Advanced nonlinear control of three phase series active power filter

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Abstract. The problem of controlling three-phase series active power filter (TPSAPF) is addressed in this paper in presence of the perturbations in the voltages of the electrical supply network. The control objective of the TPSAPF is twofold: (i) compensation of all voltage perturbations (voltage harmonics, voltage unbalance and voltage sags), (ii) regulation of the DC bus voltage of the inverter. A controller formed by two nonlinear regulators is designed, using the Backstepping technique, to provide the above compensation. The regulation of the DC bus voltage of the inverter is ensured by the use of a diode bridge rectifier which its output is in parallel with the DC bus capacitor. The Analysis of controller performances is illustrated by numerical simulation in Matlab/Simulink environment.

1 Introduction

The harmonic contamination is a harmful problem in Electric Power System. Indeed, the increasing use of rectifiers, thyristor power converters, UPS (Uninterruptible Power Supply), switching power supplies and other nonlinear loads are known to cause serious problems in electric power systems. These devices are responsible for the contamination of the line currents with the harmonics of various orders. The harmonics of the current circulating through the line impedance produce distortion in the voltages of power system. Indeed, the distortion, the unbalance and sags of the power system voltages cause several power quality problems, including the incorrect operation of some sensitive loads [1]. The TPSAPF’s are an appropriate solution to protect the sensitive load against voltage perturbations.

In most papers, the researchers often use, for the control of the TPSAPF, the method of the instantaneous power [2]. In this paper, the work focuses on the advanced nonlinear control of three-phase series active power filter in the presence of disturbances in the power system voltages by using a method based on the calculation of the references of the series voltages. A controller that is formed by two nonlinear regulators is designed, using the Backstepping technique, to ensure compensation of voltage perturbations (voltage harmonics, voltage unbalances and voltage sags) at the terminals of the sensitive loads. The regulation of DC bus voltage of the inverter is provided by the use of a diode bridge rectifier which its output is connected in parallel with the DC bus capacitor. This theoretical result is confirmed by numerical simulation.

The paper is organized as follows: the system includes the electric network and the DC/AC converter is modeled in Section 2, the control problem is formulated in Section 3 which also includes the design. Performances of controller are illustrated by simulation in Section 4. A general conclusion ends the paper.

2 Series active power filter

2.1 Series active power filter topology

Three-phase series active power filter under study has the structure of figure 1. In the AC side, the TPSAPF is inserted between the perturbed voltage source and a second order \((R_f, L_f, C_f)\) passive output filter used to connect the inverter to grid through voltages injected by three current transformers. In the DC side it has a capacitor of energy storage \(C_{dc}\). The circuit operates according to the well known Pulse Width Modulation principle (PWM) [3,4,5]. The switching function \(i\) of the inverter is defined by:

\[
  i = \begin{cases} 
  +1 & \text{if } S_i \text{ is ON and } S_{i+3} \text{ is OFF} \\
  1 & \text{if } S_i \text{ is OFF and } S_{i+3} \text{ is ON} \\
  \end{cases} \quad \text{(for } \text{i}=1,2,3) 
\]
follow their references \( v_{sd}, v_{s2} \) and \( v_{s3} \) which are defined by:

\[
[v_{sd}] = [v_{s2}] = [v_{s3}]^T
\]  

where the voltages \( v_{sd}, v_{s2} \) and \( v_{L L} \) at the terminals of the load must be sinusoidal and form a balanced three-phase system. Expressing the references voltages from equation (2) in the dq frame, it comes:

\[
\begin{bmatrix}
  x_{sd} \\ x_{sq}
\end{bmatrix} = \begin{bmatrix} v_{nd} \\ v_{rq}
\end{bmatrix} \quad \begin{bmatrix}
  v_{Ld} = \sqrt{3}/2 \sin(t) \\ v_{Lq} = \sqrt{3}/2 \cos(t)
\end{bmatrix}
\]  

To force the voltages \( x_{sd} \) and \( x_{sq} \) in order to respectively follow the references \( x_{sd}^* \) and \( x_{sq}^* \), a controller formed by two nonlinear regulators, using the Backstepping technique [6], is proposed in the follow:

