on the experimental platform which will be presented in Sect. 3. However it is possible to achieve attenuation (rejection) of narrow band disturbances without measuring them by using only a feedback approach. But since the model of the disturbance is unknown and/or time-varying an adaptive feedback approach has to be used.

So at this point one can say that we have two types of disturbances

- Single or multiple narrow band disturbances:
- Broad (finite) band disturbances.

and two approaches for doing disturbance attenuation:

- Adaptive feedback approach (which requires only a measurement of the residual force, acceleration, noise).
- Adaptive feedforward compensation requiring an additional transducer for getting a correlated measurement with the disturbance.

## 1.2. The adaptive regulation paradigm

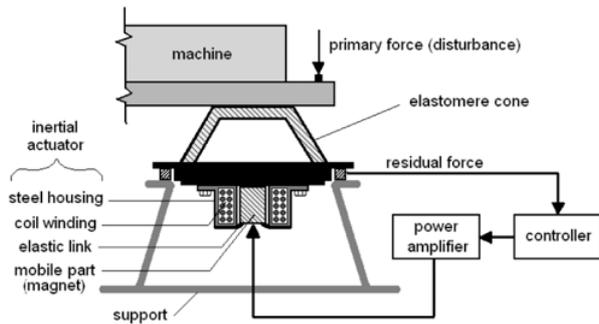
Consider the case of attenuation (rejection) of disturbances by feedback only. Since the parameters of the disturbance model are unknown and/or time-varying in order to get satisfactory performance an adaptive feedback approach has to be considered<sup>3</sup>. The classical adaptive control paradigm deals essentially with the construction of a control law when the parameters of the plant dynamic model are unknown and time-varying ([10]). However, in the present context, the plant dynamic model is almost invariant and it can be identified. The objective then is the rejection of disturbances characterized by unknown and time-varying disturbance models. It seems reasonable to call this paradigm *adaptive regulation*. In classical “adaptive control” the objective is tracking/disturbance attenuation in the presence of unknown and time-varying plant model parameters. Therefore adaptive control focuses on adaptation with respect to plant model parameter variations. The model of the disturbance is assumed to be known and invariant. Only a level of attenuation in a frequency band is imposed (with the exception of DC disturbances where the controller may include an integrator). In *adaptive regulation* the objective is to asymptotically suppress (attenuate) the effect of unknown and time-varying disturbances. Therefore, *adaptive regulation* focuses on adaptation of the controller parameters with respect to variations in the disturbance model parameters. The plant model is assumed to be known. It is also assumed that the possible small variations or uncertainties of the plant model can be handled by a robust control design. The problem of adaptive regulation as defined above has been previously addressed in a number of papers ([11–21]) among others. [3] presents a survey of the various techniques (up to 2010) used in adaptive regulation as well as a review of a number of applications.

An international benchmark has been organized on the attenuation of multiple and unknown time varying narrow band disturbances by feedback. The test bed was an active vibration control system. The results are summarized in [22]. However the adaptive regulation covers also the case of feedforward since on one hand adaptation has to deal with the change in the characteristics of the disturbances and on the other hand it is still a feedback structure as shown before.

The paper is organized as follows. Sections 2 and 3 will present two active vibration control systems:

- An active vibration control system used to test algorithms for disturbance attenuation by adaptive feedback.
- An active vibration control system used to test algorithms for combined adaptive feedforward plus feedback.

<sup>3</sup> Since it is not possible to design a robust controller which introduces a strong attenuation over a large frequency region as a consequence of the Bode Integral (water bed effect), one can not construct a single controller achieving strong attenuation of disturbances with varying frequencies.



**Figure 3.** Active vibration control using an inertial actuator (scheme).

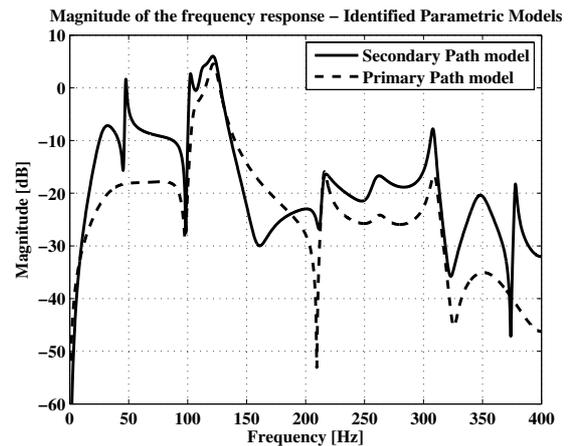


**Figure 4.** Active vibration control system (photo).

Section 4 will review the basic principles for identification of the compensator dynamic model from experimental data. Section 5 will review some basic algorithms for the attenuation of multiple unknown and time varying narrow band disturbances. Section 6 will present some experimental results concerning attenuation of narrow band disturbances with the direct adaptive regulation approach. Section 7 will discuss the adaptive attenuation of broad band disturbances by feedforward and feedback. Section 8 gives a summary of experimental results for attenuation of broad band disturbances using adaptive feedforward with or without additional feedback control.

## 2. An active vibration control system using feedback compensation

The structure of the system is presented in Fig. 3. A general view of the whole system including the testing equipment is shown Fig. 4. It consists of a passive damper, an inertial actuator, a load, a transducer for the residual force, a controller, a power amplifier and a shaker. The inertial actuator will create vibrational forces which can counteract the effect of vibrational disturbances. The equivalent control scheme is shown in Fig. 2. The system input,  $u(t)$  is the position of the mobile part (magnet) of the inertial actuator (see Figs. 3 and 2), the output  $y(t)$  is the residual force measured by a force sensor. The



**Figure 5.** Frequency response (magnitude) for the primary and the secondary paths models.

transfer function ( $q^{-d_1} \frac{C}{D}$ ), between the disturbance force,  $u_p$ , and the residual force  $y(t)$  is called *primary path*. In our case (for testing purposes), the primary force is generated by a shaker driven by a signal delivered by the computer. The plant transfer function ( $q^{-d} \frac{B}{A}$ ) between the input of the inertial actuator,  $u(t)$ , and the residual force is called *secondary path*. The control objective is to reject the effect of unknown narrow band disturbances on the output of the system (residual force), i.e. to attenuate the vibrations transmitted from the machine to the chassis. The physical parameters of the system are not available. The system has to be considered as a *black box* and the corresponding models for control design should be identified. The sampling frequency is  $F_s = 800$  Hz.

