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Nonlinear Analysis of Spur Gear Pair with Time-varying Mesh Stiffness

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Abstract. This study presents nonlinear analysis of single degree of freedom spur gear pair with time-varying mesh stiffness. The backlash is approximated using nonlinear term. The periodic steady-state solutions of the nonlinear system are obtained by closed-form expressions using the method of multiple scales. The stability and forced vibration response of the gear system are analyzed. The effect of mesh stiffness variation on the amplitude parameter of nondimensional dynamic transmission error for primary resonance is presented. The closed-form solutions in terms of mesh stiffness variations provide design guidelines for dynamic analysis of spur gear.

1 Introduction

The interest in the nonlinear behavior of the gear system vibration with backlash nonlinearity in terms of transmission error is reflected in recent studies [1-3]. Moradi and Salarieh [1] investigated primary, super-harmonic and sub-harmonic resonances of spur gear pairs including the backlash nonlinearity using multiple scale method. Chang-Jian and Chang [2] presented the nonlinear dynamic behavior of spur gear pair with nonlinear suspension system. Si-Yu and Jin-Yuan [3] investigated the parameter stability of a strong nonlinear parametric excitation system for primary resonance by using the Multiple-scales method. Ozguven and Houser [4] provided a complete literature on the mathematical modeling of the gear systems. Kahraman and Singh [5-6] studied the nonlinear dynamics of a spur gear pair with particular emphasis on the interaction between the time-varying mesh stiffness and clearance nonlinearities. Parker et al. [7] presented nonlinear behaviour of a spur gear pair including nonlinear mesh stiffness and provided a comparison between theoretical and experimental results. Theodossiades and Natsiavas [8] presented analytical and numerical methods for determining periodic steady state response of gear-pair systems with periodic stiffness and backlash.

The purpose of this study is to investigate the nonlinear analysis of spur gear pair with time-varying mesh stiffness using the method of multiple scales. Frequency response and stability are investigated for primary resonance of spur gear pair. Results of amplitude parameter of nondimensional dynamic transmission error (a) with variation of nondimensional excitation frequency (Ω) are analyzed.

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2 Analysis

A method of multiple scale analysis of spur gear pair with time varying stiffness is considered in the analysis. Figure 1 presents the schematic of dynamic model of spur gear pair.
The equation of motion of nonlinear dynamic model of spur gear pair is

$$m\ddot{x} + c\dot{x} + k_o\left(1 + \hat{\beta}\cos\omega\hat{t}\right)\left(x + gx^3\right) = f_t + f_m \tag{1}$$

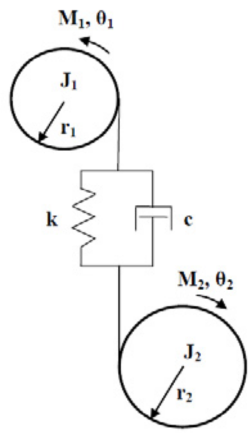

Figure 1 Dynamic model of spur gear pair

For the purpose of analysis, weak damping, weak nonlinear stiffness coefficient, weak time-varying stiffness coefficient, and weak perturbed excitation force are considered [9]. The coefficients $c, \hat{\beta}, g$ in Eq. (1) are expressed as

$$c = \varepsilon 2\mu\sqrt{mk_o}, \hat{\beta} = \varepsilon\beta, g = \varepsilon\frac{\gamma}{b^2} \tag{2}$$

The nondimensional form of Eq. (1) results in

$$\ddot{u} + 2\varepsilon\mu\dot{u} + u + \varepsilon\beta u\cos\Omega t + \varepsilon\gamma u^3 = f_o + \varepsilon f_1\cos\Omega t \tag{3}$$

Analytical approximate solution is determined using method of multiple scales. Introducing $T_o=t$ and $T_1=\varepsilon t$, partial derivatives expansions are

$$\frac{d}{dt} = D_o + \varepsilon D_1 + \ldots \quad \text{and} \quad \frac{d^2}{dt^2} = D_o^2 + 2\varepsilon D_o D_1 + \ldots \tag{4}$$

Under primary resonance, the proximity of excitation frequency (ω) to natural frequency (ω_o) is expressed in terms of nondimensional excitation frequency (Ω) and detuning parameter (σ) as $\Omega = 1 + \varepsilon\sigma$.
Considering the nonlinearity of order ε, solution of Eq. (3) is expanded to the form

$$u = u_o + \varepsilon u_1 \tag{5}$$

Substituting Eq. (4)-(5) in Eq. (3) and equating the coefficients of ε^o and ε^1 to zero, results in

$$\varepsilon^o: D_o^2 u_o + u_o = f_o \tag{6}$$

$$\varepsilon^1: \quad D_o^2 u_1 + u_1 = (f_1 - \beta u_o)\cos(\omega T_o + \sigma T_1) - 2D_o D_1 u_o - 2\mu D_o u_o - \gamma u_o^3 + cc. \tag{7}$$