**Step 1** Stabilization of the subsystem \( e_l = \begin{bmatrix} e_{ld} \\ e_{lq} \end{bmatrix} \)

Let’s introduce the tracking error \( e_l \)

\[
e_l = \begin{bmatrix} e_{ld} \\ e_{lq} \end{bmatrix} = \begin{bmatrix} x_{sd} - x_{sd}^* \\ x_{sq} - x_{sq}^* \end{bmatrix}
\]  

Given (1a), the time derivative of error \( e_l \) is:

\[
\begin{bmatrix}
  \dot{e}_{ld} \\ \dot{e}_{lq}
\end{bmatrix} = \begin{bmatrix}
  x_{sq} + (m_s x_{fd})/C_f + (m_{sq} i_{mad})/C_f \\ x_{sd} + (m_s x_{f})/C_f + (m_{sq} i_{maq})/C_f
\end{bmatrix} \quad \begin{bmatrix}
  x_{sd}^* \\ x_{sq}^*
\end{bmatrix}
\]  

Introduce the candidate Lyapunov function

\[
V_l = \frac{1}{2} e_{ld}^2 + \frac{1}{2} e_{lq}^2
\]  

Its dynamic is given by:

\[
\dot{V}_l = e_{ld} e_{ld} + e_{lq} e_{lq}
\]  

The choice \( \dot{V}_l = c_{ld} e_{ld}^2 + c_{lq} e_{lq}^2 \) ensuring the asymptotic stability of (5) with respect to the Lyapunov function (6) where \( c_{ld} > 0 \) and \( c_{lq} > 0 \) are a design parameters. Indeed, this choice would imply:

\[
\begin{bmatrix}
  \dot{e}_{ld} \\ \dot{e}_{lq}
\end{bmatrix} = \begin{bmatrix}
  x_{sq} + (m_s x_{fd})/C_f + (m_{sq} i_{mad})/C_f \\ x_{sd} + (m_s x_{f})/C_f + (m_{sq} i_{maq})/C_f
\end{bmatrix} \quad \begin{bmatrix}
  x_{sd}^* \\ x_{sq}^*
\end{bmatrix}
\]  

If we consider that \( m_s x_{fd}/C_f \) and \( m_s x_{f}/C_f \) are, respectively, the actual commands in the preceding equation (8), then the error \( e_l = \begin{bmatrix} e_{ld} \\ e_{lq} \end{bmatrix} \) will be regulated to zero if:
\[
\begin{align*}
\begin{pmatrix}
\hat{m}_x \dot{x}_d / C_f \\
\hat{m}_x \dot{x}_q / C_f 
\end{pmatrix} &= \begin{pmatrix} d \\ q \end{pmatrix}
\end{align*}
\] (9)

where \( \begin{pmatrix} d \\ q \end{pmatrix} \) are the stabilizing functions defined by:

\[
\begin{align*}
\begin{pmatrix} d \\ q \end{pmatrix} &= \begin{pmatrix} c_{id} \hat{e}_{id} + x_q \left( m^2 \hat{i}_{sd} / C_f + \hat{x}_d \right) \\
& \quad + c_{iq} \hat{e}_{iq} + x_q \left( m^2 \hat{i}_{sq} / C_f + \hat{x}_q \right)
\end{pmatrix}
\end{align*}
\] (10)

As \( m_x \dot{x}_d / C_f \) and \( m_x \dot{x}_q / C_f \) are not the actual controls, then new variable error \( \hat{e}_2 = \begin{pmatrix} \hat{e}_{2d} \\ \hat{e}_{2q} \end{pmatrix} \) between the virtual controls and their desired values \( \begin{pmatrix} d \\ q \end{pmatrix} \) are defined as follow:

\[
\begin{align*}
\hat{e}_2 &= \begin{pmatrix} \hat{e}_{2d} \\ \hat{e}_{2q} \end{pmatrix} = \begin{pmatrix} m_x \dot{x}_d / C_f \\ m_x \dot{x}_q / C_f \end{pmatrix} \begin{pmatrix} d \\ q \end{pmatrix}
\end{align*}
\] (11)

