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Reconstruction of distributed force characteristics in case of non punctual objects impacting beams

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Abstract. The inverse formulation considered to reconstruct the characteristics of an impact uses in general a technique of minimizing the root mean square error between the measured and the calculated responses. The problem takes like this the form of parametric identification. To perform this in practice, a large number of sensors or an excessive computing time are required. In this work, the characteristics of impact in case of an elastic beam with the impacting object not necessarily punctual are reconstructed. We use first the reciprocity theorem in order to decouple the localization problem from the identification problem. We solve then the localization problem by means of a particle swarm algorithm.

Keywords: Inverse Problem, Localization, Identification, Impact, Regularization, Truncation Method, Beam, Optimization, Particles swarm

1 INTRODUCTION

Structures can be subjected to impacts from various sources. Accidental impacts may cause considerable damage and threaten the structural integrity. During impacts, simple examination of apparent traces on the structure surface is not sufficient because the damage may be invisible and deep inside the structure [1,2]. The inspection by experimental means to determine the extent of damage that was endured after an impact uses specific diagnosis techniques which are too expensive: X-ray imaging, etc…

Identifying the characteristics of the force generated by impact can be used to better know the health of the structure, reducing thus to a minimum the experimental effort [3,4]. In case of simple linear elastic structures with homogeneous geometric and material properties such as beams or plates, identification of impact characteristics can be implemented through using a structural model [5,6]. This can be constructed analytically, by means of the finite element method or by experimental identification procedures. When the impact can be assumed as being punctual and the impact location is known, the impulse response functions between the impact point and the sensors placed at known positions, allows by using a regularized deconvolution to reconstruct the force signal [7, 8, 9, 10, 11]. When the point of impact is unknown, the inverse formulation uses a minimization technique between the measured and calculated responses to iteratively reconstruct the impact characteristics: point location and force time evolution.

If now the impact is not punctual, the problem is more complex because it involves identifying a distribution of pressure and not a single concentrated force. Even if the pressure can be considered to be uniform, a new parameter that represents the extent of the impacted area appears in the problem.

In this work, a technique that can determine the impact characteristics for elastic beam like structures subjected to non punctual impacts is presented. This technique relies on solving at first a minimization problem in the form of a non linear mathematical program which enables providing the impact location characteristics. Then to find the pressure time history a classical regularized deconvolution problem is used.

2 MATERIALS AND METHODS

We consider a beam having a rectangular section as shown in figure 1. The beam which is assumed to be simply supported on its ends has the dimensions: length L , width b and height e . The beam is assumed to be made from a homogeneous and isotropic elastic material with Young's modulus E and density ρ . The applied force modelling impact is assumed to result from a uniformly distributed pressure, p , applied on the beam and having a rectangular profile, figure 1. The pressure rectangle is centred on s_0 and has the length $2u_0$. The dynamic response in terms of displacement, velocity, acceleration or strains is considered at a point which is located at a given distance away from the left extremity of the beam.

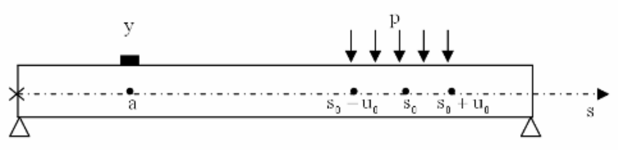

Figure 1. An elastic beam with uniform rectangular cross section loaded with a rectangular pressure.

Under the action of pressure, the dynamic response y in terms of displacement, acceleration or strain is measured at the point located at the abscissa a . Considering the time interval to be of length N , the state equation representation of the discrete linear system with multiple degrees of freedom that models the beam dynamic behaviour writes as follows

$$\begin{aligned} x(k+1) &= A\,x(k) + B\,p(k) \\ y(k) &= C\,x(k) + D\,p(k) \end{aligned} \quad k = 1,2,3...,N \tag{1}$$

with A and B representing respectively the system state matrix and state vector, C the matrix of observation, D the matrix representing the direct influence of the input p of the output y and x is the state vector describing the dynamics o f the beam system.

Equation (1) can be used to show that the response $y(k)$ can be calculated as function of the input impact pressure $p(k)$ by a linear convolution having the following form

$$y(k) = \sum_{j=1}^{k} h(j)\,p(k-j) \quad k = 1,2,3...,N \tag{2}$$

Where $y(k)$ is the discrete output as observed by the implanted sensors and $p(k)$ the discrete pressure input, h is a discrete response function of linear system considered.
The function h is given by

$$\begin{aligned} h(1) &= D \\ h(j) &= CA^{j-1}B \end{aligned} \quad j = 2,3...,N+1 \tag{3}$$