The general solution of u_o is written as

$$u_o = A(T_1)e^{iT_o} + \overline{A}(T_1)e^{iT_o} + f_o \tag{8}$$

Substituting Eq. (8) in Eq. (7) results in

$$D_o^2 u_1 + u_1 = e^{iT_o}\left[\frac{1}{2}(f_1 - \beta f_o)e^{i\sigma T_1} - 2iA'(T_1) - 2i\mu A(T_1) - 3\gamma A^2(T_1)\overline{A}(T_1) - 3\gamma A(T_1)f_o^2\right] + cc. \tag{9}$$

Eliminating the secular terms in Eq. (9) yields solvability condition as

$$\frac{1}{2}(f_1 - \beta f_o)e^{i\sigma T_1} - 2iA'(T_1) - 2i\mu A(T_1) - 3\gamma A^2(T_1)\overline{A}(T_1) - 3\gamma A(T_1)f_o^2 = 0 \tag{10}$$

Solution of Eq. (10) is obtained using the polar forms as

$$A(T_1) = \frac{1}{2}a(T_1)e^{i\alpha(T_1)} \quad \text{and} \quad \phi(T_1) = -(\sigma T_1 - \alpha) \tag{11}$$

Using the polar forms in Eq. (11) in Eq. (10) and separating the real and imaginary parts, yield

$$a' = -\mu a - \frac{1}{2}(f_1 - \beta f_o)\sin\phi \tag{12}$$

$$a\phi' = -a\sigma + \frac{3\gamma a^3}{8} + \frac{3\gamma f_o^2 a}{2} - \frac{1}{2}(f_1 - \beta f_o)\cos\phi \tag{13}$$

Using the steady state condition ($a' = \phi' = 0$) in Eqs. (12)-(13), the frequency response is determined as

$$\frac{1}{4}(f_1 - \beta f_o)^2 = \mu^2 a^2 + a^2\left(\sigma - \frac{3\gamma a^2}{8} - \frac{3\gamma f_o^2}{2}\right)^2 \tag{14}$$

Stability of the system is determined based on Jacobian matrix in Eq. (15) which is obtained from linearization of Eqs. (12)-(13) at steady state.

$$J = \begin{bmatrix} -\mu & a\left(\sigma - \frac{3\gamma a^2}{8} - \frac{3\gamma f_o^2}{2}\right) \\ \frac{1}{a}\left(-\sigma + \frac{9\gamma a^2}{8} + \frac{3\gamma f_o^2}{2}\right) & -\mu \end{bmatrix} \tag{15}$$

The negative real parts of eigen values of the characteristic equation for Jacobian matrix in Eq. (15) yield the stable solutions.

3 Results and Discussion

The nonlinear dynamic solution of spur gear pair under primary resonance is analyzed using method of multiple scales. The parameters used in the analysis are: γ=0, 1, -1; f_o=0.4, 0.9; f_1=0.3; β=0, 2; ε=0.1, 0.3; μ=0.1. The amplitude parameter response (a) curves with stable (o) and unstable (+) solutions are shown in Figs. 2a-2d. The influence of linear stiffness coefficient (γ=0) results in the linear peak of amplitude parameter response, while positive (γ=1) and negative (γ=-1) nonlinear stiffness coefficients results in the amplitude parameter response lean towards right and left respectively. Increase in force amplitude, $(f_1-\beta f_o)/2$, results in increase in the peak of amplitude parameter response. The peak of amplitude parameter shifts towards higher (lower) nondimensional excitation frequency (Ω) for positive (negative) nonlinear stiffness coefficients.

