

Analysis and control design of two cascaded boost converter

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Abstract. This work aims to study a cascade of two BOOST converters. First, a non-linear model of the whole controlled system is developed. Then, a robust non-linear controller of currents is synthesized using a backstepping design technique. A formal analysis based on Lyapunov stability and average theory is developed to describe the control currents loops performances. A classical PI controller is used for the voltages loops. The study of the stability of the system will also be discussed. Simulated results are displayed to validate the feasibility and the effectiveness of the proposed strategy.

1 Introduction

The DC / DC converters have become an essential component of industrial and military applications over the past decades. Thanks to their efficiency increasingly high, their size, their weight and reduced cost, they have replaced conventional linear power supplies, even at low power levels [1].

Obtaining a high voltage is needed in some applications which the input supply is low such as photovoltaic panel or fuel cell.

The problem can be handled either by using a simple step-up converter with high duty cycle or by using cascaded converters. The use of a single stage in performing this conversion ratio will imply working with high duty cycles and therefore will increase the losses and reduce the voltage conversion ratio. Moreover, the use of an individual power converter with a high duty cycle to obtain high voltage conversion ratios has some design limitations due to the finite commutation times of the power devices and the size of the passive elements. If galvanic isolation is not required, the cascade connection of two boost converters can be a good alternative to obtain high voltage step-up ratios.

Some previous works have presented a studies of cascaded converters, such as combination of an interleaved converter and inverter three levels [2], and a comparison between static and dynamic performances of a Z-source converter and dual-stage boost converter [3]. In this article, a study of two cascaded boost converter will be presented.

A control strategy is developed that simultaneously deals with the two cascaded boost. The control strategy is featured by its multi-loops nature. First, the two current loops are designed using the backstepping technique [4]. Then, two others loops are designed to regulate the output voltage and the voltage between the two boost converters using the classical PI regulator.

Additional actions are designed to dominate the disturbing effects while preserving the closed-loop system stability. It will be formally proved that the robust multi-loop controller thus obtained stabilizes (globally and asymptotically) the controlled system and ensures quite interesting tracking properties [5].

These theoretical results are obtained making judicious use of adequate control theory tools including averaging theory and Lyapunov stability [6]. The paper is organized as follows: the system under study (i.e. the DC-DC cascaded boost converter) is modeled and given a state space representation in Section2; the controllers design and the closed-loop system analysis are dealt with in Section3; the controller performances are illustrated through numerical simulations in Section4.

2 Modeling the two cascaded boost converter

The circuit in figure 1 consists of two cascaded Boost converter. The boost converters are according to the known pulse wide modulation (PWM) principle.

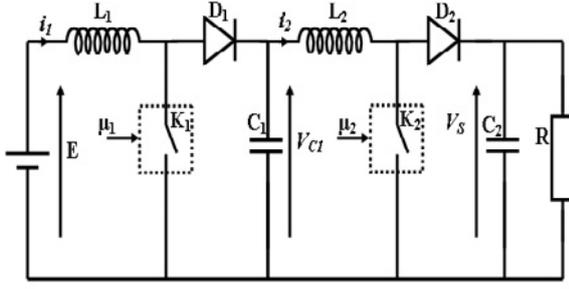


Fig. 1. Two cascaded boost converter.

The used parameter values of the present converter are depicted in Table 1.

Table 1. The used parameter values for the converter.

Parameter	Value
Inductors	$L_1 = 35\mu H$ $L_2 = 100\mu H$
Capacitors	$C_1 = 4mF$ $C_2 = 600\mu F$
Frequency	10kHz
Input voltage	$E = 12V$
Load resistor	$R = 10\Omega$
Swiches	IGBT

2.1 Switched model

By applying standard Kirchhof Voltage Law (KVL) and Kirchhof Current Laws (KCL) to the circuit depicted in Figure 1, the two cascaded Boost converter can be represented by the following differential equations:

$$L_1 \frac{di_1}{dt} = E - (1 - \mu_1)V_{c1} \quad (1a)$$

$$C_1 \frac{dV_{c1}}{dt} = (1 - \mu_1)i_1 - i_2 \quad (1b)$$

$$L_2 \frac{di_2}{dt} = V_{c1} - (1 - \mu_2)V_s \quad (1c)$$

$$C_2 \frac{dV_s}{dt} = (1 - \mu_2)i_2 - \frac{V_s}{R} \quad (1d)$$

Where:

