

Rotor-Stator Contact – Overview of Current Research

Oliver Alber^{1,a} and Richard Markert¹

Structural Dynamics, Darmstadt University of Technology, Otto-Berndt-Str. 2, 64287 Darmstadt, Germany

Abstract. The aim of this paper is to provide a compact as well as comprehensive overview of Rotor-Stator Contact in rotor dynamics. A general model is described which accounts for most phenomena of Rotor-Stator Contact observed in literature. This model is compared to different modeling approaches used in the previous literature. A glance on the variety of motion patterns including analytical approaches to the synchronous motion and Backward Whirl motion is given. As an outlook a modal reduction technique is pointed out, which is capable of reducing systems with many degrees of freedom for rotor as well as stator to the described model.

1 Introduction

In rotating machinery there is often the possibility of the rotor contacting a non-rotating device (e.g. stator, housing or retainer bearing) due to the deformation of the rotor shaft. This might lead to dangerous, mostly non-controllable vibrations. Especially when increasing the efficiency of rotating machinery by decreasing the gap between the rotor and its housing, small vibrations can bridge the gap between rotor and stator. Many reasons for Rotor-Stator Contact exist which are often inevitable. Severe damage of rotating machines is generally not caused by synchronous nonlinear motion but rather caused by asynchronous motions. These vibrations are usually related with huge contact forces and impacts. The importance of Rotor-Stator Contact in rotor dynamics is reflected in a large amount of previous literature. There are some contributions summarizing the global field of Rotor-Stator Contact like [1], [2] and [3] which in most parts describe observed phenomena, different modeling approaches and results. The described systems are all characterized by a strong non-linearity due to the contact and are therefore highly sensitive regarding modeling assumptions, system parameters and initial conditions. Many issues discussed in the literature are specific for individual applications (e.g. retainer bearings, bearings with looseness, etc.) but there are also some general phenomena occurring.

The challenge is to understand the mechanisms behind the observed phenomena and to be able to reduce the damage of potential Rotor-Stator Contact. To study the dynamics that occur during Rotor-Stator Contact it is important to focus on simple models that are on the one side capable of including all important effects and on the other side allow for an understanding of the mechanisms of these phenomena. This paper will give a quick introduction to a simple and well-established model. By applying a modal decoupling technique this model is able to describe systems consisting of a higher amount of degrees of freedom at least in a restricted regime.

2 Basic Modeling for Rotor-Stator Contact

A simple model for Rotor-Stator Contact consists of a JEFFCOTT rotor (mass m_R , stiffness k_R and mass excentricity ε_M) contacting a flexibly mounted rigid stator (mass m_S , mounting stiffness k_S), see Fig. 1. Both rotor and stator account for external viscous damping (b_S, b_R) and may allow a static stator offset r_{Sstat} . The equations of motion for rotor and stator displacement r_R and r_S are

$$m_R \ddot{r}_R + b_R \dot{r}_R + k_R r_R = -m_R \varepsilon_M (e^{i\varphi})'' - F_C, \quad (1)$$

$$m_S \ddot{r}_S + b_S \dot{r}_S + k_S (r_S - r_{Sstat}) = F_C. \quad (2)$$

The contact areas of rotor and stator are cylindrical surfaces with an average nominal gap s . The kinematic relation between s, r_R, r_S , rotor to stator instantaneous minimal gap δ and the direction ψ of the contact force shown in figure 1 is described by

$$r_R - r_S = (s - \delta) e^{i\psi} \quad (3)$$

using complex notation. Describing continuous contact ($\delta = 0$), this leads to the kinematic contact condition

$$r_R + \varepsilon_A e^{i\varphi} - r_S = s \frac{1 - i\mu_F}{\sqrt{1 + \mu_F^2}} \frac{F_C}{|F_C|} \quad (4)$$

where dry friction is taken into account by μ_F . This contact force expression is only able to reproduce continuous contact, which appears for example in the pure synchronous motion or Backward Whirl. If partial contact with temporary separation of rotor and stator occurs the contact force has to be switched to zero. Alternatively the contact force F_C can be described by a contact element with contact stiffness k_C and contact damping b_C , e.g. a pseudo-linear viscoelastic contact element [4]

$$F_C = (1 + i\mu_F) \left\langle -k_C \delta - b_C \langle \dot{\delta} \rangle \right\rangle \langle -\delta \rangle^0 e^{i\psi}, \quad (5)$$

where $\langle x \rangle = x$ only for $x > 0$.