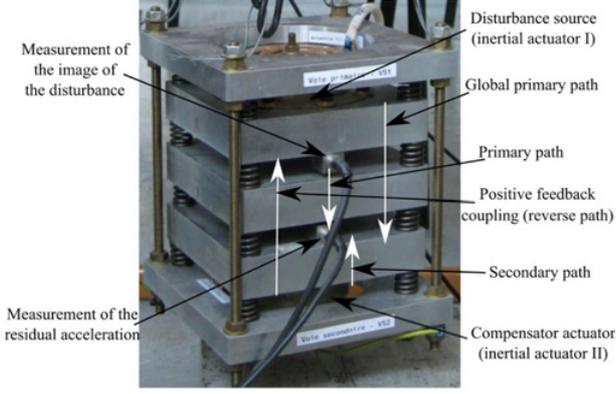
Figure 5 gives the frequency characteristics of the identified model for the primary and secondary paths<sup>4</sup> more details can be found at: ([http://www.gipsa-lab.grenoble-inp.fr/~ioandore.landau/benchmark\\_adaptive\\_regulation/index.html](http://www.gipsa-lab.grenoble-inp.fr/~ioandore.landau/benchmark_adaptive_regulation/index.html)).

## 3. An active vibration control system using feedforward-feedback compensation

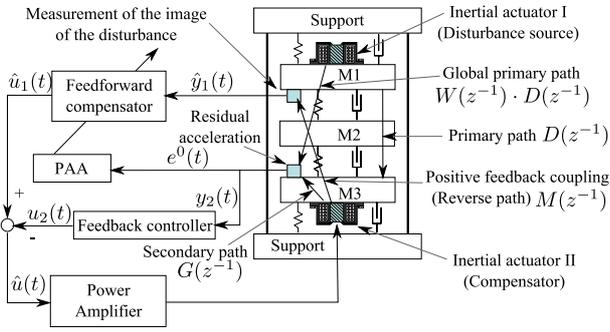
Figs. 6 and 7 show an AVC system applied to a distributed flexible mechanical structure. The corresponding block diagram is shown in Fig. 8. This mechanical structure is representative for a number of situations encountered in practice and will be used to illustrate the performance of the various algorithms which will be presented in this paper.

It consists of five metal plates connected by springs. The uppermost and lowermost ones are rigidly jointed together by four screws. The middle three plates will be labeled for easier referencing M1, M2 and M3 (see Fig. 7). M1 and M3 are equipped with inertial actuators. The one on M1 serves as disturbance generator (inertial actuator I in Fig. 7), the one at the bottom serves for disturbance compensation (inertial actuator II in Fig. 7). Inertial actuators use a similar principle as loudspeakers (see [3]).

<sup>4</sup> Primary path model is used only for simulation purposes.



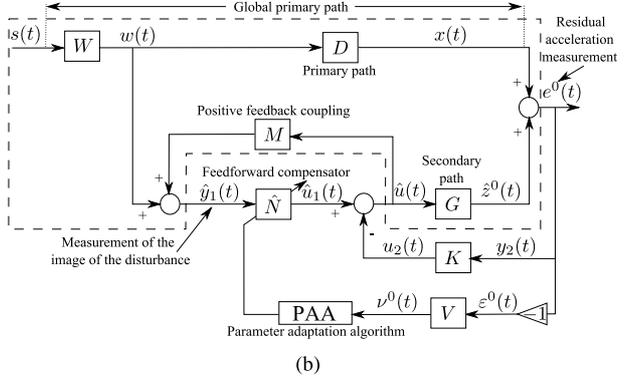
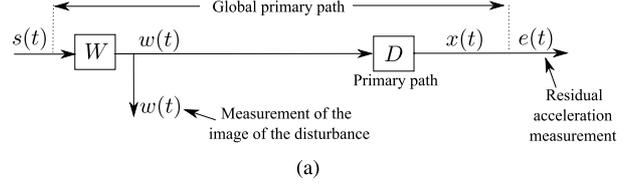
**Figure 6.** An AVC system using a feedforward compensation – photo.



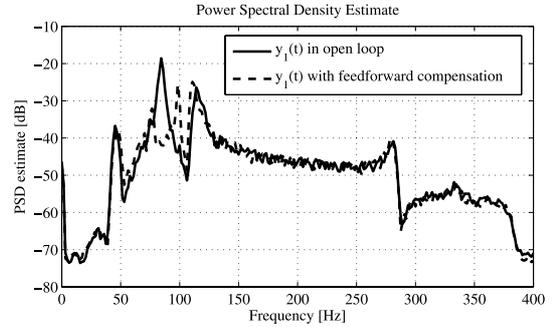
**Figure 7.** An AVC system using feedforward and feedback compensation – block diagram.

The correlated measurement with the disturbance (image of the disturbance) is obtained from an accelerometer which is positioned on plate M1. Another sensor of the same type is positioned on plate M3 and serves for measuring the residual acceleration (see Fig. 7). The objective is to minimize the residual acceleration measured on plate M3. This experimental setting allows to experiment both adaptive feedforward compensation (with or without additional feedback) as well as adaptive feedback disturbance compensation alone (without using the additional measurement upstream).

The disturbance is the position of the mobile part of the inertial actuator (see Figs. 6 and 7) located on top of the structure. The input to the compensator system is the position of the mobile part of the inertial actuator located on the bottom of the structure. When the compensator system is active, the actuator acts upon the residual acceleration, but also upon the measurement of the image of the disturbance through the *reverse path* (a positive feedback coupling). The measured quantity  $\hat{y}_1(t)$  will be the sum of the correlated disturbance measurement  $w(t)$  obtained in the absence of the feedforward compensation (see Fig. 8a) and of the effect of the actuator used for compensation. This is illustrated in Fig. 9 by the spectral densities of  $y_1(t)$  in open loop and when feedforward compensation is active (the effect of the mechanical feedback is significant).



**Figure 8.** Feedforward AVC: (a) in open loop and (b) with adaptive feedforward + fixed feedback compensator.



**Figure 9.** Spectral densities of the image of the disturbance  $y_1$  in open loop and when feedforward compensation scheme is active (experimental).