- i_1 and i_2 are respectively the currents in inductors L_1 and L_2 ,
- V_{c1} denotes the voltage in capacitor C_1 and V_s is the output voltage.
- μ_1 and μ_2 are denoted the duties ratio functions.

$$\mu_1 = \begin{cases} 1 & \text{if } K_1 \text{ is ON} \\ 0 & \text{if } K_1 \text{ is OFF} \end{cases} \quad \text{and} \quad \mu_2 = \begin{cases} 1 & \text{if } K_2 \text{ is ON} \\ 0 & \text{if } K_2 \text{ is OFF} \end{cases}$$

2.2 Averaged model

This model is defined as follows:

$$\dot{x}_1 = -\frac{1 - u_1}{L_1} x_2 + \frac{E}{L_1} \quad (2a)$$

$$\dot{x}_2 = \frac{1 - u_1}{C_1} x_1 - \frac{1}{C_1} x_3 \quad (2b)$$

$$\dot{x}_3 = \frac{1}{L_2} x_2 - \frac{1 - u_2}{L_2} x_4 \quad (2c)$$

$$\dot{x}_4 = \frac{1 - u_2}{C_2} x_3 - \frac{1}{RC_2} x_4 \quad (2d)$$

In the above model, x_1, x_2, x_3, x_4 are denoted respectively the averages currents and voltages i_1, V_{c1}, i_2, V_s and (u_1, u_2) represents the averages duties ratio function (μ_1, μ_2) .

2.3 Equilibrium points

The equilibrium point can be obtained by forcing the time derivative of the state variables of the reduced order model to be null.

$$\dot{x} = 0 \Rightarrow \begin{cases} x_1^0 = \frac{1}{(1-u_1^0)^2(1-u_2^0)^2} \frac{E}{R} & (3a) \\ x_2^0 = \frac{E}{1-u_1^0} & (3b) \\ x_3^0 = \frac{E}{R(1-u_1^0)(1-u_2^0)^2} & (3c) \\ x_4^0 = \frac{1}{(1-u_1^0)(1-u_2^0)} E & (3d) \end{cases}$$

The equilibrium point is noted:

$$X^0 = (x_1^0 \quad x_2^0 \quad x_3^0 \quad x_4^0) \quad (3e)$$

From the equilibrium point, the output power is equal to the input power and that the input voltage in a boost converter is less than its output voltage

2.4 Stability analysis

In order to study the stability of the system, the non-linear model (2a)-(2d) is linearized around the equilibrium point X^0 given by (3e). The stability of the linearized system can be studied by using the Jacobian matrix A and evaluating it at the equilibrium point X^0 . This matrix can be expressed as follows:

$$A = \left. \frac{\partial f}{\partial x} \right|_{\substack{U = U^0 \\ X = X^0}}$$

$$A = \begin{pmatrix} 0 & -\frac{(1-u_1^0)}{L_1} & 0 & 0 \\ \frac{(1-u_1^0)}{C_1} & 0 & -\frac{1}{C_1} & 0 \\ 0 & \frac{1}{L_2} & 0 & -\frac{(1-u_2^0)}{L_2} \\ 0 & 0 & \frac{(1-u_2^0)}{C_2} & -\frac{1}{RC_2} \end{pmatrix}$$

The local stability analysis of the system can be carried out by using the characteristic polynomial equation: $p(s) = \det(A - sI) = 0$ of the linearized system, where I is the unitary matrix.

Using Routh-Hurwitz criterion, four conditions for stability are obtained.

If we consider the choice equilibrium point:

$$X^0 = (7,5 \quad 18 \quad 5 \quad 30)^T$$

$$U = (1/3 \quad 0,4)^T$$

$p(s)$ can be written in the following form:

$$\det(sI - A) = s^4 + 166,7s^3 + 3174,10^3s^2 + 546,2,10^6s + 39,7,10^{12} = 0$$

The roots are located in the left plane and hence, the system is asymptotically stable.

