

Nonlinear System Identification via Basis Functions Based Time Domain Volterra Model

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Abstract. This paper proposes basis functions based time domain Volterra model for nonlinear system identification. The Volterra kernels are expanded by using complex exponential basis functions and estimated via genetic algorithm (GA). The accuracy and practicability of the proposed method are then assessed experimentally from a scaled 1:100 model of a prototype truss spar platform. Identification results in time and frequency domain are presented and coherent functions are performed to check the quality of the identification results. It is shown that results between experimental data and proposed method are in good agreement.

1 Introduction

Floating structures is known as a structural system that exhibit significant nonlinear behavior [1-2]. In certain case, it is difficult to conduct a rigorous mathematical modeling, especially when it involves too complex system such as a floating structure. For these reasons, it is convenient to use alternative representation of the model that avoids mathematical formulations. Empirical models can be used instead by utilizing measured system input-output from models tests. By acquiring the measured time series of wave height and motion responses experimentally, empirical models could be derived to describe the dynamic behavior of the system. If the interest is in the amplitude and statistics of the floating structure motions, the empirical models are usually presented in term of transfer function (TF) or response amplitude operator (RAO) [3]. The generated transfer function is classified as wave-to-motion TF [4] and consists of linear and nonlinear TFs. One of the versatile empirical models that are used to generate the TFs of nonlinear dynamic system is Volterra model. References [5-6] had implemented the method successfully. They concluded that second-order Volterra model was adequate to model the nonlinear relationship between the wave height and surge motion of mini TLP. Frequency domain Volterra model was utilized in their works. In estimating the TFs, frequency domain Volterra model needs higher-order spectral analysis (HOSA).

However, HOSA introduces bias and higher variance in the estimation process. Frequency resolution will be trade off in the TFs estimation. Sometimes, it leads to untrustworthy, especially for non-Gaussian wave. In this regard, the objective of this paper is to avoid the use of HOSA by using time-domain Volterra model as an alternative. The proposed method is named as basis functions based time domain Volterra model (BFVM) and then applied to the floating structures as case study.

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2 Mathematical formulations

In this section, mathematical frameworks as a pavement to achieve the objective of this paper are presented sequentially.

2.1 First and second-order Volterra model

Nonlinear relationship between system input u and output y may be approached with Volterra model. The theory has been mature and available in many literatures. In this paper, the second-order Volterra model is utilized. To revisit the theory shortly, Volterra model for causal nonlinear system with nonlinearity degree P , maximum memory length $M-1$ in discrete time index n along the data length N can be written as follows [7],

$$\hat{y}(n) = h_0 + \sum_{i=0}^{M-1} h_1(i)u(n-i) + \sum_{j=0}^{M-1} \sum_{k=j}^{M-1} h_2(j,k)u(n-j)u(n-k). \quad (1)$$

As shown in Equation (1), a specific advantage in the use of this type of model is that it can decompose the system response into linear and nonlinear part separately. Term h_0 is the first-order Volterra kernels while h_1 is the first-order Volterra kernels, known as linear impulse response function (LIRF). Further, h_2 is the second-order Volterra kernels, known as quadratic impulse response function (QIRF).

2.2 Representation of Volterra kernels in basis functions

In order to increase the accuracy of time domain Volterra model, Volterra kernel coefficients $h_1(i)$ and $h_2(j,k)$ are expanded onto a set of basis function $\pi(n)$:

$$h_{1(i)}(n) = \sum_{\ell=0}^V \alpha_{1(i)}(\ell) \pi_{\ell}(n) \quad (2)$$

$$h_{2(j,k)}(n) = \sum_{\ell=0}^V \beta_{2(j,k)}(\ell) \pi_{\ell}(n). \quad (3)$$

Terms $\alpha_{1(i)}(\ell)$ and $\beta_{2(j,k)}(\ell)$ represent the expansion parameters with V as the maximum number of basis sequences. The stationary portion of the model $\pi(n)=1$ is included, because the basis function π_0 has a constant value for all time n . Substituting Equation (2) and (3) into Equation (1), equation (4) can be obtained as follows,

$$\hat{y}(n) = h_0 + \sum_{i=0}^{M-1} \sum_{\ell=0}^V \alpha_{1(i)}(\ell) \pi_{\ell}(n) u(n-i) + \sum_{j=0}^{M-1} \sum_{k=j}^{M-1} \sum_{\ell=0}^V \beta_{2(j,k)}(\ell) \pi_{\ell}(n) u(n-j) u(n-k). \quad (4)$$

The next stage is to select the proper basis functions so that the Volterra kernels can be estimated. Once the basis functions are chosen, new variables can be defined as follows:

$$u_{\ell}(n-i) = \pi_{\ell}(n) u(n-i) \quad (5)$$

$$u_{q\ell}(n-j)u_{q\ell}(n-k) = \pi_{\ell}(n)u(n-j)u(n-k). \quad (6)$$

Substituting Equation (6) and (7) into Equation 5) results in the Equation (8),

$$\hat{y}(n) = h_0 + \sum_{i=0}^{M-1} \sum_{\ell=0}^V \alpha_{1(i)}(\ell) u_{\ell}(n-i) + \sum_{j=0}^{M-1} \sum_{k=j}^{M-1} \sum_{\ell=0}^V \beta_{2(j,k)}(\ell) u_{q\ell}(n-j) u_{q\ell}(n-k). \quad (7)$$