The corresponding block diagrams, in open loop operation and with the adaptive compensator system, are shown in Figs. 8a and 8b, respectively.

$$D = \frac{q^{-d_D} B_D}{A_D}, G = \frac{q^{-d_G} B_G}{A_G}, M = \frac{q^{-d_M} B_M}{A_M} \quad (1)$$

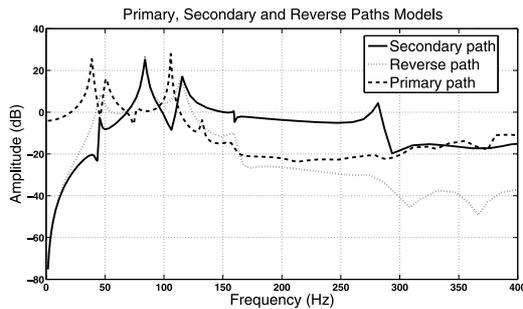
represent the transfer operators associated with the primary (D), secondary (G) and reverse (M) paths (all asymptotically stable), with

$$B_X(q^{-1}) = b_1^X q^{-1} + \dots + b_{n_{B_X}}^X q^{-n_{B_X}} = q^{-1} B_X^*(q^{-1}), \quad (2)$$

$$A_X(q^{-1}) = 1 + a_1^X q^{-1} + \dots + a_{n_{A_X}}^X q^{-n_{A_X}}. \quad (3)$$

for  $X \in D, G, M$  and  $d_X$  is the plant pure time delay in number of sampling periods<sup>5</sup>.

<sup>5</sup> The complex variable  $z^{-1}$  will be used to characterize the system's behaviour in the frequency domain and the delay operator  $q^{-1}$  will be used for the time domain analysis.



**Figure 10.** Frequency characteristics of the primary, secondary and reverse paths.

The frequency characteristics of the identified models of the primary, secondary and reverse paths are shown in Fig. 10.

At this stage it is important to make the following remarks, when the feedforward filter is absent (open loop operation):

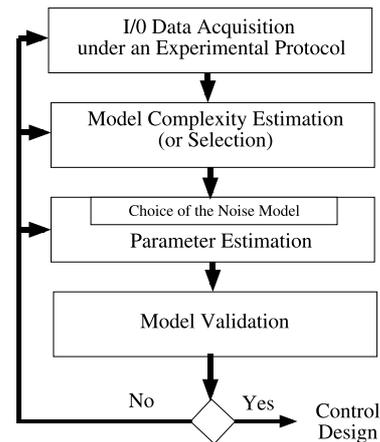
- very reliable models for the secondary path and the “positive” feedback path can be identified;
- design of a fixed model based stabilizing feedforward compensator requires the knowledge of the reverse path model only;
- knowledge of the disturbance characteristics and of the primary, secondary and reverse paths models is mandatory for the design of an optimal fixed model based feedforward compensator ([4,5]);
- adaptation algorithms do not use information upon the primary path whose characteristics may be unknown.

#### 4. Identification of the active vibration control system

As indicated in the Introduction, to design an active control one needs the dynamical model of the compensator systems (from the control to be applied to the measurement of the residual acceleration or force). Model identification from experimental data is a well established methodology see [2]. Identification of dynamic systems is an experimental approach for determining a dynamic model of a system. It includes four steps:

1. Input-output data acquisition under an experimental protocol.
2. Estimation of the model complexity (structure).
3. Estimation of the model parameters.
4. Validation of the identified model (structure of the model and values of the parameters).

A complete identification operation must comprise the four stages indicated above. The typical input excitation sequence is a PRBS (pseudo random binary sequence). The type of model which will be identified will be a discrete time parametric model which will allow later to directly design a control algorithm straightforwardly implementable on a computer. In our case, we will focus on the identification of discrete time input-output



**Figure 11.** The identification methodology.

models described by difference equations using recursive parameter estimation methods. Model validation is the final key point.

It is important to emphasize that it does not exist one single algorithm that can provide in all the cases a good model (i.e. which passes the model validation tests). System identification should be viewed as an iterative process as illustrated in 11 which has as objective to obtain a model which passes the model validation test and then can be used safely for control design. The software iREG ([http://www.adapttech.com/IMG/pdf/iREG\\_FT-2.pdf](http://www.adapttech.com/IMG/pdf/iREG_FT-2.pdf) and <http://tudor-bogdan.airimitoaiie.name/ireg.html>) for system identification and controller design has been extensively used for identification of mechanical structures including those considered in this paper.

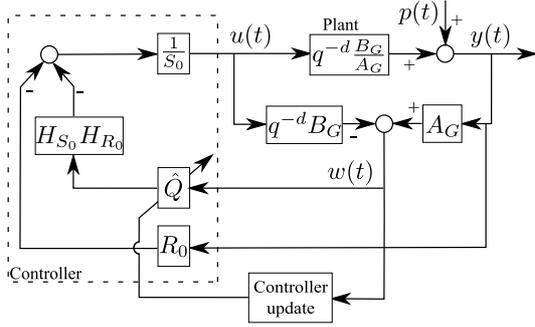
#### 5. Adaptive feedback attenuation of multiple unknown and time-varying narrow band disturbances

##### 5.1. Background

The objective is to reject asymptotically or strongly attenuate multiple narrow band disturbances which have unknown or time-varying spikes in the frequency domain. To asymptotically reject the disturbance, the Internal Model Principle (IMP) has to be applied. As a consequence, the controller should include a model of the disturbance. Since the disturbances are unknown, two approaches can be considered:

- Indirect adaptive regulation (one has to identify the model of the disturbance and recompute the controller which will include the estimated model of the disturbance).
- Direct adaptive regulation (the controller parameters will be directly adapted).

We will focus on the direct adaptive control approach since the recent ECC benchmark (see [22]) has shown that this approach allows to meet sharp performance specification and it is much less demanding in terms of computer power compared with the indirect approach). The Youla-Kucera parametrization of the controller (see [16]) is a key point



**Figure 12.** Direct adaptive regulation scheme for rejection of unknown disturbances.

for an efficient application of the Internal Model Principle as well as for developing a direct adaptive regulation algorithm.

## 5.2. Controller structure

The structure of the LTI discrete time model of the plant, also called *secondary path* ( $G$ ), used for controller design is given in (1), (2) and (3). For simplifying the writing one takes  $d_G = d$ .