We have:

$$B = \begin{pmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} \\ \frac{\partial f_3}{\partial u_1} & \frac{\partial f_3}{\partial u_2} \\ \frac{\partial f_4}{\partial u_1} & \frac{\partial f_4}{\partial u_2} \end{pmatrix} = \begin{pmatrix} \frac{x_2^0}{L_1} & 0 \\ \frac{x_1^0}{C_1} & 0 \\ 0 & \frac{x_4^0}{L_2} \\ 0 & \frac{x_3^0}{C_2} \end{pmatrix}$$

$$C = \begin{pmatrix} \frac{\partial y}{\partial x_1} & \frac{\partial y}{\partial x_2} & \frac{\partial y}{\partial x_3} & \frac{\partial y}{\partial x_4} \end{pmatrix} = (0 \quad 0 \quad 0 \quad 1)$$

$$y = x_4$$

The transfer function defined by:

$$H_s(s) = C (sI - A)^{-1}B + D$$

We can justify that the system is a non- minimum phase because it contains some zeros dynamics

The output is given by:

$$Y(s) = \frac{N_1(s)}{\det(sI - A)}U_1(s) + \frac{N_2(s)}{\det(sI - A)}U_2(s)$$

Then:

$$N_1(s) = -(5860s + 122.45)10^{12}$$

$$N_2(s) = (0,0083s^3 + 7800s^2 - 69000s - 2481010^6)10^6$$

3 Control design

3.1 Control objectives

There are four operational control objectives:

- (i) Current regulation: the input current x_1 (respectively the current x_3 in inductor L_2) must track ,as closely as possible, a given reference signal x_1^* (respectively x_3^*).
- (ii) Controlling the voltage x_2 making it track a given reference signal x_2^* .
- (iii) Regulating the output voltage x_4 to a reference value x_4^* .
- (iv) Ensure the global stability of the system.

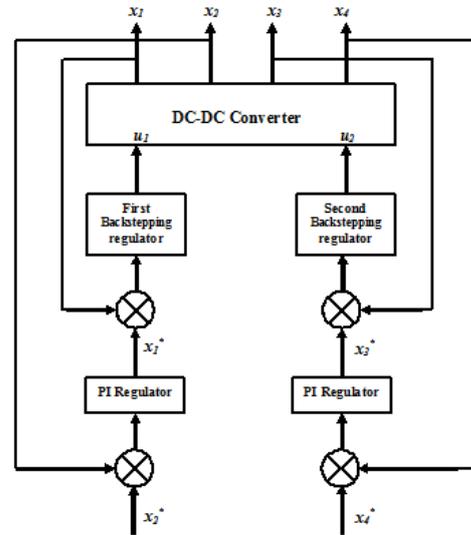


Fig. 2. Regulators based structure.

As shown in figure 2 we define two current loops (inner loop) and two voltage loops (outer loop).

3.2 Inner loop

Consider the tracking errors z_1 and z_3 defined by:

$$z = \begin{pmatrix} z_1 \\ z_3 \end{pmatrix} = \begin{pmatrix} L_1(x_1 - x_1^*) \\ L_2(x_3 - x_3^*) \end{pmatrix} \quad (4)$$

Its derivative is given by:

$$\dot{z} = \begin{pmatrix} \dot{z}_1 \\ \dot{z}_3 \end{pmatrix} = \begin{pmatrix} L_1(\dot{x}_1 - \dot{x}_1^*) \\ L_2(\dot{x}_3 - \dot{x}_3^*) \end{pmatrix} \quad (5)$$

Consider the quadratic Lyapunov function:

$$V = \frac{1}{2}z^T z \quad (6)$$

Its derivative is done:

$$\dot{V} = \frac{1}{2}(\dot{z}^T z + z^T \dot{z}) = z_1 \dot{z}_1 + z_3 \dot{z}_3 \quad (7)$$

To get a stabilizing control laws, the time-derivative \dot{V} must be a negative definite function of z_1 and z_3 . Then \dot{V} is chosen :

$$\dot{V} = -k_1 z_1^2 - k_3 z_3^2 < 0 \quad (8)$$

Where k_1 and k_3 are the constants positives parameters synthesis of the controllers.