This leads to the following system of algebraic equations written in the time domain

$$Y = H\,P \tag{4}$$

With

$$H = \begin{bmatrix} h(1) & 0 & \cdots & 0 \\ h(2) & h(1) & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ h(N) & h(N-1) & \cdots & h(1) \end{bmatrix} \tag{5}$$

$$\begin{aligned} Y &= \left[y(1) \quad y(2) \qquad y(N) \right]^t \\ P &= \left[p(1) \quad p(2) \qquad p(N) \right]^t \end{aligned} \tag{6}$$

where H is the Toeplitz like transfer matrix connecting the pressure p to the measured signal y . The matrix H can be constructed analytically, but can also be obtained by means of the finite element method or by using an experimental identification procedure.
Denoting Δt the time step used in discretization, M the number of modes of truncation that are retained to model the beam dynamical response, the impulse response function giving the deformation of the top fibre of a section of abscissa a writes explicitly as

$$G_{kj}(s_0, u_0, a) = \sum_{m=1}^{M} \Bigg[\sin\left(\frac{m\pi s_0}{L}\right) \sin\left(\frac{m\pi u_0}{L}\right) \sin\left(\frac{m\pi a}{L}\right) g(\omega_m, \xi_m, (j-k)\Delta T) \Bigg] \tag{7}$$

With

$$g(\omega_m, \xi_m, (j-k)\Delta T) = \left[-\frac{mh\pi}{2\omega_m\sqrt{1-\xi_m^2}} \begin{matrix} \sin\left(\omega_m\sqrt{1-\xi_m^2}\,(j-k)\Delta T\right) \\ \exp\left(-\xi_m\omega_m(j-k)\Delta T\right) \end{matrix} \right] \tag{8}$$

Where $\omega_m = \dfrac{m\pi}{L}\sqrt{\dfrac{EI}{\rho S}}$ and ξ_m are respectively the circular eigenfrequency and damping ratio for a given eigenmode m .
The problem of locating the impact requires to identify the impact position s_0 and parameter u_0 defining the extent of the impacted zone. The responses calculated or measured by two strain sensors placed in points having respectively the abscissa a_i and a_j can be expressed under the following form

$$Y_i = G(s_0,u_0,a_i)P \quad Y_j = G(s_0,u_0,a_j)P \qquad (9)$$

The commutative property $G(s_0,u_0,a_i)G(s_0,u_0,a_j) = G(s_0,u_0,a_j)G(s_0,u_0,a_i)$ resulting form Maxwell-Betti theorem for elastic systems yields as shown in [10, 11] to the following relation

$$G(s_0,u_0,a_j)Y_i = G(s_0,u_0,a_i)Y_j \qquad (10)$$

The interest of this relation is that it does not depend on the time history of the applied pressure P.

The tow parameters that define the impact location s_0 and u_0 can be found by minimizing the following loss function

$$(s_0,u_0) = \underset{(s,u)}{Arg\min}\left\{\phi(s,u) = \sum_{i=1}^{N_c}\sum_{\substack{j=1\\ j\neq i}}^{N_c}\alpha_{ij}\left\|G(s,u,a_j)Y_i - G(s,u,a_i)Y_j\right\|^2\right\} \qquad (11)$$

where $N_c \geq 2$ denotes the number of used sensors and α_{ij} are weights that avoid trivial solutions to be obtain such as the point located on the borders of the beam.

The following expressions can be taken for the coefficients α_{ij}

$$\alpha_{ij} = \frac{1}{\left\|G(s,u,a_i)Y_j\right\|^2 + \eta} \qquad (12)$$

Where η is sufficiently small constant.

The minimization of the functional ϕ is subjected to the following constraints

$$\begin{aligned} &0 \leq u_0 \leq L/2 \\ &u_0 \leq s_0 \leq L - u_0 \end{aligned} \qquad (13)$$

The problem defined by equations (10), (11) and (12) takes the form of a nonlinear mathematical program for which it is not easy to explicit the objective function. To solve it, an evolutionary algorithm based on particles swarm is used [12, 13, 14].

Particles swarm is a heuristic optimization method that mimics the social behavior. Its implementation needs to specify the protocol of cooperation and competition among the potential solutions [12-14]. In this technique, the domain of the objective function to be minimized is chosen randomly and each particle has an index i ranging form 1 up to N_p. It occupies the position $x_i(t)$ and has the velocity $v_i(t)$.

In each generation t, the value of the objective function in each position $x_i(t)$ is calculated and the following updating rules are applied

$$\begin{aligned} &V_{ij}(t+1) = wV_{ij}(t) + c_1r_1\left(P_{ij}(t) - x_{ij}(t)\right) + c_2r_2\left(g_j(t) - x_{ij}(t)\right) \\ &x_{ij}(t+1) = x_{ij}(t) + V_{ij}(t) \\ &i = 1,\dots,N_p \\ &j = 1,\dots,D \end{aligned} \qquad (14)$$

where D is the dimension of the search space, w the coefficient of inertia, c_1 and c_2 designate acceleration coefficients, r_1 and r_2 are two random numbers drawn uniformly at each iteration in interval [0,1], $P_i = \left(P_{i1},\dots,P_{iD}\right)$ is the best position reached by the particle having index i, $g_i = \left(g_{i1},\dots,g_{iD}\right)$ the best overall position reached by all particles of the considered swarm.