Then, by using (10) and (11), equation (5) becomes:

\[
\begin{align*}
\begin{pmatrix} \hat{e}_{id} \\ \hat{e}_{iq} \end{pmatrix} &= \begin{pmatrix} c_{id} \hat{e}_{id} + \hat{e}_{2d} \\ c_{iq} \hat{e}_{iq} + \hat{e}_{2q} \end{pmatrix}
\end{align*}
\] (12)

Thereafter, the derivative of equation (6) becomes:

\[\dot{V}_i = c_{id} \hat{e}_{id}^2 + c_{iq} \hat{e}_{iq}^2 + \hat{e}_{id} \hat{e}_{2d} + \hat{e}_{iq} \hat{e}_{2q}\] (13)

end of step 1.

**Step 2** Stabilization of the subsystem \( (e_1, e_2) \)

To achieve the above objective, the controller forcing the errors \( (e_1, e_2) \) to tend to zero, one needs the dynamics of \( e_2 = \begin{pmatrix} \hat{e}_{2d} \\ \hat{e}_{2q} \end{pmatrix} \). We derive (11), using (1b) we obtain:

\[
\begin{align*}
\begin{pmatrix} \hat{e}_{2d} \\ \hat{e}_{2q} \end{pmatrix} &= \begin{pmatrix} m_R x_d / C_f \hat{x}_d + m_x \dot{x}_d + m_x \dot{x}_q + \frac{1}{C_f} X_d \hat{x}_d \\ m_R \hat{x}_d / C_f x_d + m_x \dot{x}_q + \frac{1}{C_f} X_q \hat{x}_q \end{pmatrix} \begin{pmatrix} \hat{e}_{id} \\ \hat{e}_{iq} \end{pmatrix}
\end{align*}
\] (14)

Introduce the candidate Lyapunov function

\[V_2 = V_1 + \frac{1}{2} \hat{e}_2^2 \hat{e}_2 \] (15)

The derivative of (15) is obtained by using (13):

\[\dot{V}_2 = c_{id} \hat{e}_{id}^2 + c_{iq} \hat{e}_{iq}^2 + \hat{e}_{id} \hat{e}_{2d} + \hat{e}_{iq} \hat{e}_{2q} + \hat{e}_{id} \hat{e}_{2d} + \hat{e}_{iq} \hat{e}_{2q} \] (16)

To ensure the negativity of \( \dot{V}_2 \), it is necessary that:

\[
\begin{pmatrix} \hat{e}_{2d} \\ \hat{e}_{2q} \end{pmatrix} = \begin{pmatrix} e_{id} \\ e_{iq} \\ e_{2d} \\ e_{2q} \end{pmatrix}
\] (17)

where \( e_{2d} > 0 \) and \( e_{2q} > 0 \) are design parameters then,

From the equations (14) and (17) we deduce the expressions of the laws of actual control in thedq reference

\[
\begin{align*}
\begin{pmatrix} u_d \\ u_q \end{pmatrix} &= \frac{1}{L} \begin{pmatrix} 2L_i C_d (e_{id} \hat{e}_{id} + e_{2d} \hat{e}_{id}) + 2R_x x_d \hat{x}_d + \frac{2}{m} x_d \hat{x}_d \\ \frac{1}{L} \begin{pmatrix} 2L_i C_d (e_{id} \hat{e}_{id} + e_{2d} \hat{e}_{id}) + 2R_x x_d \hat{x}_d + \frac{2}{m} x_d \hat{x}_d \end{pmatrix}
\end{pmatrix}
\end{align*}
\] (18)

### 3.2 Voltage of DC bus loop design

The regulation of the DC bus voltage is provided by the use of a full bridge rectifier which its output is connected in parallel with the capacitor of the DC bus. The voltage \( v_{dc} \) of the DC bus is regulated to the average value of the output voltage of the rectifier bridge: \( U_{moy} = 3\sqrt{3} E_f \).

