

## The effective diffusion coefficient of boron in the Fe<sub>2</sub>B layers formed on the iron substrate

M. Keddam<sup>1</sup>, B. Bouarour<sup>1</sup>, Z. Nait Abdellah<sup>2</sup>, and R. Chegroune<sup>1</sup>

<sup>1</sup>Laboratoire LTM, Faculté G.M et G.P. USTHB, B.P. 32 El-Alia 16111, Alger, Algérie

<sup>2</sup>Département de Chimie, Faculté des Sciences, Université Mouloud Mammeri, 15000, Tizi-Ouzou, Algérie

**Abstract.** In this current work, the boron diffusion coefficient in Fe<sub>2</sub>B was firstly evaluated using a diffusion model. It considers the effect of the incubation times required to form the Fe<sub>2</sub>B layers by the paste-boriding process on the iron substrate.

This model solves the mass balance equation at the (Fe<sub>2</sub>B/substrate) interface under certain assumptions. Afterwards, the effective boron diffusion coefficient in Fe<sub>2</sub>B was evaluated through an application of a simple equation. As a result, the estimated value of boron activation energy in the presence of chemical stresses was found to be equal to 146.5 kJ mol<sup>-1</sup> on the basis of experimental data taken from the literature.

### 1 Introduction

The boriding is a diffusion-related surface treatment with the purpose of improving the tribological properties, the fatigue endurance and the corrosion resistance of ferrous and non-ferrous alloys. This thermochemical treatment can be carried out between 1123 and 1323 K with the exposure times varying from 0.5 to 10 h [1]. The diffused boron atoms react with the substrate surface to form two types of borides FeB and Fe<sub>2</sub>B. The suggested model takes into the effect of chemical stresses through the determination of effective diffusion coefficient of boron in Fe<sub>2</sub>B grown on the iron substrate. The objective of this work was to evaluate the diffusivity of boron in Fe<sub>2</sub>B (with and without the presence of chemical stresses) in the temperature range of 1223-1323 K by applying a recent diffusion model [2, 3] for the Fe<sub>2</sub>B layer development.

### 2 Diffusion model

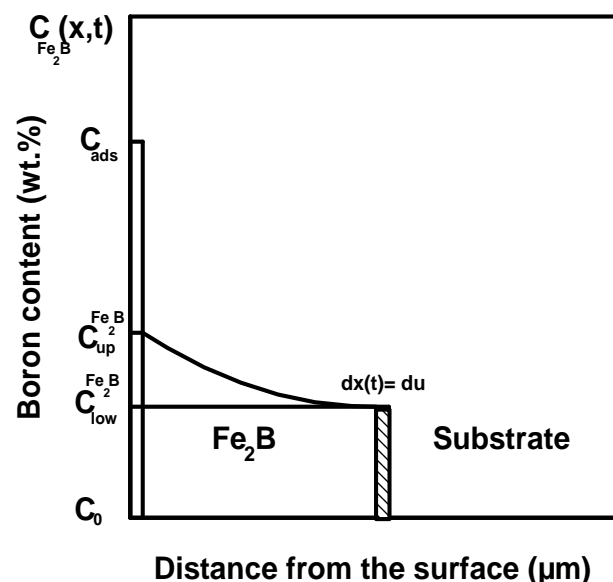
The diffusion model considers the growth of Fe<sub>2</sub>B layer on a saturated substrate with boron atoms. A schematic non-linear concentration profile of boron through the Fe<sub>2</sub>B layer is illustrated in Figure 1.

The constant  $C_{up}^{Fe_2B}$  ( $= 59.64 \times 10^3 \text{ mol.m}^{-3}$ ) is the upper boron concentration in Fe<sub>2</sub>B.

$C_{low}^{Fe_2B}$  ( $= 59.2 \times 10^3 \text{ mol m}^{-3}$ ) represents the lower boron concentration in Fe<sub>2</sub>B and  $t_0(T)$  is the boride incubation time depending upon the boriding temperature.  $C_{ads}$  is defined as the effective boron concentration [4]. The distance  $u$  is the layer depth at the

(Fe<sub>2</sub>B /substrate) interface as a function of the treatment time  $t$ .

$C_0$  is the boron solubility in the matrix. The assumptions taken during the formulation of the diffusion model can be found elsewhere [2].