The term $wV_{ij}(t)$ represents the physical component of displacement; the particle tends to follow its current direction of travel. The term $c_1r_1\left(P_{ij}(t) - x_{ij}(t)\right)$ is the cognitive component of displacement; the particle tends to move towards the best site in which it has already passed. The term $c_2r_2\left(g_j(t) - x_{ij}(t)\right)$ is the social component part of displacement; the particle tends to rely on the experience of its own and, thus, to move towards the best quality position already achieved by its neighbours.

3 RESULTS AND DISCUSSIONS

The direct problem is considered with the following material and geometric parameters:

$E = 7.06\times10^{10}\ Pa$; $L = 0.5\ m$; $c = 5\times10^{-3}\ m$;
$b = 5\times10^{-3}\ m$; $\rho = 2660\ \mathrm{kg.m^{-3}}$; $\xi_m = 2\%$;
$s_0 = 0.417\ m$; $u_0 = 0.0417\ m$; $a = 0.25\ m$.

The selected computing time duration $T_c = 0.5\ s$.

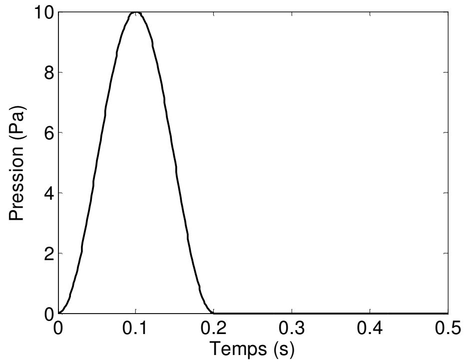

Fig. 2. The imposed impact pressure time history.

Figure 2 shows the pressure signal corresponding to the uniform impact. Figure 3 shows the resulting axial strain on the fiber in the upper fiber of the beam cross section having the abscissa a .

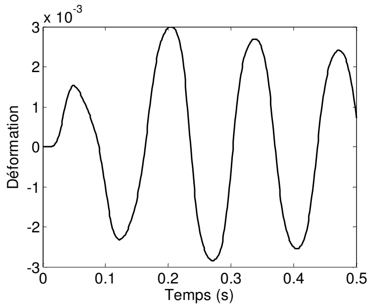

Fig. 3. Deformation calculated in the top fiber of the section of abscissa $a = 0.25\,m$

A modified version of the classical PSO algorithm was developed under Matlab environment in order to solve the nonlinear program defined by equations (10), (11) and (12). Convergence has been achieved after only performing 2 iterations.

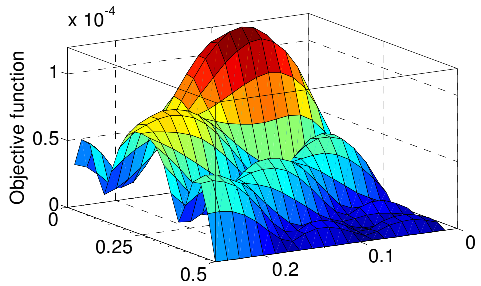

Fig. 4. Surface representing the objective function to be minimised

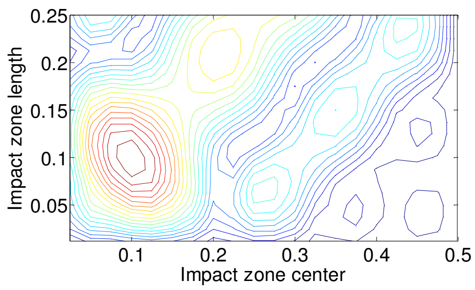

Fig. 3. Fig. 5. Isovaleurs of the objective function to be minimised

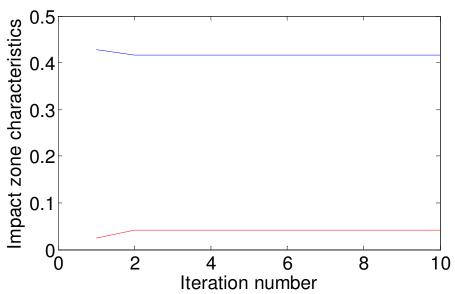

Fig. 6. Evolution of impact zone characteristics as function of the generation order

4 CONCLUSIONS

The obtained results demonstrate the possibility of reconstructing the characteristics of a non punctual impact occurring on an elastic beam where the force of impact can be considered to be a uniform applied pressure. The reciprocity theorem was used in order to first locate the impacted zone. This was performed by means of the particles swarm algorithm that was used to solve the obtained non linear mathematical program.

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