^a e-mail: alber@sdy.tu-darmstadt.de

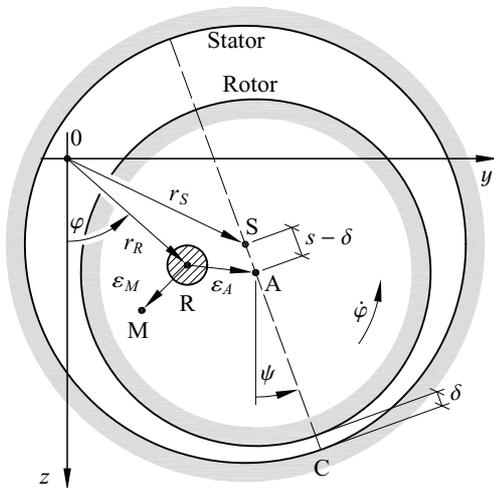
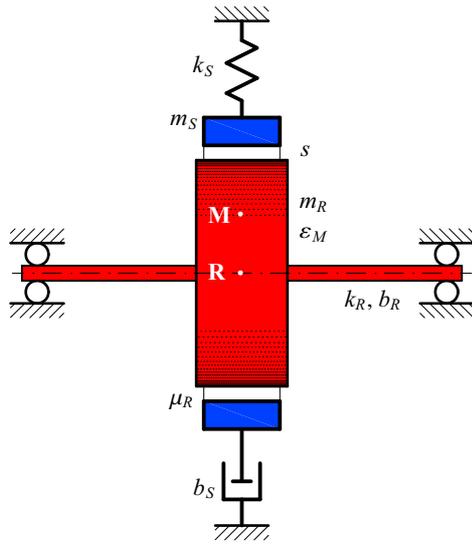


Fig. 1. Model and kinematics of rotor and stator [4]

3 Assignment of the Model to Literature

The model of section 2 shall be compared to well-known literature. Some papers like e.g. Black [5] and Childs [6] use an identical model only investigating continuous contact ($\delta = 0$) and therefore using the contact condition (4). This simplification gives rise to analytical solutions for continuous contact (see 4.1 and 4.2). Ehrich [7] neglects friction and stator damping ($\mu_F = 0, b_S = 0$) but also includes the idea of switching the contact force to zero if there is no contact. Most of the literature assumes COULOMB friction between the contact surfaces, like it is described in section 2. Isaksson [8] and Jiang et al. [9] neglect stator mass and stator damping ($m_S = b_S = 0$) but also take into account whether there is contact or no contact. Muszynska [2] generally neglects the stator mass ($m_S = 0$), which gives only rise to systems with infinite high stator natural frequency. She introduces a contact formulation similar to eq. (5), but assumes the contact damping to be proportional to the rotor velocity and not the penetration depth δ . Bartha [10] takes into account a relative tangential velocity $v_{t,rel}$ into and against the direction of rotation in Fig. 1. As in some other papers he uses a non-linear contact formulation by HUNT-CROSSLEY. In [13] different contact mod-

els are compared. It is concluded that all contact models observed show a similar global transient behaviour during impact if the energy dissipation of the different models is equivalent. Gasch [11] uses the presented formulas including an elastic contact element similar to eq. (5) but without contact damping ($b_C = 0$).