The output of the plant  $y(t)$  and the input  $u(t)$  in the absence of the Youla Kucera filters, may be written as (see Fig. 12 with  $\hat{Q}$ ):

$$y(t) = \frac{q^{-d} B_G(q^{-1})}{A_G(q^{-1})} \cdot u(t) + p(t), \quad (4)$$

$$S_0(q^{-1}) \cdot u(t) = -R_0(q^{-1}) \cdot y(t). \quad (5)$$

In (4),  $p(t)$  is the effect of the disturbances on the measured output<sup>6</sup> and  $R_0(z^{-1})$ ,  $S_0(z^{-1})$  are polynomials in  $z^{-1}$  having the following expressions<sup>7</sup>:

$$S_0 = 1 + s_1^0 z^{-1} + \dots + s_{n_{S_0}}^0 z^{-n_{S_0}} = S'_0 \cdot H_{S_0}, \quad (6)$$

$$R_0 = r_0^0 + r_1^0 z^{-1} + \dots + r_{n_{R_0}}^0 z^{-n_{R_0}} = R'_0 \cdot H_{R_0}, \quad (7)$$

where  $H_{S_0}(q^{-1})$  and  $H_{R_0}(q^{-1})$  represent pre-specified parts of the controller (used for example to incorporate the internal model of a disturbance or to open the loop at certain frequencies) and  $S'_0(q^{-1})$  and  $R'_0(q^{-1})$  are computed. The characteristic polynomial, which specifies the desired closed loop poles of the system is given by<sup>8</sup> (see also [2]):

$$P_0(z^{-1}) = A_G(z^{-1})S_0(z^{-1}) + z^{-d} B_G(z^{-1})R_0(z^{-1}). \quad (8)$$

In what follows the Youla-Kučera parametrization ([23]) is used.

<sup>6</sup> The disturbance passes through a so called *primary path* which is not represented in this figure, and  $p(t)$  is its output.

<sup>7</sup> The argument ( $z^{-1}$ ) will be omitted in some of the following equations to make them more compact.

<sup>8</sup> It is assumed that a reliable model identification is achieved and therefore the estimated model is assumed to be equal to the true model.

Selecting a FIR structure for the  $Q$  filter associated to the Youla-Kucera parametrization, the controller's polynomials become:

$$R = R_0 + A_G Q H_{S_0} H_{R_0}, \quad (9)$$

$$S = S_0 - z^{-d} B_G Q H_{S_0} H_{R_0}, \quad (10)$$

where  $R_0$  and  $S_0$  define the central controller which verifies the desired specifications in the absence of the disturbance. The characteristic polynomial of the closed loop is still given by (8). We define the output sensitivity function (the transfer function between the disturbance  $p(t)$  and the output of the system  $y(t)$ ) as

$$S_{yp}(z^{-1}) = \frac{A_G(z^{-1})S(z^{-1})}{P_0(z^{-1})} \quad (11)$$

and the input sensitivity function (the transfer function between the disturbance  $p(t)$  and the control input  $u(t)$ ) as

$$S_{up}(z^{-1}) = -\frac{A_G(z^{-1})R(z^{-1})}{P_0(z^{-1})}. \quad (12)$$

## 5.3. Direct adaptive regulation using Youla-Kučera parametrization

A key aspect of this methodology is the use of the Internal Model Principle (IMP). It is supposed that  $p(t)$  is a deterministic disturbance given by

$$p(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t), \quad (13)$$

where  $\delta(t)$  is a Dirac impulse and  $N_p$ ,  $D_p$  are coprime polynomials of degrees  $n_{N_p}$  and  $n_{D_p}$ , respectively<sup>9</sup>. In the case of stationary narrow-band disturbances, the roots of  $D_p(z^{-1})$  are on the unit circle.

**Internal Model Principle:** The effect of the disturbance given in (13) upon the output

$$y(t) = \frac{A_G(q^{-1})S(q^{-1})}{P(q^{-1})} \cdot \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t), \quad (14)$$

where  $D_p(z^{-1})$  is a polynomial with roots on the unit circle and  $P(z^{-1})$  is an asymptotically stable polynomial, converges asymptotically towards zero *iff* the polynomial  $S(z^{-1})$  in the RS controller has the form (based on Eq. (6))

$$S(z^{-1}) = D_p(z^{-1})H_{S_0}(z^{-1})S'(z^{-1}). \quad (15)$$

Thus, the pre-specified part of  $S(z^{-1})$  should be chosen as  $H_S(z^{-1}) = D_p(z^{-1})H_{S_0}(z^{-1})$  and the controller is computed solving

$$P = A_G D_p H_{S_0} S' + z^{-d} B_G H_{R_0} R', \quad (16)$$

where  $P$ ,  $D_p$ ,  $A_G$ ,  $B_G$ ,  $H_{R_0}$ ,  $H_{S_0}$  and  $d$  are given<sup>10</sup>.

<sup>9</sup> Throughout the paper,  $n_X$  denotes the degree of the polynomial  $X$ .

<sup>10</sup> Of course, it is assumed that  $D_p$  and  $B_G$  do not have common factors.

For the purpose of direct adaptive regulation,  $Q(z^{-1})$  is considered to be a FIR filter ( $A_Q(z^{-1}) = 1$  and  $Q(z^{-1}) = B_Q(z^{-1})$ )

$$Q(z^{-1}) = q_0 + q_1 z^{-1} + \dots + q_{n_Q} z^{-n_Q}. \quad (17)$$

To compute  $Q(z^{-1})$  in order that the polynomial  $S(z^{-1})$  given by (10) incorporates the internal model of the disturbance (15), one has to solve the diophantine equation

$$S' D_p + z^{-d} B_G H_{R_0} Q = S'_0, \quad (18)$$

where  $D_p$ ,  $d$ ,  $B_G$ ,  $S'_0$ , and  $H_{R_0}$  are known and  $S'$  and  $Q$  are unknown. Eq. (18) has a unique solution for  $S'$  and  $Q$  with:  $n_{S'_0} \leq n_{D_p} + n_{B_G} + d + n_{H_{R_0}} - 1$ ,  $n_{S'} = n_{B_G} + d + n_{H_{R_0}} - 1$ ,  $n_Q = n_{D_p} - 1$ . One sees that the order  $n_Q$  of the polynomial  $Q$  depends upon the structure of the disturbance model. The use of the Youla-Kučera parametrization, with  $Q$  given in (17), is interesting in this case because it allows to maintain the closed loop poles as given by the central controller but at the same time introduces the parameters of the internal model into the controller. Using the Q-parametrization, the output of the system in the presence of a disturbance can be expressed as