**Figure 1.** A schematic non-linear concentration profile of boron through the Fe<sub>2</sub>B layer.

The initial condition of the diffusion problem is expressed by Equation (1):

$$C_{Fe_2B}(x,0) = C_0 \quad (1)$$

The boundary conditions of the diffusion problem are given by Equations (2) and (3):

$$C_{Fe_2B} \left\{ x \left[ t = t_0(T) \right] = 0, t_0(T) \right\} = C_{up}^{Fe_2B} \text{ for } C_{ads} > 59.2 \times 10^3 \text{ mol. m}^{-3} \quad (2)$$

$$C_{Fe_2B} \left\{ x(t=t) = u, t \right\} = C_{low}^{Fe_2B} \text{ for } C_{ads} < 59.2 \times 10^3 \text{ mol. m}^{-3} \quad (3)$$

for  $0 \leq x \leq u$ .

The Fick's second law of diffusion [5], relating the change in boron concentration through the  $Fe_2B$  layer with time  $t$  and location  $x(t)$  is:

$$\frac{\partial C_{Fe_2B}(x,t)}{\partial t} = D_B^{Fe_2B} \frac{\partial^2 C_{Fe_2B}(x,t)}{\partial x^2} \quad (4)$$

where  $D_B^{Fe_2B}$  is the diffusion coefficient of boron in  $Fe_2B$  free of chemical stresses and  $C_{Fe_2B}(x,t)$  is the boron concentration through the  $Fe_2B$  layer. The distribution of boron concentration as a function of the depth satisfies Equation (5):

$$C_{Fe_2B}(x,t) = C_{up}^{Fe_2B} + \left( \frac{C_{low}^{Fe_2B} - C_{up}^{Fe_2B}}{\text{erf}\left(\frac{u}{2\sqrt{D_B^{Fe_2B}t}}\right)} \right) \text{erf}\left(\frac{x}{2\sqrt{D_B^{Fe_2B}t}}\right) \quad (5)$$

for  $0 \leq x \leq u$ .

The continuity equation at the ( $Fe_2B$ /substrate) interface is expressed by Equation (6):

$$[0.5(C_{up}^{Fe_2B} + C_{low}^{Fe_2B}) - C_0]k = 2\sqrt{\frac{D_B^{Fe_2B}}{\pi}} \cdot \frac{(C_{up}^{Fe_2B} - C_{low}^{Fe_2B})}{\text{erf}\left(\frac{k\beta(T)}{2\sqrt{D_B^{Fe_2B}}}\right)} \exp(-\beta^2(T)k^2/4D_B^{Fe_2B})\beta(T) \quad (6)$$

$$\text{with } \beta(T) = [1 - (t_0(T)/t)]^{0.5} \quad (6)$$

The  $Fe_2B$  layer thickness  $u$  can be expressed by Equation (7):

$$u = k[t - t_0(T)]^{1/2} \quad (7)$$

where  $k$  represents the parabolic growth constant at the ( $Fe_2B$ /substrate) interface. As stated by Brakman et al. [6], the boride incubation time is decreased with an increase of the boriding temperature.

The  $\beta(T)$  parameter can be approached by a linear relationship [2] (Equation (8)) on the basis of experimental data taken from [7]:

$$\beta(T) = (5 \cdot 15 \times 10^{-4} T + 0.306637) \quad (8)$$

It is possible to numerically evaluate the diffusion coefficient of boron in  $Fe_2B$   $D_B^{Fe_2B}$  using the Newton-Raphson method [8]. For this purpose, a computer program was written in Matlab (version 6.5) to find the roots of Equation (6). By ignoring the pressure effect on the diffusion, the effective diffusion coefficient of boron in  $Fe_2B$  can be evaluated from Equation (9) as follows [3]:

$$D_B^{eff} = D_B^{Fe_2B} \left[ 1 + \frac{\bar{V}^2 (C_{up}^{Fe_2B} + C_{low}^{Fe_2B}) E}{9RT(1-\nu)} \right] \quad (9)$$

where  $D_B^{eff}$  and  $D_B^{Fe_2B}$  are the diffusion coefficients of boron in  $Fe_2B$  with and without the presence of chemical stresses, respectively.