Equation (7) shows that the Volterra kernels can now be considered to be time-invariant kernel coefficients, since $\alpha_{1(i)}(\ell)$ and $\beta_{2(j,k)}(\ell)$ do not depend on time. Identification process is simplified to solve the kernels $\alpha_{1(i)}(\ell)$ and $\beta_{2(j,k)}(\ell)$, which can be estimated using proposed estimation techniques. By a simple calculation, the total of Volterra kernels is

$$LL = \frac{(P+M)!}{P!M!} (V+1). \quad (8)$$

This value is much more than the original Volterra kernels. In the system modeling, more adjustable parameters are always able to provide better modeling capacity. This is the advantage of expanding the Volterra model with basis function.

2.3 Volterra kernels estimation using genetic algorithm

The final procedure is to estimate the Volterra kernels $\alpha_{1(i)}(\ell)$ and $\beta_{2(j,k)}(\ell)$ in Equation (11) and calculate the time-varying Volterra kernels $h_{1(i)}(n)$ and $h_{2(j,k)}(n)$ using Equation (6) and (7). By manipulating Equation (7) and including model error $e(n)$, system can be expressed :

$$y(n) = \varphi^T \cdot \theta + e(n). \quad (9)$$

At this stage, Equation (9) can be seen as linear system parameter estimation. Standard least square (LS) technique may be used for such a system. However, as in linear case, the regressor matrix φ can be numerically ill-conditioned, so that the LS is not reliable. Genetic algorithm (GA) method is proposed for this case. The summary of using GA method in estimating the Volterra kernels is below:

- (1) Set the proper size of Volterra kernels using Equation (8) and initiate each value randomly.
- (2) Rank the values with following fitness function,

$$MSE = \frac{1}{N} \sum_{n=1}^N \left(y(n) - \hat{y}(n) \right)^2. \quad (10)$$

- (3) Create new kernels starting from existing ones by biological operators such as selection, cross-over and mutation.
- (4) Create new population starting from the current one by an evolutionary strategy such as successive generations of kernels.
- (5) Stop the process by a termination criterion. The termination is based on a maximum number of generations or a threshold on the fitness function.

3 Experimental works

The method is then applied to the truss spar model, made from galvanized steel. The model was designed with scale of 1:100 to fit the wave tank by following the Froude's law of similitude. For stationary keeping, the test model was moored with four mooring lines. The linear springs were

connected to load cells to measure the mooring system loads. The model was then tested in the wave tank of the offshore engineering laboratory, Universiti Teknologi PETRONAS. The wave tank has 22 m length, 10 m width and 1.5 m depth. The rigid body motions were measured by optical tracking system, while wave heights were measured with wave probes. The scheme of the model and experimental setup are displayed in Figure 1 and 2.

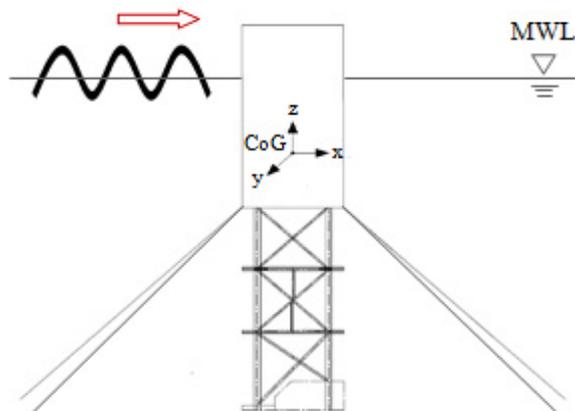


Figure 1. Scheme of the model.

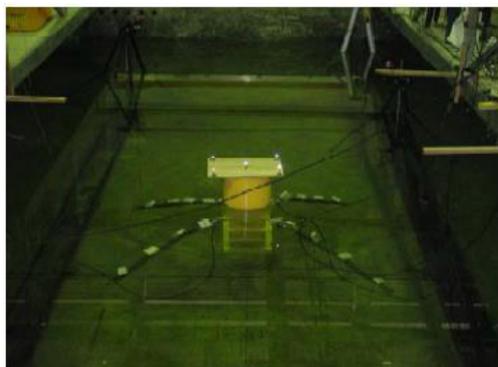


Figure 2. Experimental setup.

The JONSWAP spectrum was used to generate the random wave. The model was then subjected a unidirectional random wave in head seas configuration. The sea state is $H_s = 4.5\text{m}$, $f_p = 0.1\text{ Hz}$ and $\gamma = 3$ in full scale. The tests were recorded for 3000 seconds at a sampling frequency of 10 Hz (prototype scale).

4 Results

Volterra kernel coefficients are expanded using twenty two-term basis functions. Three types of basis function: polynomial, complex exponential and Legendre basis functions are tested. A criterion adopted for determining the proper number of basis functions is based on the MSE values. It is found out that complex exponential is suitable for this case. Ten complex exponential have been chosen because this provides one of the lowest MSE values.