$$\begin{aligned} y(t) &= \frac{A_G [S_0 - q^{-d} B_G H_{S_0} H_{R_0} Q]}{P} \cdot \frac{N_p}{D_p} \cdot \delta(t) \\ &= \frac{S_0 - q^{-d} B_G H_{S_0} H_{R_0} Q}{P} \cdot w(t), \end{aligned} \quad (19)$$

where  $w(t)$  is given by (see also Fig. 12)

$$w(t) = \frac{A_G N_p}{D_p} \cdot \delta(t) = A_G \cdot y(t) - q^{-d} \cdot B_G \cdot u(t). \quad (20)$$

From these equations one obtains finally the following expression for the residual error (acceleration or force, for details see [16]):

$$\varepsilon(t+1) = [\theta^T - \hat{\theta}^T(t+1)] \cdot \phi(t) + v(t+1). \quad (21)$$

where  $\theta = [q_0, \dots, q_{n_Q}]^T$ ,  $\hat{\theta}(t) = [\hat{q}_0, \dots, \hat{q}_{n_Q}]^T$ ,  $\phi^T(t) = [w_2(t) \ w_2(t-1) \ \dots \ w_2(t-n_Q)]$ .  
and

$$w_1(t+1) = \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t+1), \quad (22)$$

$$w_2(t) = \frac{q^{-d} B^*(q^{-1})}{P(q^{-1})} \cdot w(t), \quad (23)$$

$$w(t+1) = A_G(q^{-1}) \cdot y(t+1) - q^{-d} B_G^*(q^{-1}) \cdot u(t). \quad (24)$$

One can remark that  $\varepsilon(t+1)$  corresponds to an *a posteriori* adaptation error ([10]).

From (19), one obtains the *a priori* adaptation error

$$\varepsilon^0(t+1) = w_1(t+1) - \hat{\theta}^T(t) \phi(t), \quad (25)$$

where  $B_G(q^{-1})u(t+1) = B_G^*(q^{-1})u(t)$ .

The *a posteriori* adaptation error is obtained from (19)

$$\varepsilon(t+1) = w_1(t+1) - \hat{\theta}^T(t+1) \phi(t). \quad (26)$$

As a consequence of the form of the above equation the following PAA (Parameter Adaptation Algorithm) is used ([10]) for the estimation of the parameters of  $\hat{Q}(t, q^{-1})$ :

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F(t) \phi(t) \varepsilon(t+1), \quad (27)$$

$$\varepsilon(t+1) = \frac{\varepsilon^0(t+1)}{1 + \phi^T(t) F(t) \phi(t)}, \quad (28)$$

$$\varepsilon^0(t+1) = w_1(t+1) - \hat{\theta}^T(t) \phi(t), \quad (29)$$

$$F(t+1) = \frac{1}{\lambda_1(t)} \left[ F(t) - \frac{F(t) \phi(t) \phi^T(t) F(t)}{\frac{\lambda_1(t)}{\lambda_2(t)} + \phi^T(t) F(t) \phi(t)} \right], \quad (30)$$

$$1 \geq \lambda_1(t) > 0, \quad 0 \leq \lambda_2(t) < 2, \quad (31)$$

where  $\lambda_1(t)$ ,  $\lambda_2(t)$  allow to obtain various profiles for the evolution of the adaption gain  $F(t)$  (for details see [2, 10]). For a stability proof under the hypothesis *model=plant*<sup>11</sup> see [16].

#### 5.4. Robustness considerations

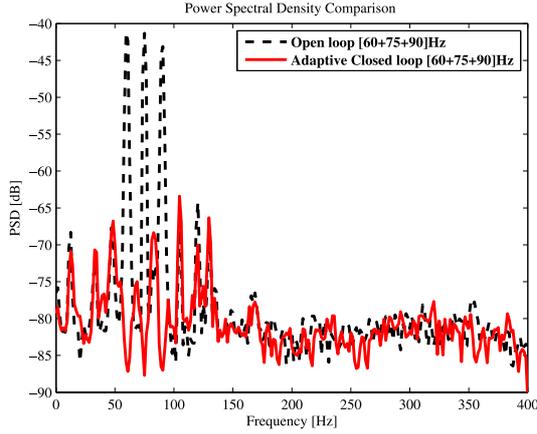
The introduction of the internal model for the perfect rejection of the disturbance (asymptotically) will have as effect to raise the maximum value of the modulus of the output sensitivity function  $S_{yp}$ . This may lead to unacceptable values for the modulus margin ( $|S_{yp}(e^{-j\omega})|_{max}^{-1}$ ) and the delay margin if the controller design is not appropriately done (see [2]). As a consequence, a robust control design should be considered assuming that the model of the disturbance and its domain of variation in the frequency domain are known. The objective is that for all situations, acceptable modulus margin and delay margin are obtained. If the number of narrow band disturbances is too high or a finite band disturbance covers a too wide frequency region, it may not be possible to cancel their effect assuring in the mean time robustness of the closed loop (the well known ‘‘water bed’’ effect resulting from the Bode integral of  $S_{yp}$  in the frequency domain). In such cases, one has to consider only an attenuation of the disturbance to an acceptable level.

Furthermore, at the frequencies where perfect rejection of the disturbance is achieved one has  $S_{yp}(e^{-j\omega}) = 0$  and

$$|S_{up}(e^{-j\omega})| = \left| \frac{A_G(e^{-j\omega})}{B_G(e^{-j\omega})} \right|. \quad (32)$$

Equation (32) corresponds to the inverse of the gain of the system to be controlled. The implication of Eq. (32) is that cancelation (or in general an important attenuation) of disturbances on the output should be done only in frequency regions where the system gain is large enough. If the gain of the controlled system is too low,  $|S_{up}|$  will be large at these frequencies. Therefore, the robustness vs additive plant model uncertainties will be reduced and the stress on the actuator will become important.

