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Buckling of Functionally Graded Nanobeams Based on the Nonlocal New First-Order Shear Deformation Beam Theory

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Abstract. In this work, the size-dependent buckling behavior of functionally graded (FG) nanobeams is investigated on the basis of the nonlocal continuum model. The material properties of FG nanobeams are assumed to vary through the thickness according to the power law. In addition, Poisson's ratio is assumed constant in the current model. The nanobeams is modelled according to the new first order shear beam theory with small deformation and the equilibrium equations are derived using the Hamilton's principle. The Navier-type solution is developed for simply-supported boundary conditions, and exact formulas are proposed for the buckling load. The effects of nonlocal parameter, aspect ratio, various material compositions on the stability responses of the FG nanobeams are discussed.

Keywords. Buckling, nonlocal elasticity theory, functionally graded nanobeams.

1 Introduction

Functionally graded materials (FGMs) are made from a mixture of two materials to achieve a composition that provides certain functionality. Recently, the application of FGMs has broadly been spread in micro-and nano-scale devices and systems such as thin films [1]. This is achieved by gradually varying the volume fraction of the constituent materials. During the past decade, FGMs have been widely used in various aspects of engineering sciences, such as aerospace, nuclear, civil, automotive, optical, biomechanical, electronic, chemical, mechanical, and shipbuilding industries. With the development of the material technology, FGMs have been employed in micro/nano-electro-mechanical system (MEMS/NEMS) [2]. Stability studies of nanostructure made of functionally graded materials are of great significance for estimating the performance of FG nanobeams. In recent years, various investigations have been carried out to study the buckling [3], bending analysis [4,6], and free vibration [5] of the FG nanobeams in which the classical continuum mechanics have been employed.

This paper focuses on the buckling of the FG nanobeam based on the nonlocal refined first order shear beam theory. The material properties of FG nanobeams are assumed to vary through the thickness according to the power law. In addition, Poisson's ratio is assumed constant in the current model. The nanobeam is modelled according to the new Timoshenko beam theory with small deformation and the equilibrium equations are derived using the Hamilton's principle. The Navier-type solution is developed for simply-supported boundary conditions, and exact formulas are proposed for the buckling load. The effects of nonlocal parameter, aspect ratio, various material compositions on the stability responses of the FG nanobeams are discussed.

2 Theoretical formulations

2.1 Basic assumptions of the refined first shear deformation beam theory

The displacement field of the proposed theory is chosen based on the following assumptions:

(i) The displacements are small in comparison with the plate thickness and, therefore, strains involved are infinitesimal.

(ii) The transverse displacement w components are functions of coordinates x, y only.

$$w(x,y,z)=w(x,y) \tag{1}$$

(iii) The transverse normal stress σ_z is negligible in comparison with in-plane stresses σ_x and σ_y.

(iv) The displacements u in x-direction consist of extension and bending components.

$$U = u_0 + u_b \tag{2}$$

The bending components (u_b) and (v_b) are assumed to be similar to the displacements given by the classical plate theory. Therefore, the expression for (u_b) and (v_b) can be given as

$$u_b = -z\frac{\partial \phi}{\partial x} \tag{3}$$

2.2 Kinematics and constitutive equations based on the nonlocal elasticity theory

Based on the assumptions made in the preceding section, the displacement field can be obtained using Eqs. (1,3) as

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\frac{\partial \phi}{\partial x} \\ w(x,y,z) &= w(x,y) \end{aligned} \tag{4}$$

The strains associated with the displacements in Eq. (4) are

$$\begin{aligned} \varepsilon_x &= \varepsilon_x^0 + z\,k_x \\ \gamma_{xz} &= \gamma_{xz} \end{aligned} \tag{5}$$

Where

$$\varepsilon_x^0 = \frac{\partial u_0}{\partial x},\quad k_x = -\frac{\partial^2 \phi}{\partial x^2},\quad \gamma_{xz} = \frac{\partial w}{\partial x} - \frac{\partial \phi}{\partial x} \tag{6}$$

2.3 Constitutive relations

The FG microbeam is is made of two different Materials and the effective material properties (Young modulus) of the FG nanobeam vary continuously in the thickness direction (in the z direction). According to the rule of mixture, the effective material properties $E(z)$ can be expressed as:

$$E(z) = E_1 + (E_2 - E_1)\left(\frac{2z+h}{2}\right)^k$$

It is easily seen that $E(z) = E_2$, when $z = +h/2$, and $E(z) = E_1$, when $z = -h/2$, $\nu_1 = \nu_2 = \nu = 0.3$.

k represent the volume fraction exponent which takes values greater than or equal to zero. The above power-law assumption reflects a simple rule of mixtures used to obtain the effective properties of the E_1 and E_2 material of FG nanobeams.