The described model of section 2 does not account for thermal effects like spiral vibrations or *soft rubbing* [12]. These vibrations usually occur on a much slower time scale than the vibrations discussed in section 2. Also a non-linear stator stiffness characteristic [13] and anisotropic rotor stiffness [2] are neglected. Gyroscopic influences on such a system are described in [14]. If the contact surfaces are non-cylindrical a more complicated contact kinematic has to be taken into account (e.g. [15]). A model approach for including torsional effects is given in [16].

4 Motion Patterns for Rotor-Stator Contact

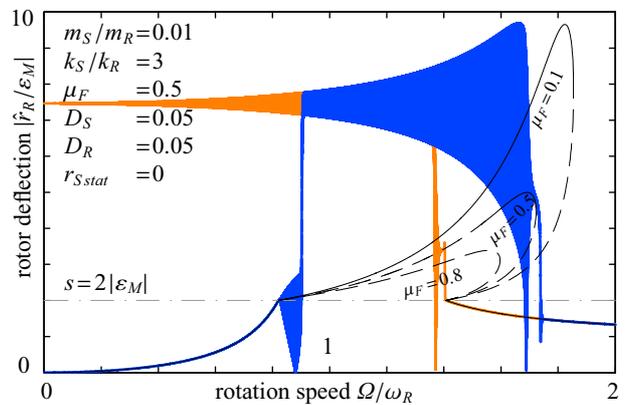


Fig. 2. Unbalance response of a JEFFCOTT rotor contacting the stator for $\mu_F = 0.5$; analytical solution for different values of μ_F [17]

For the system described different motion patterns have been observed and in some parts validated experimentally [4]. Depending on the initial conditions different motion patterns may develop for the same parameter set. For example in Fig. 2 at rotor speed $\Omega = 0.84 \omega_R$ Forward or Backward Whirl motion arise. The regular unbalance response is the synchronous motion. If however the synchronous motion is unstable (dashed line in Fig. 2), asynchronous motion occurs. In order to identify the motion patterns it is recommended to monitor the following state variables: From the time signal of the contact force it can be deduced whether a continuous or partial contact is present. This influences the decision which contact model assumptions have to be taken. An explicit differentiation between the motion patterns can only be achieved by analyzing the frequency spectrum of rotor or stator deflection amplitudes and identifying the frequency components in relation to the rotation speed. In addition the orbits or POINCARÉ maps of the rotor or stator deflection amplitudes can be analyzed, but in this way no unique identification is possible [17]. For illustration in Fig. 3 orbits as well as POINCARÉ maps of the synchronous (a), subharmonic (b) and superharmonic (c) motion, Forward (d) and Backward (e) Whirl, motion with sidebands around the synchronous motion (f) and chaotic

