The effects of large vibration amplitudes on the mode shapes and natural frequencies of thin isotropic skew plates

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Abstract. The present work concerns the nonlinear dynamic behavior of skew plates at large vibration amplitudes with fully clamped edges. The large amplitude free vibration problem is modelled by a set of non-linear algebraic equations using Hamilton's principle and spectral analysis. The natural frequencies and modes of vibration have been obtained for fully clamped edges for various values of the skew angle. The relationship between the amplitude and frequencies is studied for isotropic skew plates. It was found that the frequencies of non linear vibration increase with increasing amplitude. Numerical details are presented and results are given, corresponding to the linear case and to the nonlinear case for various values of θ and various amplitudes of vibration. The results, compared to the previous ones available in the literature, show a reasonably good agreement.

1 Introduction

In contemporary structures, skew plates are used in diverse domains, such as stiffened plate, used as floors in bridges, ship hulls, buildings, etc. The simulation of static and dynamic behavior of skew plates is an interesting area of work for researchers. Alwar and Rao (1973) [5] presented a nonlinear analysis of orthotropic skew plates of constant thickness subjected to a uniform transverse load. G. Prathap and T. K. Varadan (1978) [4] studied the large amplitude free flexural vibrations of thin, elastic anisotropic skew plates. Chia (1980) [6] considered the moderately large deflection elastic behaviour of homogenous isotropic and laminated anisotropic rectangular as well as skew plates by analytical methods. Xiang et al. (1995) [7] studied the elastic buckling behaviour of skew Mindlin plate under a shear load. M.K. Singha and Rupesh Daripa (2007) [8] investigate the large amplitude free flexural vibration behavior of symmetrically laminated composite skew plates.
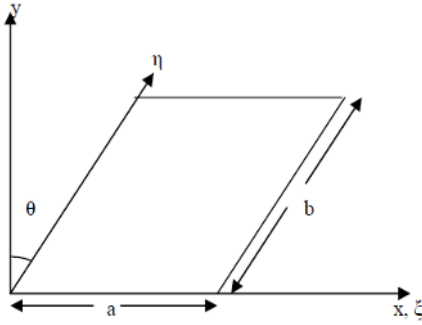
In the present study, we apply the theoretical model developed in [1] to analyze the geometrically nonlinear free dynamic response of an isotropic fully clamped skew plate in order to investigate the effect of nonlinearity on the nonlinear resonance frequency at large vibration amplitudes.

In the present paper, the general formulation of the model for nonlinear vibration of isotropic skew plates at large vibration amplitudes is presented first, and the relationships between the nonlinear resonance frequency

ratio and the vibration amplitudes is given for values of the angle θ varying from 0° to 45°. While there are many papers dealing with rectangular plate, as listed in reference, only few papers exist that deal with the large amplitude free flexural vibration of skew plates [4]. In reference [4], the authors obtained results for the large amplitude free flexural vibrations of thin, elastic anisotropic skew plates, on specializing for an isotropic skew plate and an orthotropic rectangular plate by using the von Karman field equations and the Berger approximation to study a skew plate.

2 Theoretical formulations

A skew plate with a skew angle θ is shown in Figure 1. For the large displacement formulation, it is assumed that the material of the plate is elastic, isotropic and homogeneous. The thickness of the plate is considered to be sufficiently small so as to avoid the effect of shear deformation. Also the stress and strain measures are based on the original dimensions of the plate.

Fig. 1. Skew plate in x-y and ξ-η co-ordinate system.

Consider the transverse vibration of the skew plate shown in figure 1 having the following characteristics: *a, b, S*: length, width and area of the plate; *x-y*: plate co-ordinates in the length and the width directions; *ξ-η, H*: Skew plate co-ordinate and thickness of the plate; E,ν: Young's modulus and Poisson's ratio; *D, ρ*: bending stiffness of the plate and mass per unit volume of the plate.

The theoretical formulation of the skew plate large vibration amplitude problem is presented, which leads to a numerical model. By assuming harmonic motion and expanding the transverse displacement in the form of finite series of functions, the bending strain energy V_b, the axial strain energy V_a and the kinetic energy T are expressed. By discretization of the expressions for V_a, V_b and T, the tensors m_{ij}, k_{ij} and b_{ijkl} are defined. The frequency equation at large vibration amplitudes is developed. Finally, the non dimensional formulation of the problem and details of the numerical solution are presented.