<sup>11</sup> No positive real condition required for asymptotic stability.



**Figure 13.** PSD comparison between the open loop measured disturbance and the residual accelerations obtained with direct adaptation.

## 6. Some experimental results – Attenuation of multiple narrow band disturbances

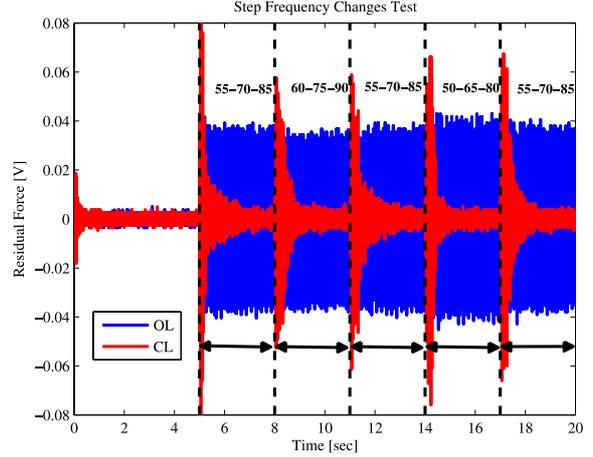
Performance of the algorithm from Sect. 5.3 for adaptive attenuation of unknown and time-varying disturbances will be illustrated on the experimental platform described in Sect. 2.

### *Direct adaptive feedback attenuation of unknown and time-varying multiple narrow band disturbances*

The experimental platform described in Sect. 2 was used for an International Benchmark in Adaptive regulation [22]. The objective is the rejection of unknown/time-varying narrowband disturbances in three levels. The most difficult level corresponds to three narrowband disturbances. The disturbances are located between 50 and 95 Hz. For a constant disturbance frequency, three main indicators were defined<sup>12</sup> to describe the performance of the adaptive scheme: 1) *Global Attenuation (GA)*, 2) *Disturbance Attenuation (DA)* and 3) *Maximum amplification (MA)*. Each one of these indicators has a benchmark specification at this level, e.g. for  $GA \geq 30$  dB, for  $DA \geq 40$  dB and for  $MA \leq 9$  dB. Figure 13 shows the comparison between the power spectral density (PSD) estimates in closed loop (CL) and open loop (OL). The case corresponds to three disturbances located at 60, 75 and 90 Hz. The PSD is computed after the adaptation process has converged toward an almost constant controller from the real-time results. The benchmark specifications for *DA* and *MA* are also depicted. In this case the benchmark specifications are fulfilled and in general, according to [24] and [22], the direct adaptive scheme (Sect. 5.3) presents one of the best performances with lower complexity (execution time) for the third level among the seven contributions presented.

When the disturbance frequency is time-varying, a transient evaluation is considered. Figure 14 shows the comparison between the OL system response and the

<sup>12</sup> For more details about the definition of such indicators and performance index see [22].



**Figure 14.** Performance comparison in the presence of a three sinusoidal disturbances.

CL system response. The disturbance frequency changes each three seconds after the application. In the figure the disturbance frequencies at each step are shown. In general the direct adaptive scheme shows a fast convergence, nevertheless the proximity of plant low damped complex zeros affects the speed of the convergence.

## 7. Adaptive Feedforward + Fixed feedback compensation of broad band disturbances

For broad band disturbance attenuation the use of the feedforward compensation is mandatory (as explained in the Introduction). However adding feedback should improve the global performance. Of course the algorithms for the hybrid feedforward+feedback control can be particularized for the case without feedback control.

### 7.1. Basic equations and notations

The block diagram associated with an AVC system using an hybrid (feedback + adaptive feedforward) control is shown in Fig. 8. The transfer operators characterizing the primary path (D), the secondary path (G) and the reverse path (M) are given in (1), (2) and (3). To simplify the writing, the delay  $d_X$  is incorporated in  $B_X$  where  $X \in D, G, M$ . The optimal feedforward filter (unknown) is defined by

$$N(q^{-1}) = \frac{R(q^{-1})}{S(q^{-1})}, \quad (33)$$

where

$$R(q^{-1}) = r_0 + r_1 q^{-1} + \dots + r_{n_R} q^{-n_R}, \quad (34)$$

$$S(q^{-1}) = 1 + s_1 q^{-1} + \dots + s_{n_S} q^{-n_S} = 1 + q^{-1} S^*(q^{-1}). \quad (35)$$

The estimated feedforward filter is denoted by

$$\hat{N}(q^{-1}) = \frac{\hat{R}(q^{-1})}{\hat{S}(q^{-1})}. \quad (36)$$

The vector of optimal feedforward filter parameters is

$$\theta^T = [s_1, \dots, s_{n_S}, r_0, \dots, r_{n_R}]^T \quad (37)$$

and the vector of estimated feedforward filter coefficients is

$$\hat{\theta}^T(t) = [\hat{s}_1(t), \dots, \hat{s}_{n_S}(t), \hat{r}_0(t), \dots, \hat{r}_{n_R}(t)]^T. \quad (38)$$

The fixed RS controller  $K$ , computed on the basis of the model  $\hat{G}$  to reject broadband disturbances on the output  $e(t)$ , is characterized by the asymptotically stable transfer function

$$K(q^{-1}) = \frac{B_K(q^{-1})}{A_K(q^{-1})}, \quad (39)$$

where

$$B_K(q^{-1}) = b_0^K + b_1^K q^{-1} + \dots + b_{n_{B_K}}^K q^{-n_{B_K}}, \quad (40)$$

$$A_K(q^{-1}) = 1 + a_1^K q^{-1} + \dots + a_{n_{A_K}}^K q^{-n_{A_K}}. \quad (41)$$

The input of the feedforward filter (called also reference) is denoted by  $\hat{y}_1(t)$  and it corresponds to the measurement provided by the primary transducer (force or acceleration transducer in AVC or a microphone in ANC). The output of the feedforward compensator is denoted by  $\hat{u}_1(t+1) = \hat{u}_1(t+1|\hat{\theta}(t+1))$  (*a posteriori* output). The measured input to the feedforward filter can also be written as

$$\hat{y}_1(t+1) = w(t+1) + \frac{B_M^*(q^{-1})}{A_M(q^{-1})} \hat{u}(t), \quad (42)$$

where

$$\hat{u} = \hat{u}_1(t) - u_2(t), \quad (43)$$

$\hat{u}_1(t)$  and  $u_2(t)$  are the outputs given by the adaptive feedforward and the fixed feedback compensator, respectively.  $\hat{u}$  is the effective input sent to the control actuator.