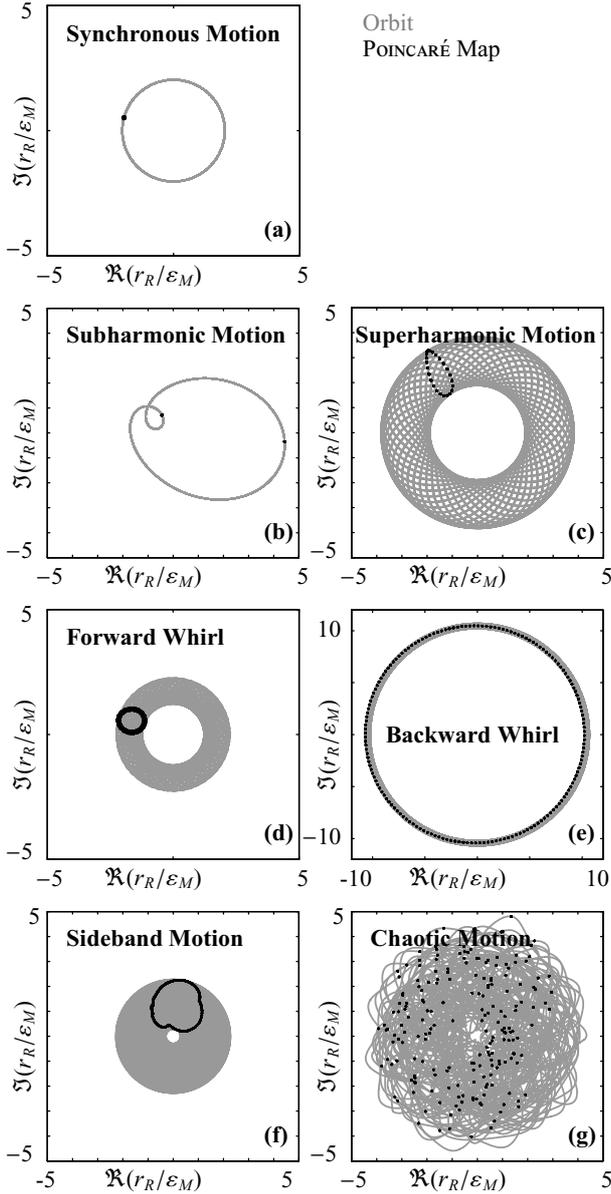


Fig. 3. Orbits & POINCARÉ maps for different motion patterns [17]

motion (g) are displayed. The occurrence of different motion patterns is generally dependent on different parameters: For example a static offset generally induces sub- and superharmonic motions as well as chaotic motions. An centrally mounted disk with low friction and high damping usually leads to synchronous motion. The relation between the uncoupled resonance frequencies of rotor and stator influences rotor and stator amplitudes, contact force amplitude and resonance frequency as well as the stability of the synchronous motion. Dry friction in the contact surfaces causes backward components in the frequency spectrum and therefore Backward Whirl and/or Forward Whirl results. A comprehensive parameter study can be found in [4].

4.1 Synchronous Motion and its Stability

The purely synchronous motion during stationary rotation ($\Omega = \text{const.}$) can be calculated analytically. Inserting a synchronous approach

$$r_R = \widehat{r}_R e^{i\Omega t}, \quad r_S = \widehat{r}_S e^{i\Omega t}, \quad F_C = \widehat{F}_C e^{i\Omega t} \quad (6)$$

into the equations of motion (1), (2) and the kinematic contact condition (4), the contact force amplitude \widehat{F}_C as well as the rotor deflection amplitude \widehat{r}_R can be eliminated. The resulting equation

$$\left[(a_R + a_S) |\widehat{r}_S| + a_{RS} \frac{1 - i\mu_F}{\sqrt{1 + \mu_F^2}} \frac{a_S}{|a_S|} \right] \frac{\widehat{r}_S}{|\widehat{r}_S|} = \dots \dots a_S \varepsilon_A + m_R \varepsilon_S \Omega^2 \quad (7)$$

with the dynamic stiffness of rotor and stator

$$a_R = k_R - \Omega^2 m_R + i\Omega b_R \quad (8)$$

$$a_S = k_S - \Omega^2 m_S + i\Omega b_S \quad (9)$$

can be solved for the stator deflection amplitudes \widehat{r}_S . This method is described for example in [18], [5] and [2]. By using LJAPUNOW'S method a stability analysis of the synchronous motion can be performed (e.g. [13] or [2]).

4.2 Backward Whirl Motion

If a high friction coefficient in the contact surface is present, this may drive the rotor to move in the opposite direction of the rotation speed with a negative asynchronous frequency Ψ . Two cases can be separated depending on the relative tangential velocity v_{trel} of rotor and stator at the contact position. v_{trel} is approximately

$$v_{trel} \approx \Psi s + \Omega \frac{d_R}{2}, \quad (10)$$

assuming that the propagation speed of the contact point is primarily dependent on the asynchronous frequency Ψ [4].

a) Rolling of the rotor in the stator ($v_{trel} = 0$)