This choice implies that:

$$\begin{cases} \dot{z}_1 = -k_1 z_1 \\ \dot{z}_3 = -k_3 z_3 \end{cases} \quad (9)$$

$$\quad (10)$$

We replace \dot{z}_1 and \dot{z}_3 in (4), we obtain:

$$\begin{cases} -k_1 z_1 = L_1(\dot{x}_1 - \dot{x}_1^*) \\ -k_3 z_3 = L_2(\dot{x}_3 - \dot{x}_3^*) \end{cases} \quad (11)$$

$$\quad (12)$$

Substituting \dot{x}_1 and \dot{x}_3 respectively by the relationships of the expressions (2a) and (2c), then:

$$\begin{cases} -k_1 z_1 = E - (1 - u_1)x_2 - L_1 \dot{x}_1^* \\ -k_3 z_3 = x_2 - (1 - u_2)x_4 - L_2 \dot{x}_3^* \end{cases}$$

Then the control laws to satisfy a stabilization are defined by:

$$\begin{cases} u_1 = 1 + \frac{L_1 \dot{x}_1^* - k_1 z_1 - E}{x_2} \\ u_2 = 1 + \frac{L_2 \dot{x}_3^* - k_3 z_3 - x_2}{x_4} \end{cases} \quad (15)$$

$$\quad (16)$$

The properties of these control laws are summarized in the proposition 1.

Proposition1. Consider the system composed of the currents equations (2a) and (2c), and the control laws (15) and (16) where $k_1 > 0$ and $k_3 > 0$ are arbitrarily chosen by the user. If the references x_1^* and x_3^* and their first time derivatives are available, then one has the following properties:

- The current loop undergoes the equations (9) and (10) with (5). As k_1 and k_3 are positives, these equations are exponentially stables.
- The input current and the current in inductor L_2 tends to their references.

3.3 Outer loop

The aim is now to design a tuning laws for the x_1^* and x_3^* .

3.3.1 Relationship between x_1^* and x_2

The first step in designing such a loop is to establish the relation between x_1^* (control input current) and the output voltage x_2 .

Using the equation (2b) and replacing u_1 by (15) and x_1, x_3 by the relations deduced for (4), we obtain:

$$2C_1 x_2 \dot{x}_2 = 2(k_1 z_1 + E - L_1 \dot{x}_1^*) \left(\frac{z_1}{L_1} + x_1^* \right) - 2x_2 \left(\frac{z_3}{L_2} + x_3^* \right) \quad (17)$$

If we consider: $y_2 = x_2^2$ then: $\dot{y}_2 = 2x_2 \dot{x}_2$

$$C_1 \dot{y}_2 = 2(k_1 z_1 + E - L_1 \dot{x}_1^*) \left(\frac{z_1}{L_1} + x_1^* \right) - 2x_2 \left(\frac{z_3}{L_2} + x_3^* \right)$$

Finally:

$$\dot{y}_2 = \frac{2E}{c_1} x_1^* + \frac{2z_1 k_1}{c_1} x_1^* - \frac{L_1}{c_1} \frac{d}{dt} (x_1^{*2}) - \frac{2z_1}{c_1} \dot{x}_1^* + X(x, t) \quad (18)$$

With:

$$X(x, t) = \frac{1}{c_1} \left(-2\sqrt{y_2} x_3^* + \frac{2k_1}{L_1} z_1^2 + \frac{2E}{L_1} z_1 - \frac{2z_3}{L_2} \sqrt{y_2} \right) \quad (19)$$

Proposition 2. Consider the system described by (2a) and (2b) together with the control law (15). Under the same assumptions as in Proposition 1, one has the following properties:

- (i) The output voltage x_2 varies, in response to the reference, according to the equation (17).
- (ii) The squared voltage $y_2 = x_2^2$ varies, in response to the reference, according to the equation (18) with (19).

3.3.2 Controller PI design

Note that the model (18) can be seen as an integrator disturbed by the signal $p(t)$. The development of the controller is based on the following average model:

$$\dot{\bar{y}}_2 = \alpha \bar{x}_1^* + \bar{p}(t) \quad (20)$$

with: $\alpha = \frac{2E}{c_1}$

The system (20) can be stabilized using a simple PI corrector whose transfer function is given by:

$$C(s) = k_{p2} + \frac{k_{i2}}{s}$$

The first derivative of the reference x_1^* must be available (Proposition 1), the low pass filter is adopted, then:

$$x_1^* = \left[k_{p2}(y_2^* - y_2) + k_{i2} \int (y_2^* - y_2) dt \right] \frac{b}{s + b} \quad (21)$$

Then:

$$x_1^* = \frac{b}{s + b} (k_{p2}z_2 + k_{i2}z_{22}) \quad (22)$$

$$\text{With: } \begin{cases} z_2 = y_2^* - y_2 \\ z_{22} = \int z_2 dt \end{cases}$$

The block diagram of the controlled system is shown in Figure 3.