### 4 Numerical simulations

In order to simulate the behavior of the three-phase series active power filter shown in figure 1, the chosen nonlinear load is a three-phase bridge rectifier which supplies an inductive load comprising a resistor \( R_L \) and an inductor \( L_k \). The coil \( L_o \) is inserted to the input of three-phase bridge rectifier to limit the \( di_{in}/dt \) \( k \{1,2,3\} \).

The performances of the proposed controller are now numerically evaluated with the following characteristics:

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Network</td>
<td>( E, f )</td>
<td>6220, 2 ( V ), 50 Hz</td>
</tr>
<tr>
<td>DC bus</td>
<td>( C_{dc} )</td>
<td>9000 ( F )</td>
</tr>
<tr>
<td>TPSAPF</td>
<td>( R_f, L_f, C_f )</td>
<td>80 ( m ), 3 ( mH ), 1200 ( F )</td>
</tr>
<tr>
<td>Rectifier</td>
<td>( R_{L_k}, L_{L_k}, L_o )</td>
<td>20 ( \Omega ), 500 ( mH ), 5 ( mH )</td>
</tr>
<tr>
<td>Voltages regulators</td>
<td>( c_{id} = c_{2d} )</td>
<td>3000 ( s^{-1} )</td>
</tr>
<tr>
<td></td>
<td>( c_{iq} = c_{2q} )</td>
<td>6000 ( s^{-1} )</td>
</tr>
</tbody>
</table>

### 4.1 Voltage harmonics compensation

The three-phase source voltages are balanced but contain the 5\( \text{th} \) and 7\( \text{th} \) harmonic components. Their expressions are given by:

\[
\begin{align*}
v_{n1}(t) &= E_1 \sin (5 \omega t) + E_1 \sin (7 \omega t) \\
v_{n2}(t) &= E_1 \sin \left( \frac{2}{3} \omega t \right) + E_1 \sin \left( \frac{4}{3} \omega t \right) + E_1 \sin \left( \frac{1}{3} \omega t \right)
\end{align*}
\]
\[ v_{n,i}(t) = E_i \sin \left( \frac{\omega t + \frac{2}{3}}{3} \right) + E_s \sin \left( 5 \omega t + \frac{2}{3} \right) \]

4.2 Voltage unbalance compensation

The three-phase source voltages are unbalanced, but do not contain harmonic components. Their expressions are given by:

\[ v_{x,i}(t) = E_i \sin \left( \omega t + \frac{2}{3} \right) + 0.1E_s \sin \left( \omega t + \frac{2}{3} \right) \]

\[ v_{x,j}(t) = E_i \sin \left( \omega t + \frac{2}{3} \right) - 0.1E_s \sin \left( \omega t + \frac{2}{3} \right) \]

4.3 Voltage sags compensation

The performances of the controller are illustrated by figures 2 to 8. The figure 2 shows the voltage of DC bus \( v_{dc} \) converges, in the mean to the average value of the output voltage of the rectifier bridge: \( U_{moy} = 3\sqrt{3}E_f \).

Figure 3 shows the distortion in source voltages \( v_{a1} v_{a2} v_{a3} \). Figure 4 clearly shows that the load voltages \( v_{L1} v_{L2} v_{L3} \) after compensation are balanced and sinusoidal. Figure 5 shows source voltages unbalanced \( v_{a1} v_{a2} v_{a3} \). Figure 6 shows the load voltages \( v_{L1} v_{L2} v_{L3} \) after compensation are balanced and sinusoidal. Figure 7 shows the sags in source voltages \( v_{a1} v_{a2} v_{a3} \). Figure 8 shows the load voltages \( v_{L1} v_{L2} v_{L3} \) after compensation are balanced and sinusoidal after the voltages sags.

5 Conclusion

The problem of controlling three-phase series active power filter is addressed in this paper. The control objective is to compensate all voltage perturbations (voltage harmonics, voltage unbalance and voltage sags) caused by nonlinear loads. Indeed the harmonics of the current generated by the nonlinear load, cause voltage distortion of the power source. The solution of the problem is processed using a controller formed by two nonlinear regulators by using the Backstepping technique to compensate the voltage perturbations. The simulation results show that it performs perfectly the objectives.

References