This motion is for example described in [5] and [10]. The rolling frequency depends on the average gap size s , the contact surface diameter d_R and the rotation speed Ω :

$$\Psi_{roll} = -\Omega \frac{d_R}{2s} \quad (11)$$

b) Sliding of the rotor in the stator ($v_{trel} > 0$)

Eq. (10) shows, that for positive v_{trel} the inequality

$$-\frac{\Psi}{\Omega} < \frac{d_R}{2s} \quad (12)$$

has to be fulfilled. As the gap size s is usually very small compared to the contact surface diameter d_R , Eq. (12) is only violated for very small rotation speeds [4].

In this case a multi-frequent approach

$$r_R = \widehat{r}_{R\Omega} e^{i\Omega t} + \widehat{r}_{R\Psi} e^{i\Psi t}, \quad (13)$$

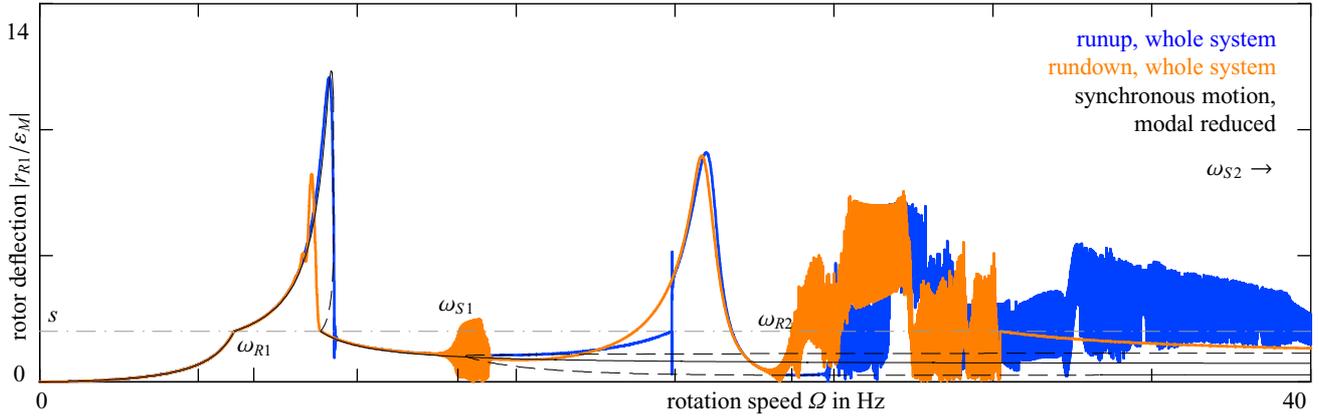


Fig. 4. Modal decoupling of the synchronous motion [20]

$$r_S = \widehat{r}_{S\Omega} e^{i\Omega t} + \widehat{r}_{S\Psi} e^{i\Psi t}, \quad (14)$$

$$F_C = \widehat{F}_{C\Omega} e^{i\Omega t} + \widehat{F}_{C\Psi} e^{i\Psi t} \quad (15)$$

leads to a semi-analytical calculation of the Backward Whirl frequency Ψ by forcing the imaginary part of the approximated asynchronous contact force amplitude

$$|\widehat{F}_{C\Psi}| = \frac{-s(1-i\mu_F)}{\sqrt{1+\mu_F^2}} \frac{1}{\frac{1}{k_R - \Psi^2 m_R + ib_R} + \frac{1}{k_S - \Psi^2 m_S + ib_S}} \quad (16)$$

to be zero. Stability of the Backward Whirl has to be investigated numerically, for example by using the calculated motion as an initial condition for a simulation with fixed rotation speed and investigating the steady-state motion [19].