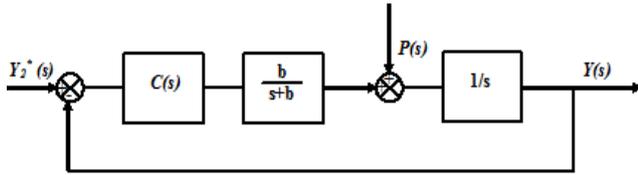


Fig. 3. Voltage equivalent loop.

In the closed loop, the output signal y depends on the reference and the disturbance signal $p(t)$ using the following equation:

$$Y_2(s) = F(s)Y_2^*(s) + G(s)P(s) \quad (23)$$

With:

$$F(s) = \frac{b(k_{i2} + k_{p2}s)}{s^3 + bs^2 + k_{p2}bs + k_{i2}b} \quad (24)$$

$$G(s) = \frac{s(s + b)}{s^3 + bs^2 + k_{p2}bs + k_{i2}b} \quad (25)$$

Using Routh-Hurwitz criterion, the conditions for stability are given by:

$$\begin{cases} b > 0 \\ k_{i2} > 0 \\ k_{p2}b > k_{i2} \end{cases}$$

The transfer function $F(s)$ shows that the regulator guarantees perfect tracking $\lim_{t \rightarrow +\infty} (y_2^* - y_2) = 0$ and $G(s)$ guarantees a disturbance rejection because it contains a derivative effect.

The parameters values chosen are depicted in Table 2.

The BODE diagram in figure 4 shows the frequency behavior of $F(s)$.

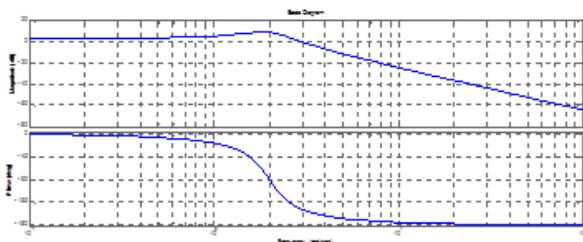


Fig. 4. Bode diagram of $F(s)$.

3.3.3 Relationship between x_3^* and x_4 :

Proceed by the same approach that paragraph 3.3.2, we obtain:

$$\dot{y}_4 = \frac{2}{C_2} \left[(k_3 z_3 + x_2 - L_2 \dot{x}_3^*) \left(\frac{z_3}{L_2} + x_3^* \right) - \frac{2}{R} y_4 \right]$$

$$x_3^* = \left[k_{p4}(y_4^* - y_4) + k_{i4} \int (y_4^* - y_4) dt \right] \frac{b'}{s + b'}$$

$$x_1^* = \frac{b}{s + b} (k_{p2}z_2 + k_{i2}z_{22})$$

$$\text{With: } \begin{cases} z_2 = y_2^* - y_2 \\ z_{22} = \int z_2 dt \end{cases}$$

4 Simulations

In order to verify the theoretical results predicted in Sections 2 and 3, the two cascaded boost converter has been simulated by using SIMPOWER of Matlab-Simulink software with the set of parameter values depicted in Tables 1 and 2.

Table 2. Parameter values for the simulations.

Parameters	Values
k_1, k_3	200
Integral: k_{i2}, k_{i4}	4
Proportional: k_{p2}, k_{p4}	2
Low filter: b, b'	3
Reference: x_2^*	{ Initial value: 18 Final value: 36
Reference: x_4^*	{ Initial value: 30 Final value: 60

The converter has been simulated to check the previous stability conditions.

The figures 5-6 and 7-8 present respectively the forms of currents and voltages. We observe that the tracking is satisfied and the stability is guaranteed.

The forms of the errors are showed in figures 9 and 10.

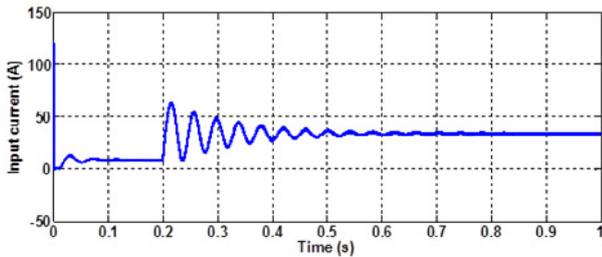


Fig. 5. Input current.