The *a priori* output of the estimated feedforward filter is given by

$$\begin{aligned} \hat{u}_1^0(t+1) &= \hat{u}_1(t+1|\hat{\theta}(t)) \\ &= -\hat{S}^*(t, q^{-1})\hat{u}_1(t) + \hat{R}(t, q^{-1})\hat{y}_1(t+1) \\ &= \hat{\theta}^T(t)\phi(t) = [\hat{\theta}_S^T(t), \hat{\theta}_R^T(t)] \begin{bmatrix} \phi_{\hat{y}_1}(t) \\ \phi_{\hat{u}_1}(t) \end{bmatrix} \end{aligned} \quad (44)$$

where  $\hat{\theta}^T(t)$  has been given in (38) and

$$\begin{aligned} \phi^T(t) &= [-\hat{u}_1(t), \dots, -\hat{u}_1(t-n_S+1), \dots \\ &\quad \dots, \hat{y}_1(t+1), \dots, \hat{y}_1(t-n_R+1)] \\ &= [\phi_{\hat{u}_1}^T(t), \phi_{\hat{y}_1}^T(t)]. \end{aligned} \quad (45)$$

The input to the feedback (fixed) compensator is given by the performance variable, therefore  $y_2(t) = e(t)$ . Its output will be  $u_2(t) = K \cdot y_2(t)$ . The unmeasurable value of the output of the primary path (when the compensation is active) is denoted  $x(t)$ . The *a priori* output of the secondary path is denoted  $\hat{z}^0(t+1) = \hat{z}(t+1|\hat{\theta}(t))$  while its input is  $\hat{u}(t)$ . One has

$$\hat{z}^0(t+1) = \frac{B_G^*(q^{-1})}{A_G(q^{-1})} \hat{u}(t) = \frac{B_G^*(q^{-1})}{A_G(q^{-1})} \hat{u}(t|\hat{\theta}(t)). \quad (46)$$

The measured residual acceleration (or force) satisfies the following equation

$$e^0(t+1) = x(t+1) + \hat{z}^0(t+1). \quad (47)$$

The *a priori* and *a posteriori* adaptation error are defined as

$$\varepsilon^0(t+1) = \varepsilon(t+1|\hat{\theta}(t)) = -e^0(t+1) \quad (48)$$

and

$$\begin{aligned} \varepsilon(t+1) &= \varepsilon(t+1|\hat{\theta}(t+1)) \\ &= -e(t+1) = -x(t+1) - \hat{z}(t+1) \end{aligned} \quad (49)$$

where the *a posteriori* value of the output of the secondary path  $\hat{z}(t+1)$  (dummy variable) is given by

$$\hat{z}(t+1) = \hat{z}(t+1|\hat{\theta}(t+1)) = \frac{B_G^*(q^{-1})}{A_G(q^{-1})} \hat{u}(t|\hat{\theta}(t+1)). \quad (50)$$

For compensators with constant parameters  $\varepsilon^0(t) = \varepsilon(t)$ ,  $e^0(t) = e(t)$ ,  $\hat{z}^0(t) = \hat{z}(t)$ ,  $\hat{u}^0(t) = \hat{u}(t)$ .

## 7.2. Development of the algorithms

The algorithms for adaptive feedforward compensation in presence of feedback controller will be developed under the following hypotheses:

H1) The signal  $w(t)$  is bounded, *i.e.*,

$$|w(t)| \leq \alpha, \quad \forall t \quad (0 \leq \alpha < \infty) \quad (51)$$

H2) Perfect matching condition – There exists a filter  $N(q^{-1})$  of finite dimension such that

$$\frac{N(z^{-1})}{1 - N(z^{-1})M(z^{-1})} G(z^{-1}) = -D(z^{-1}) \quad (52)$$

and the characteristic polynomials:

- of the “internal” positive coupling loop

$$P(z^{-1}) = A_M(z^{-1})S(z^{-1}) - B_M(z^{-1})R(z^{-1}), \quad (53)$$

- of the closed loop (G-K)

$$P_{cl}(z^{-1}) = A_G(z^{-1})A_K(z^{-1}) + B_G(z^{-1})B_K(z^{-1}), \quad (54)$$

- and of the coupled feedforward-feedback loop

$$P_{fb-ff} = A_M S[A_G A_K + B_G B_K] - B_M R A_K A_G \quad (55)$$

are Hurwitz polynomials.

H3) Deterministic context – The effect of the measurement noise upon the measured residual error is neglected.

H4) The primary path model  $D(z^{-1})$  is unknown and constant.

A first step in the development of the algorithms is to establish a relation between the errors on the estimation of the parameters of the feedforward filter and the measured residual error (acceleration or force).

This residual error for a fixed feedforward filter can be expressed by [25]:

$$\varepsilon(t + 1) = \frac{A_M A_G A_K G}{P_{fb-ff}} [\theta - \hat{\theta}]^T \phi(t) \quad (56)$$

where

$$\theta^T = [s_1, \dots, s_{n_S}, r_0, r_1, \dots, r_{n_R}] = [\theta_S^T, \theta_R^T] \quad (57)$$

is the vector of parameters of the optimal filter  $N$  assuring perfect matching,

$$\hat{\theta}^T = [\hat{s}_1 \dots \hat{s}_{n_S}, \hat{r}_0 \dots \hat{r}_{n_R}] = [\hat{\theta}_S^T, \hat{\theta}_R^T] \quad (58)$$

is the vector of constant estimated parameters of  $\hat{N}$ , and  $\phi^T(t)$  is defined in (45)

Filtering the vector  $\phi(t)$  through an asymptotically stable filter  $L(q^{-1}) = \frac{B_L}{A_L}$ , Eq. (56) for  $\hat{\theta} = \text{constant}$  becomes:

$$\varepsilon(t + 1) = \frac{A_M A_G A_K G}{P_{fb-ff} L} [\theta - \hat{\theta}]^T \phi_f(t) \quad (59)$$

$$\phi_f(t) = L(q^{-1})\phi(t). \quad (60)$$

Equation (59) will be used to develop the adaptation algorithms neglecting the non-commutativity of the operators when  $\hat{\theta}$  is time-varying (however an exact algorithm can be derived in such cases – see [10]).