5 Approximative Modal Decoupling

The simple model for Rotor-Stator Contact of section 2 is able to predict the dynamics of even more complicated systems containing many resonance frequencies for rotor and stator, at least in particular regimes. In [20] a transformation to the modal coordinates $\mathbf{p}(t)$ of the uncoupled rotor respective stator

$$\mathbf{r}_R(t) = \mathbf{Q}_R \mathbf{p}_R(t) = \sum_{n=1}^N \widehat{\mathbf{q}}_{Rn} p_{Rn}(t) \quad (17)$$

$$\mathbf{r}_S(t) = \mathbf{Q}_S \mathbf{p}_S(t) = \sum_{m=1}^M \widehat{\mathbf{q}}_{Sm} p_{Sm}(t) \quad (18)$$

is performed using the modal matrices \mathbf{Q}_R and \mathbf{Q}_S . Taking into account only the n -th rotor mode and the m -th stator mode, normalizing the contacting coordinates of these modes to one ($\widehat{\mathbf{q}}_{RnC} = \widehat{\mathbf{q}}_{SmC} = 1$) and neglecting all other modes, one obtains the reduced equations

$$m_{Rn} \ddot{\tilde{r}}_R(t) + b_{Rn} \dot{\tilde{r}}_R(t) + k_{Rn} \tilde{r}_R(t) = \dots \quad (19)$$

$$\dots - \left(\mathbf{Q}_R^T \mathbf{M}_R \boldsymbol{\varepsilon}_M \right)_n \left(e^{i\varphi} \right)^- \tilde{F}_C(t),$$

$$m_{Sm} \ddot{\tilde{r}}_S(t) + b_{Sm} \dot{\tilde{r}}_S(t) + k_{Sm} \tilde{r}_S(t) = \dots \quad (20)$$

$$\dots \left(\mathbf{Q}_S^T \mathbf{K}_S \mathbf{r}_{Sstat} \right)_m = \tilde{F}_C(t),$$

$$\tilde{r}_R(t) + \varepsilon_A e^{i\varphi} - \tilde{r}_S(t) = s \frac{1-i\mu_F}{\sqrt{1+\mu_F^2}} \frac{\tilde{F}_C(t)}{|\tilde{F}_C(t)|} \quad (21)$$

with the modal parameters of the n -th rotor mode $\langle \rangle_{Rn}$ and the m -th stator mode $\langle \rangle_{Sm}$. $\langle \rangle_n$ and $\langle \rangle_m$ are the n -th and m -th matrix rows and $\langle \rangle$ are the approximated state variables. This form does exactly match to eq. (1), (2) and (4). In a similar way a reduction using eq. (5) is possible. If the neglected modes are not effecting the motion in the relevant rotating speed regime, the motion of the multimode system can be approximated quite well by the presented approach. This assumption has showed to be valid for the synchronous as well as for most of the asynchronous motions. Problems occur when reducing a Backward Whirl motion because in this case additional modes can not be neglected.

Fig. 4 shows an example system from [20] with two resonance frequencies for rotor (ω_{R1} , ω_{R2}) and for stator (ω_{S1} , ω_{S2}). Runup (blue) and rundown (orange) of the whole nonlinear system are compared to the approximative modal reduced system (black). For the reduced system the synchronous motion and its stability is calculated according to section 4.1. The dashed lines represent unstable synchronous motions. In the rotation speed regime up to 12 Hz only the first modes of rotor and stator are contributing to the overall motion of the system, the second modes are located at higher frequencies. In this regime a modal decoupling by the described way is appropriate. Above 12 Hz the higher modes are contributing significantly to the motion; in this regime a modal decoupling is not suitable.

6 Conclusions

In many cases the dynamics of Rotor-Stator Contact in rotating machinery can be reduced to a JEFFCOTT rotor contacting a flexibly mounted rigid stator. For this system a wide repertoire of literature is available, dealing with a huge variety of specific issues and applications. Analytical concepts for the usual synchronous motion as well as for the dangerous Backward Whirl motion are present. Most well-known literature is using exactly the same or at least a similar or even simpler model for Rotor-Stator Contact.

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