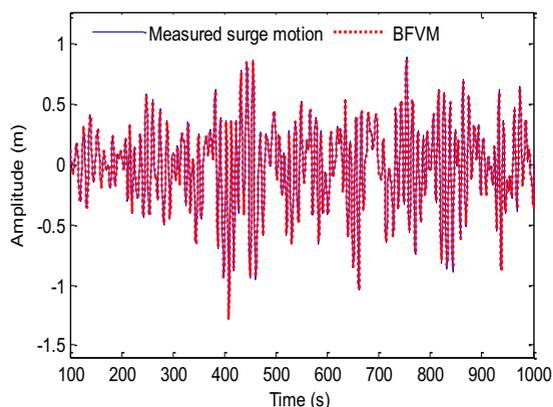


Figure 3. Identification results in time domain.

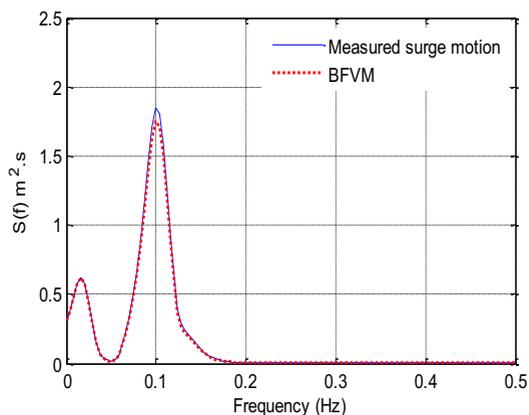


Figure 4. Identification results in frequency domain.

Based on that, identification results in time and frequency domain are presented in Figure 3 and 4.

Those figures show the agreement between the proposed method, basis functions based Volterra model (BFVM) and experimental data is found. In the Figure 4, surge motion spectrum has two principal frequency peaks. The first peak is known as low frequency region (LFR) at around 0.016 Hz. This low frequency represents the second-order slowly varying motion created through the difference frequency interaction. This value is above the surge natural frequency of the prototype at 0.008 Hz. The second peak is around 0.1 Hz, corresponds to the frequency exist in the random wave spectrum. And known as wave frequency region (WFR). Most of wave energy is located in this frequency region. These finding results are also found by many prior researches, [1-2] and [5-6]. Results above suggest that truss spar platform can be classified as nonlinear system. To check the quality of identification results by the proposed method, coherence functions are presented in Figure 5.

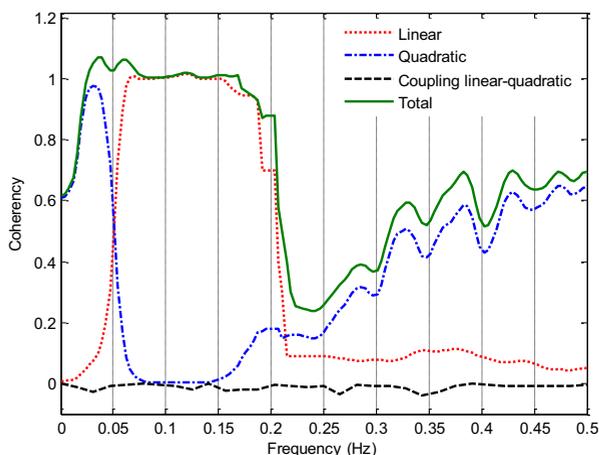


Figure 5. Coherence functions.

Those coherence functions are generated based on [6]. As can be seen, system coherency is divided into two regions as purpose of system identification. The linear coherency seems to be good in the WFR. The frequency range is around 0.07-0.16 Hz. Quadratic coherency seems also to be good in the LFR. The coupled coherence spectrum at all frequencies is approximately zero. The total coherence spectrum is close to unity up to 0.18 Hz, meaning that the linear and nonlinear surge motion in random waves is adequately modeled as second-order Volterra model. Figure 5 also suggests that the surge motion of truss spar is linearly correlated with random waves at WFR and quadratically correlated at LFR, which is similar with [5-6]. Thus, identification results are trustworthy.

References

1. P.D. Spanos, R. Ghosh, L.D. Finn and J.E. Halkyard, OMAE. J., ASME, 127 (2005).
2. Y.M. Low and R.S. Langley, OMAE. J., ASME, 130 (2008).
3. B.M. Suleiman. *Identification of finite-degree-of-freedom models for ship motion* (Ph.D. dissertation, Virginia Polytechnic Institute and State Univ., America, 2004)
4. R. Taghipour, T. Perez and T. Moan, Ocean Eng. J. 35 (2008).
5. N. Kim, *Extraction of the second-order nonlinear response from model test data in random seas and comparison of the Gaussian and non-Gaussian models* (Ph.D. dissertation, Texas A&M Univ., America, 2004)
6. Y. Birkelund, E.J. Power and A. Hansen, *On the estimation of nonlinear Volterra models in offshore engineering* (Proc. ISOPE, pp. 180-187, 2002)
7. C. Luigi and K. Ahsan, Eng. Mech. J., 136 (2010).