Response of materials at the nanoscale is different from those of their bulk counterparts. Nonlocal elasticity is first considered by Eringen [7]. He assumed that the stress at a reference point is a functional of the strain field at every point of the continuum. Eringen [7] proposed a differential form of the nonlocal constitutive relation as

$$\sigma_x - \mu\frac{d^2\sigma_x}{dx^2} = Q_{11}\varepsilon_x$$

$$\tau_{xz} - \mu\frac{d^2\tau_{xz}}{dx^2} = Q_{55}\gamma_{xz}$$

Where $Q_{11} = E$ and $Q_{55} = G$ are the elastic modulus and shear modulus of the nanobeam, respectively; $\boldsymbol{\mu} = (\boldsymbol{e}_0\boldsymbol{a})^2$ is the nonlocal parameter, $\boldsymbol{e}_0$ is a constant appropriate to each material and $\boldsymbol{a}$ is an internal characteristic length.

2.4 Governing equations

In order to obtain the equations of motion, the principal of the Hamilton's principle is required. The principle can be stated in analytical form as

$$\delta\int_0^T (U + V)\,dt = 0$$

Where δU is the virtual variation of the strain energy; δV is the virtual variation of the potential energy. The variation of the strain energy of the beam can be stated as

$$\begin{aligned} \delta U &= \int_0^L\int_A (\sigma_x\delta\varepsilon_x + \tau_{xz}\delta\gamma_{xz})\,dA\,dx \\ &= \int_0^L\left(N_x\frac{\partial \delta u}{\partial x} - M_x\frac{\partial^2\delta\phi}{\partial x^2} + Q_{xz}\left(\frac{\partial \delta w}{\partial x} - \frac{\partial \delta\phi}{\partial x}\right)\right)dx \end{aligned}$$

Where (N), (M) and (Q) are the stress resultants defined as

$$(N_x, M_x) = \int_A (1,z)\,\sigma_x\,dA \quad \text{and} \quad Q_{xz} = \int_A k_s\tau_{xz}\,dA$$

Where k_s is the shear correction factor.

The variation of the potential energy by the applied loads can be written as

$$\delta V = \int_0^L q\,\delta w\,dx + \int_0^L N_0\frac{dw}{dx}\frac{d\delta w}{dx}dx$$

Where (q) and (N_0) are the transverse and axial loads, respectively.

Substituting the expressions for (δU) and (δV) from Eqs. (10), (12) and (13) into Eq. (9) and integrating by parts, and collecting the coefficients of (δu_0), (δw), and ($\delta \phi$), the following equations of motion of the proposed beam theory are obtained

$$\boldsymbol{\delta u}: \ \frac{\partial N_x}{\partial x} = 0$$

$$\boldsymbol{\delta \phi}: \ \frac{\partial^2 M_x}{\partial x^2} - \frac{\partial Q_{xz}}{\partial x} = 0$$

$$\boldsymbol{\delta w}: \ \frac{\partial Q_{xz}}{\partial x} + q - N_{cr}\frac{d^2 w}{dx^2} = 0$$

By substituting Eq. (6) into Eq. (8) and the subsequent results into Eq. (11), the stress resultants are obtained as

$$N - \mu\frac{d^2 N}{dx^2} = A_{11}\frac{du_0}{dx} - B_{11}\frac{d^2\phi}{dx^2}$$

$$M - \mu\frac{d^2 M}{dx^2} = B_{11}\frac{du_0}{dx} - D_{11}\frac{d^2\phi}{dx^2}$$

$$Q - \mu\frac{d^2 Q}{dx^2} = A_{55}\frac{dw_s}{dx}$$

Where

$$(A_{11}, B_{11}, D_{11}) = \int_A Q_{11}(1, z, z^2)dA \text{ , } A_s = \int_A k_s Q_{55}\, dA$$

By substituting Eq. (14) into Eq. (13), the nonlocal equations of motion can be expressed in terms of displacements (u_0, w, ϕ) as

$$A_{11}\frac{d^2 u_0}{dx^2} - B_{11}\frac{d^3\phi}{dx^3} = 0$$

$$B_{11}\frac{d^3 u_0}{dx^3} - D_{11}\frac{d^4\phi}{dx^4} - A_{55}\frac{d^2 w_s}{dx^2} = 0$$

$$A_{55}\frac{d^2 w_s}{dx^2} + q - \mu\frac{d^2 q}{dx^2} - N_{cr}\left(\frac{d^2 w}{dx^2} - \mu\frac{d^4 w}{dx^4}\right) = 0$$