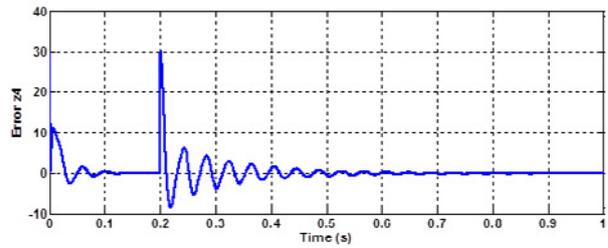


Fig. 9. Tracking error z_4 .

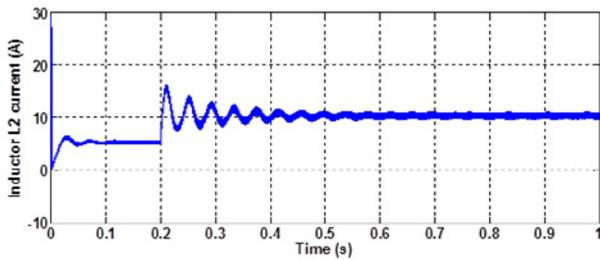


Fig. 6. Inductor L_2 current.

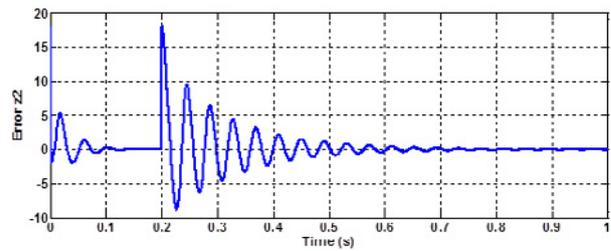


Fig. 10. Tracking error z_2 .

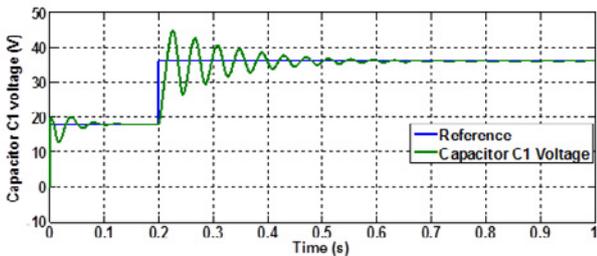


Fig. 7. Capacitor C_1 voltage.

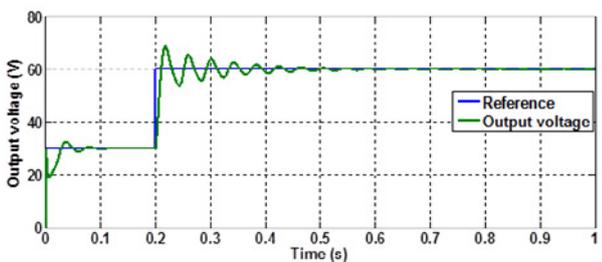


Fig. 8. Output voltage.

5 Conclusion

In some applications, high conversion ratios are needed. Different techniques and converter topologies can be obtained to handle this problem. In this paper, the two cascaded boost converter has been studied for his potential use in this kind of applications. It has been analyzed theoretically and by numerical simulations using Matlab software. The dynamical models and stability analysis of this system has been presented. The controller design have been studied also. The results of simulations have been satisfied.

References

1. H. Sira-Ramirez, M. Garcia-Esteban, A.S.I. Zinober, *PWM DC-DC power converter* .Int. J. E **65**, 205 (1996)
2. A. Shahin, B. Huang, J.P. Martin., S. Pierfederici., B. Davat.*New non linear control strategy for DC/DC converter* E.C.M **51**, 56 (2010)
3. A. Elaroudi, R. Haroun, A. Cid-Pastor, A. Kouzou, L. Martinez-Salamero.,*Z-source and dual-stage boost comparison* Energie Procedia **42**, 587 (2013)
4. J. Alvarez-Ramirez, G. Espinosa-Perez, D. Noriega-Pineda. *Banaszak Backstepping approach*.IEEE., 190 (2001)
5. A. Elmaguiri, F. Giri, A. Abouloifa, F.Z. Chaoui. *Robust control* , Elsevier (2010)
6. H. Elfadil, F. Giri, F.Z. Chaoui., M. Haloua, H. Ouadi , *Non-linear and adaptive control*. IEEE **42**,4475 (2003)