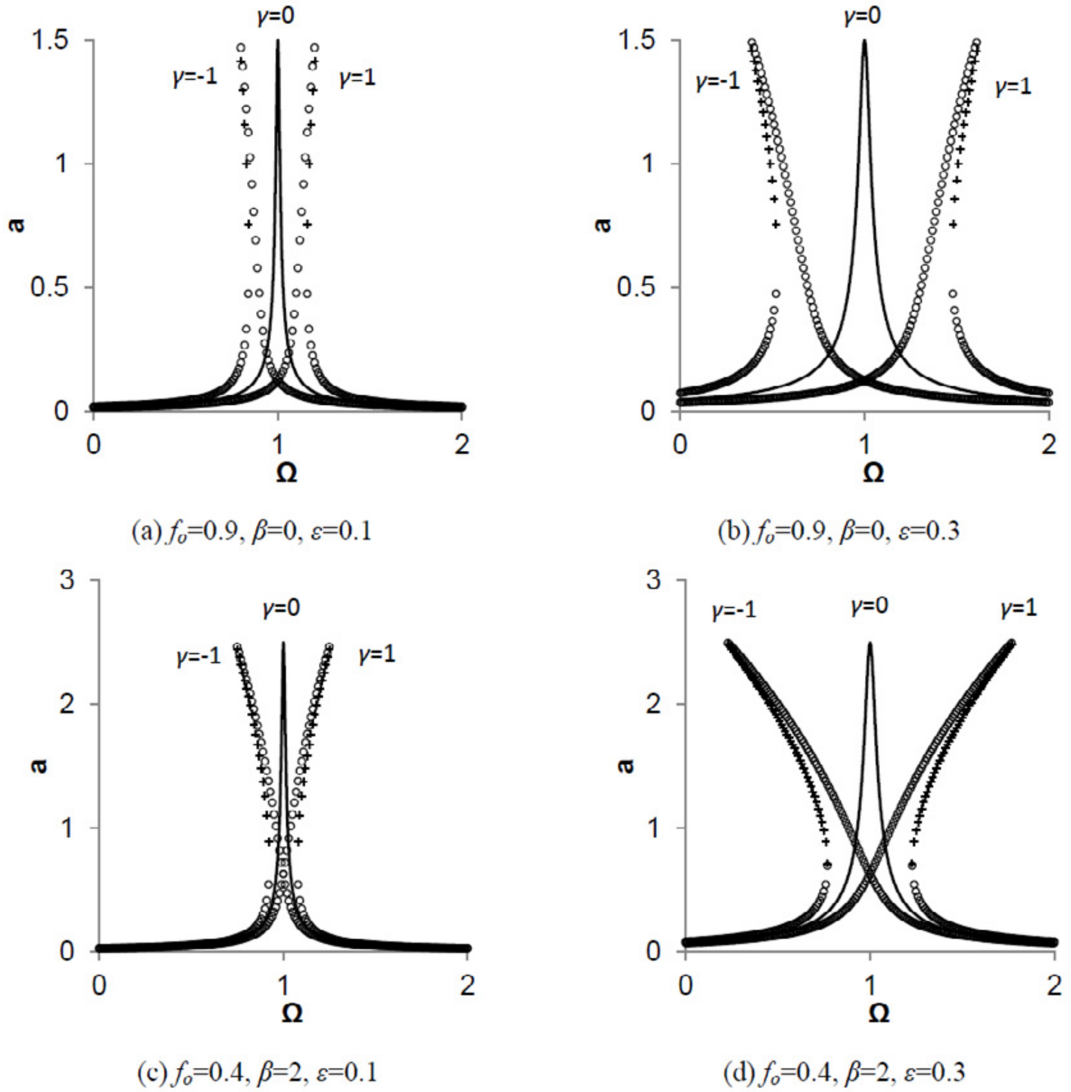

Figure 2 Amplitude parameter of nondimensional dynamic transmission error

4 Conclusion

The present study investigates nonlinearity of spur gear pair with time-varying mesh stiffness. Analytical approximate solutions of the nonlinear system for primary resonance are obtained by closed-form expressions using the method of multiple scales. The maximum amplitude parameter response (a) increases with increase in force amplitude. As small parameter (ε) increases, there is shift in the peak of amplitude parameter.

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Nomenclature

a — Amplitude of A in polar form
A — Amplitude parameter
b — Half of gear backlash
c — Damping coefficient
$c.c$ — Complex congugate
e — Static transmission error
f_t — Load; $f_t = \dfrac{J_2 r_1 M_1 + J_1 r_2 M_2}{J_1 r_2^2 + J_2 r_1^2}$
f_m — Excitation; $f_m = m\ddot{e}$
f_o, f_1 — Nondimensional mean and perturbed excitation force
J_1, J_2 — Mass moment of inertia of gears
k_o — Stiffness coefficient of gear mesh
m — Mass; $m = \dfrac{J_1 J_2}{J_1 r_2^2 + J_2 r_1^2}$
M_1, M_2 — Moments transmitted through gears
r_1, r_2 — Base circle radius of gears
x — Dynamic transmission error; $x = r_1\varphi_1 - r_2\varphi_2 - e$
u — Nondimensional dynamic transmission error; $u = x/b$
t — Nondimensional time; $t = \omega_o \hat{t}$
β — Time-varying mesh stiffness coefficient
ε — Small parameter
γ — Nonlinear stiffness coefficient
ϕ — Phase of A in polar form
φ_1, φ_2 — Angular displacement of gears
θ — Phase parameter
μ — Nondimensional damping coefficient
ζ — Nondimensional damping coefficient; $\zeta = c/2\sqrt{mk_o}$
σ — Detuning parameter
ω — Excitation frequency
ω_o — Natural frequency; $\omega_o = \sqrt{k_o/m}$
Ω — Nondimensional excitation frequency; $\Omega = \omega/\omega_o$

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