3 Analytical Solutions for critical buckling of simply supported FG nanobeam

The above equations of motion are analytically solved for buckling problems. The Navier solution procedure is used to determine the analytical solutions for a simply supported FG nanobeam. The solution is assumed to be of the form

$$\begin{Bmatrix} u_0 \\ w_b \\ w_s \end{Bmatrix} = \sum_{m=1}^{\infty} \begin{Bmatrix} U_n \cos(\alpha x) \\ W_{bn}\sin(\alpha x) \\ W_{sn}\sin(\alpha x) \end{Bmatrix}$$

Where (U_n), (W_{bn}), and (W_{sn}) are arbitrary parameters to be determined, and $\alpha = n\pi / L$.
Substituting the expansions of $\boldsymbol{u}_0$, $\boldsymbol{w}_b$, $\boldsymbol{w}_s$, and $\boldsymbol{q}$ from Eqs. (17) into Eq. (16), the analytical solutions can be obtained from the following equations

$$\left(\begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22}-k & 0 \\ 0 & 0 & S_{33}-k \end{bmatrix} - \right)\begin{Bmatrix} U_n \\ \Phi_n \\ W_n \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

Where

$$S_{11} = A_{11}\alpha^2, \ S_{12} = -B_{11}\alpha^3, \ S_{22} = D_{11}\alpha^4,$$

$$S_{33} = A_{55}\alpha^2, \ \boldsymbol{k} = \boldsymbol{\lambda N_{cr}\alpha^2}, \ \lambda = 1 + \mu\alpha^2$$

4 Numerical results

For the purpose of verification, the present model is used to find the first critical buckling load for a simply supported beam made up of a homogenous material where the nonlocal effect is taken into consideration and compare the results with those of Simsek [8]. The non-dimensional buckling load defined is defined as:

$$N_{cr} = \overline{N_{cr}}\frac{L}{EI}$$

The nondimensional critical buckling load $\overline{N_{cr}}$ of a simply supported beam are presented in Tables 1 for various values of thickness ratio L/h and nonlocal parameter μ . The nonlocal parameters $\mu = (e_0 a)^2$ are taken as 0, 1, 2, 3, and 4 nm^2 . These values are taken because $e_0 a$ should be smaller than 2 nm for carbon nanotubes as described by Wang and Wang [9].
Table 1 shows that the present buckling loads agree very well with the solutions of Simsek [8] and the solutions of Aydogdu [10, 11] for first order shear deformation beam theory.

Table 1. Dimensionless critical buckling load, $\overline{N_{cr}}$ of the homogenous nanobeam.

L/h	μ	Aydogdu [27]	Simsek [27]	Present
10	0	9.6227	9.6226	9.6226
	1	8.7583	8.7582	8.7582
	2	8.0364	8.0363	8.0363
	3	7.4244	7.4244	7.4244
	4	6.8990	6.8990	6.8990
20	0	9.8067	9.8066	9.8066
	1	9.5705	9.5705	9.5705
	2	9.3455	9.3455	9.3455
	3	9.1308	9.1308	9.1308
	4	8.9258	8.9258	8.9258
50	0	9.8595	9.8594	9.8594
	1	9.8207	9.8207	9.8207
	2	9.7822	9.7822	9.7822
	3	9.7440	9.7440	9.7440
	4	9.7062	9.7062	9.7062

To investigate the significance of using functionally graded materials on the buckling of nanobeams, the FG nanobeams have the following material properties: $E_1 = 1TPa$, $E_2 = 0.25TPa$, $\nu_1 = \nu_2 = 0.3$ and where they change across the beam's thickness according to a power law as presented earlier. The shear correction factor is taken $k_s = 5/6$.

Table 2 shows the nondimensional critical buckling load $\overline{N_{cr}}$ of a simply supported nano FGM beam. The nonlocal parameter $e_0 a$ varies from 0 to 2 nm and the material distribution parameter k varies from 0 to 10. The obtained results are compared with those reported by [Simsek] based on nonlocal Timoshenko beam theory. It can be seen that the results of present theory are in excellent agreement with those predicted by [Simsek] for all values of small scale coefficient, material distribution parameter k, and length-to-depth ratio of beams.

Table 2. Dimensionless critical buckling load, $\left(P_{cr} = \overline{P_{cr}} L^2 / (E_1 I)\right)$ of the FG nanobeam.