The partial molar volume  $\bar{V}$  is taken equal to  $(1.01 \times 10^{-5} \text{ m}^3 \text{ mol}^{-1})$  [9].  $E$  ( $=290 \text{ GPa}$ ) and  $\nu$  ( $=0.3$ ) are the Young's modulus and Poisson's ratio of the  $Fe_2B$  layer, respectively [10, 11].

### 3 Simulation results

The experimental results available in reference [7] were firstly used to evaluate the diffusion coefficient of boron in  $Fe_2B$  using Equation (6), and secondly to determine the effective diffusion coefficient of boron in  $Fe_2B$  based on Equation (9). The past-boriding process was carried out on the iron substrate under an argon atmosphere in a conventional furnace at 4 temperatures (1223, 1253, 1273 and 1323 K) and for variable times (2, 4, 6 and 8 h). Due to the saw-tooth morphology of the (boride layer/substrate) interface, twenty five measurements were done on different cross-sections of the borided samples to estimate the  $Fe_2B$  layer thickness. In Figure 2 is plotted the variation of the squared value of experimental boride layer thickness [7] as a function of the treatment time.

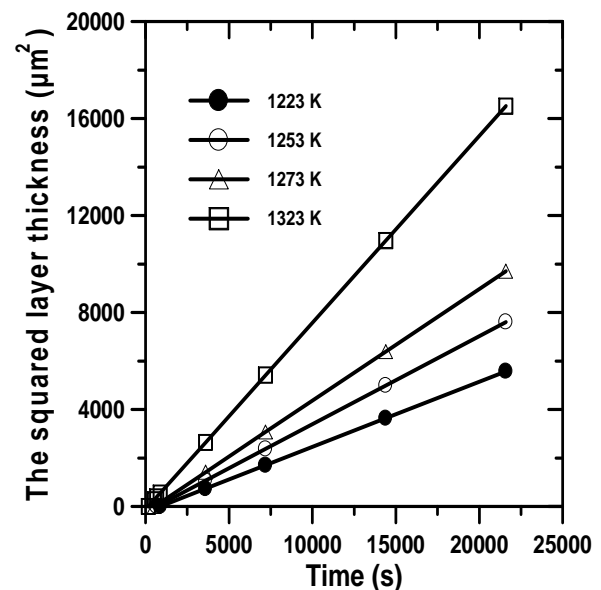


Figure 2. Variation of the squared value of the experimental boride layer thickness [7] versus the boriding time at increasing temperatures.

The model uses as input data the following parameters: the time, the temperature, the upper and lower boron concentrations in the Fe<sub>2</sub>B iron boride and the experimental values of the parabolic growth constants at the (Fe<sub>2</sub>B/substrate) interface. Table 1 provides the experimental value of parabolic growth constant  $k_{exp}$  at each boriding temperature with the corresponding incubation boride time.

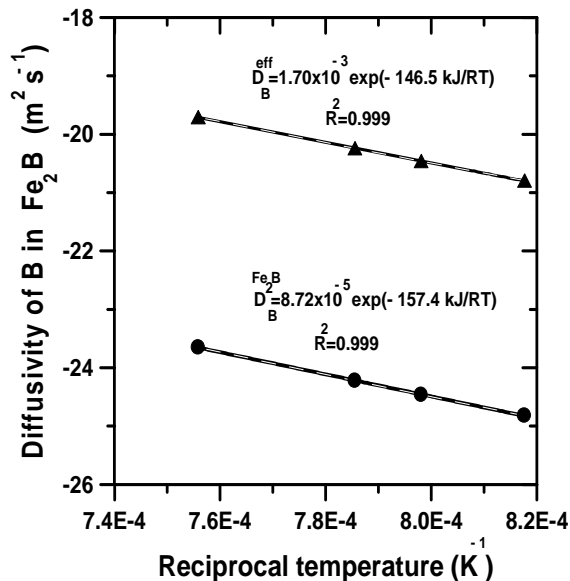
**Table 1.** Experimental parabolic growth constants  $k_{exp}$  at the (Fe<sub>2</sub>B/substrate) interface with the corresponding boride incubation times  $t_0(T)$ .