Replacing the fixed estimated parameters by the current estimated parameters, Eq. (59) becomes the equation of the *a-posteriori* residual (adaptation) error  $\varepsilon(t + 1)$  (which is computed):

$$\varepsilon(t + 1/\hat{\theta}(t + 1)) = \frac{A_M A_G A_K G}{P_{fb-ff} L} [\theta - \hat{\theta}(t + 1)]^T \phi_f(t). \quad (61)$$

Equation (61) has the standard form for an *a-posteriori* adaptation error ([10]), which immediately suggests to use the same parametric adaptation algorithm given in Eq. (27) through (30). The system is asymptotically stable for any initial conditions  $\hat{\theta}(0)$ ,  $\varepsilon^0(0)$ ,  $F(0)$ , provided that:

$$H'(z^{-1}) = H(z^{-1}) - \frac{\lambda_2}{2}, \max_t [\lambda_2(t)] \leq \lambda_2 < 2 \quad (62)$$

is a strictly positive real (SPR) transfer function where:

$$H(q^{-1}) = \frac{A_M A_G A_K G}{P_{fb-ff} L} G, \psi = \phi_f. \quad (63)$$

Various choices can be considered for the filter  $L$  in order to satisfy the positive real condition. See [25,26]. It is important to remark that the positive real condition is strongly influenced by the presence of the feedback controller and its design. The best performances are in general obtained by taking  $L$  as an estimation of  $H$  (see Eq. (63)). Relaxation of the positive real condition by

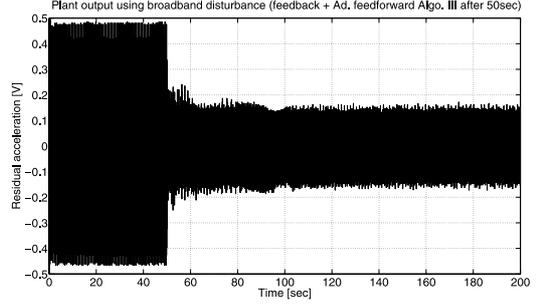


Figure 15. Real time results obtained with feedback controller and adaptive feedforward Algorithm III.

averaging arguments is discussed in [26] and by adding proportional adaptation in [27]. Filtering of the residual error can also be considered for satisfying the positive real condition, but this will modify the criterion which is minimized ([27,28]). Analysis of the algorithms when hypotheses H2 and H3 are violated can be found in [26].

### 7.3. Use of the Youla-Kučera parametrization for adaptive feedforward disturbance compensation

Since most of the adaptive feedforward vibration (or noise) compensation systems feature an internal “positive feedback” coupling between the compensator system and the correlated disturbance measurement which serves as reference, one may think building a stabilizing controller for this internal loop to which an additional filter will be added with the objective to enhance the disturbance attenuation capabilities while preserving the stabilization properties of the controller. In order to achieve this, instead of a standard IIR feedforward compensator one can use an Youla Kučera parametrization of the adaptive feedforward compensator. The central compensator will assure the stability of the internal positive feedback loop and its performance are enhanced in real-time by the direct adaptation of the parameters of the Youla-Kučera Q-filter. See [29]. Comparisons between IIR, FIR YK, and IIR YK adaptive feedforward have been done [29].

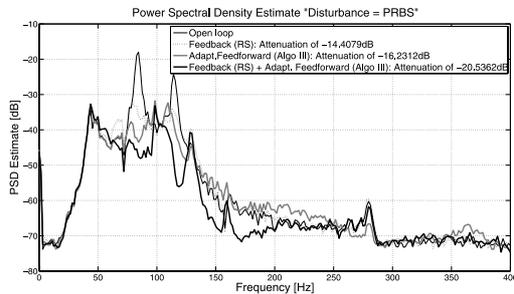
## 8. Some experimental results-Attenuation of broad band disturbances

A summary of various results obtained on the system described in Sect. II will be presented next. The adaptive feedforward filter structure for all the experiments has been  $n_R = 9$ ,  $n_S = 10$  (total of 20 parameters) and this complexity does not allow to verify the “perfect matching condition” (which requires more than 40 parameters). A feedback RS controller has been also introduced to test the potential improvement in performance.

Table 1 summarizes the global attenuation results for various configurations. Clearly, hybrid adaptive feedforward-feedback scheme brings a significant improvement in performance with respect to adaptive

**Table 1.** Global attenuation for various configurations.

	Feedback only	Feedforward only ( $H_\infty$ )	Adaptive Feedforward only	Feedback & Adaptive Feedforward	Feedforward ( $H_\infty$ ) & Feedback
<b>Attenuation [dB]</b>	-14.40	-14.70	-16.23	-20.53	-18.42

**Figure 16.** Power spectral densities of the residual acceleration (Disturbance = PRBS).

feedforward compensation alone. This can be also seen on the power spectral densities shown in Fig. 16<sup>13</sup>.

It is important to point out that the design of a linear feedforward+feedback requires not only the perfect knowledge of the disturbance characteristics but also of the model of the primary path, while an adaptive approach does not require these informations. Real-time results for the hybrid configuration obtained on the system described in Sect. 2 are presented next. A pseudo-random binary sequence (PRBS) excitation on the global primary path has been considered as the disturbance.

Time domain results obtained in open loop and with the hybrid control on the AVC system are shown in Fig. 15.

The experiments have been carried on by first applying the disturbance in open loop during 50s and after that closing the loop with the hybrid adaptive feedforward-feedback algorithms.

## 9. Conclusion

In our opinion, looking to adaptive active vibration control systems from the automatic control perspective allows a better understanding of the problems and as a consequence solutions for a variety of situations encountered in practice can be provided. The present paper has presented several algorithms for AVC systems which have been validated by extensive experimental tests.

The control concepts which are important for the development of algorithms for AVC are:

- The sensitivity functions.
- The Bode integral.
- Internal Model Principle.
- Youla - Kučera parametrization.
- System (model) identification.
- Parameter Adaptation Algorithms.
- Stability analysis.

<sup>13</sup> For the adaptive schemes the PSD is evaluated after the adaptation transient has settled.

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