L/h	k	Nonlocal parameter, $e0a(nm)$: 0, FSDT [27]	0, Present	0.5, FSDT [27]	0.5, Present	1, FSDT [27]	1, Present	1.5, FSDT [27]	1.5, Present	2, FSDT [27]	2, Present
10	0	2.4056	2.4056	2.3477	2.3477	2.1895	2.1895	1.9685	1.9685	1.7247	1.7247
	0.3	3.9921	3.9921	3.8959	3.8959	3.6335	3.6335	3.2667	3.2667	2.8621	2.8621
	1	5.3084	5.3084	5.1805	5.1805	4.8315	4.8315	4.3437	4.3437	3.8059	3.8059
	3	6.6720	6.6720	6.5113	6.5113	6.0727	6.0727	5.4596	5.4596	4.7835	4.7835
	10	8.1289	8.1289	7.9332	7.9332	7.3987	7.3987	6.6518	6.6518	5.8281	5.8281
30	0	2.4603	2.4603	2.4536	2.4536	2.4336	2.4336	2.4011	2.4011	2.3570	2.3570
	0.3	4.0811	4.0811	4.0699	4.0699	4.0368	4.0368	3.9828	3.9828	3.9096	3.9096
	1	5.4146	5.4146	5.3998	5.3998	5.3559	5.3559	5.2843	5.2843	5.1871	5.1871
	3	6.8011	6.8011	6.7825	6.7825	6.7273	6.7273	6.6373	6.6373	6.5153	6.5153
	10	8.2962	8.2962	8.2735	8.2735	8.2062	8.2062	8.0964	8.0964	7.9476	7.9476

The variation of buckling responses of FG nanobeam with the aspect ratio is presented in Figure 1 for local and non local case with ($k = 1$, $e_0 a = 1$) and the aspect ratio varies from L/h = 5 to L/h = 50. The critical buckling load predicted by the local (classical) theory is larger than those of the nonlocal results due to the small scale effects. The nonlocal effect decreases the buckling loads especially at high values of nonlocal parameter.

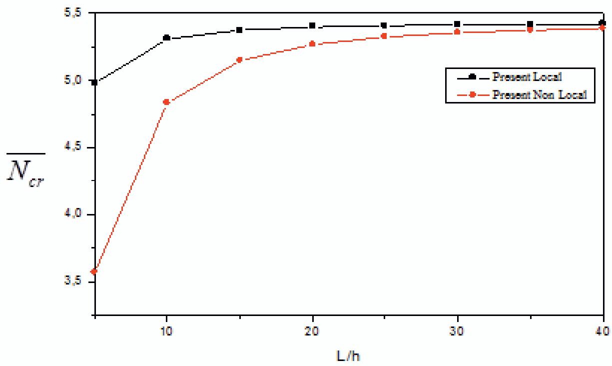

Fig. 1. Effect of the aspect ratio on the dimensionless buckling load for $k = 1$, e_0a = 1 nm.

The effect of the nonlocal parameter on buckling responses of FG nanobeam is presented in Fig. 2 using present model ($k = 1$) and the aspect ratio varies from L/h = 10 to L/h = 50. According to this figure, the responses of critical buckling load vary nonlinearly with the nonlocal parameter. From this conclusion obtained above, the critical buckling load decreases as the nonlocal parameter increases.

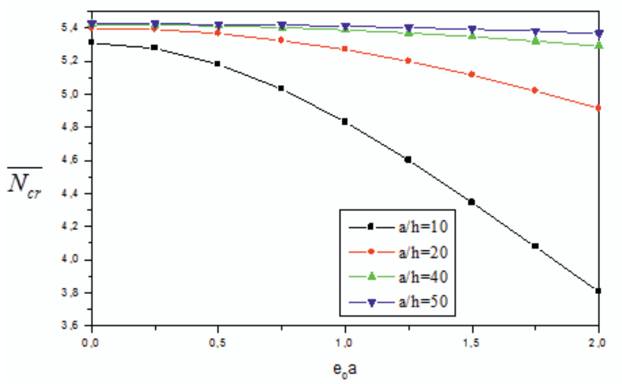

Fig. 2. Effect of the nonlocal parameter on the dimensionless buckling load for $k = 1$.

Conclusions

A simple refined first order shear deformation beams theory was presented for critical buckling of FG nanobeams. Equations of motion are derived from Hamilton's principle. Analytical solutions are obtained for simply supported beams. The effects of nonlocal parameter, aspect ratio, various material compositions on the critical buckling responses of the FG nanobeam are presented and discussed. Numerical results show that the critical buckling load decreases as the nonlocal parameter increases.

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