T (K)	$k_{exp}$ ( $\mu ms^{-1/2}$ )	$t_0(T)$ (s)
1223	0.518	884.3
1253	0.603	675
1273	0.678	533
1323	0.878	172

In Table 2, are gathered the computed values of  $D_B^{Fe_2B}$  and  $D_B^{eff}$  obtained at each boriding temperature using Equations (6) and (9) for  $C_{up}^{Fe_2B} = 59.64 \times 10^3 \text{ mol m}^{-3}$ .

**Table 2.** Determination of the diffusion coefficient of boron in Fe<sub>2</sub>B and the effective diffusion coefficient for each boriding temperature for  $C_{up}^{Fe_2B} = 59.64 \times 10^3 \text{ mol m}^{-3}$ .

T(K)	$D_B^{Fe_2B}$ ( $m^2 s^{-1}$ )	$D_B^{eff}$ ( $m^2 s^{-1}$ )
1223	$1.66 \times 10^{-11}$	$9.30 \times 10^{-10}$
1253	$2.38 \times 10^{-11}$	$1.30 \times 10^{-9}$
1273	$3.02 \times 10^{-11}$	$1.62 \times 10^{-9}$
1323	$5.34 \times 10^{-11}$	$2.76 \times 10^{-9}$



**Figure 3.** Temperature dependence of the diffusivity of boron in Fe<sub>2</sub>B.

It is noticed that the presence of chemical stresses enhances the diffusion coefficient of boron in Fe<sub>2</sub>B by a factor of 52 to 56. The dependence between the diffusivity of boron in Fe<sub>2</sub>B and the boriding temperature is expressed by the Arrhenius equation. The activation energy of boron (with and without the presence of chemical stresses) can be obtained from the corresponding curves displayed in Figure 3.

As a result, the diffusion coefficient of boron in Fe<sub>2</sub>B ( $m^2.s^{-1}$ ) in the temperature range of 1223-1323 K was found to be equal to:

$$D_B^{Fe_2B} = 8.72 \times 10^{-5} \exp\left(\frac{-157.4 \text{ kJ/mol}}{RT}\right) \quad (10)$$

The effective diffusion coefficient of boron in Fe<sub>2</sub>B ( $m^2.s^{-1}$ ) was also determined as:

$$D_B^{eff} = 1.7 \times 10^{-3} \exp\left(\frac{-146.5 \text{ kJ/mol}}{RT}\right) \quad (11)$$

where  $R$  is the universal gas constant ( $= 8.314 \text{ J.mol}^{-1}.\text{K}^{-1}$ ), and  $T$  represents the boriding temperature in Kelvin.

The obtained value of boron activation energy ( $=157.4 \text{ kJ.mol}^{-1}$ ), without the chemical stresses, is close to that determined by Campos et al. [12] (i.e.  $151 \text{ kJ mol}^{-1}$ ). This value represents the required energy to stimulate the boron diffusion in the preferential crystallographic direction [001] (i.e. in the direction perpendicular to the sample substrate). In this direction, it is reported that the boron element diffuses more speedily with a minimum resistance during the preferential growth of boride crystals.

In the direction parallel to the growth direction, the active flux of boron atoms accelerates the diffusion rate leading to a decrease in activation energy due to the chemical stresses effect. The deduced value of activation energy of boron ( $= 146.5 \text{ kJ.mol}^{-1}$ ) under the chemical stresses is lower than that of  $157.4 \text{ kJ.mol}^{-1}$  due to enhancement of the boron diffusion.

## 4 Conclusion

In the present work, the diffusivity of boron in the Fe<sub>2</sub>B layers grown on the iron substrate was firstly evaluated without the presence of chemical stresses through an application of a kinetic approach. This approach was based on solving the mass balance equation at the (Fe<sub>2</sub>B/substrate) interface under certain assumptions. For the case of incorporating chemical stress effects, the boron effective diffusion coefficient in Fe<sub>2</sub>B was secondly evaluated by applying a simple equation.

A lower value of activation energy for the effective diffusivity ( $= 146.5 \text{ kJ.mol}^{-1}$ ) was obtained for an upper boron content in Fe<sub>2</sub>B equal to  $59.64 \times 10^3 \text{ mol.m}^{-3}$ . This result is a consequence of the chemical stresses enhancing the boron diffusion through the Fe<sub>2</sub>B layer.

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