2.1 Expression for the bending strain, axial and kinetic energies in the skew plate

The potential energies due to bending and axial strains and the kinetic energy are given in x-y co-ordinate by:

$$V_b = \frac{D}{2}\int_S \left[\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 + 2(1-\nu)\left(\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \left(\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right)\right)\right] dS \tag{1}$$

$$V_a = \frac{3D}{2H^2}\int_S \left[\left(\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right)^2\right] dS \tag{2}$$

$$T = \frac{\rho H}{2}\int_S \left(\frac{\partial w}{\partial t}\right)^2 dS \tag{3}$$

The skew co-ordinates are related to the rectangular co-ordinate by: $\xi = x - y\,tan\theta$; $\eta = y/cos\theta$.

The potential energies due to the bending and axial strains and the kinetic energy are given in the *ξ-η* co-ordinate system by:

$$V_b = \frac{D}{2}\int_A \left[\frac{1}{cos^4\theta}\left(\frac{\partial^2 w}{\partial \xi^2} + \frac{\partial^2 w}{\partial \eta^2} - 2sin\theta\frac{\partial^2 w}{\partial \xi \partial \eta}\right)^2 + \frac{2(1-\nu)}{cos^2\theta}\left(\left(\frac{\partial^2 w}{\partial \xi \partial \eta}\right)^2 - \left(\frac{\partial^2 w}{\partial \xi^2}\frac{\partial^2 w}{\partial \eta^2}\right)\right)\right] dA \tag{4}$$

$$V_a = \frac{3D}{2H^2}\int_A \left[\frac{1}{cos^4\theta}\left(\left(\frac{\partial w}{\partial \xi}\right)^2 + \left(\frac{\partial w}{\partial \eta}\right)^2 - 2sin\theta\frac{\partial^2 w}{\partial \xi \partial \eta}\right)^2\right] dA \tag{5}$$

$$T = \frac{\rho H}{2}\int_A \left(\frac{\partial w}{\partial t}\right)^2 dA \tag{6}$$

where: $dA = cos\theta\, d\xi d\eta$ and $W(\xi, \eta, t)$ is the transverse displacement function.

The plate functions used in the discretisation process are obtained as product of beam functions. The transverse displacement can be written as:

$$w(\xi, \eta, t) = w(\xi, \eta)sin\omega t \tag{7}$$

where:

$$w(\xi, \eta) = a_i w_i(\xi, \eta) \tag{8}$$

The discretization and the non dimensional formulation is made as follows:

$$k_{ij}^* = \int_{A^*}\left\{\alpha^4 \frac{\partial^2 w_i^*}{\partial \xi^2}\frac{\partial^2 w_j^*}{\partial \xi^2} + \frac{\partial^2 w_i^*}{\partial \eta^{*2}}\frac{\partial^2 w_j^*}{\partial \eta^{*2}} + \alpha^2\frac{\partial^2 w_i^*}{\partial \xi^2}\frac{\partial^2 w_j^*}{\partial \eta^{*2}} + \alpha^2\frac{\partial^2 w_i^*}{\partial \eta^{*2}}\frac{\partial^2 w_j^*}{\partial \xi^2} - 2sin\theta\,\alpha^3\frac{\partial^2 w_i^*}{\partial \xi^2}\frac{\partial^2 w_j^*}{\partial \xi^* \partial \eta^*} - 2sin\theta\,\alpha\frac{\partial^2 w_i^*}{\partial \eta^{*2}}\frac{\partial^2 w_j^*}{\partial \xi^* \partial \eta^*} - 2sin\theta\,\alpha^3\frac{\partial^2 w_i^*}{\partial \xi^* \partial \eta^*}\frac{\partial^2 w_j^*}{\partial \xi^2} - 2sin\theta\,\alpha\frac{\partial^2 w_i^*}{\partial \xi^* \partial \eta^*}\frac{\partial^2 w_j^*}{\partial \eta^{*2}} + 4sin^2\theta\,\alpha^2\frac{\partial^2 w_i^*}{\partial \xi^* \partial \eta^*}\frac{\partial^2 w_j^*}{\partial \xi^* \partial \eta^*} + 2(1-\nu)cos^2\theta\,\alpha^2\left[\left(\frac{\partial^2 w_i^*}{\partial \xi^* \partial \eta^*}\frac{\partial^2 w_j^*}{\partial \xi^* \partial \eta^*}\right) - \left(\frac{\partial^2 w_i^*}{\partial \xi^2}\frac{\partial^2 w_j^*}{\partial \eta^{*2}}\right)\right]\right\} d\eta^* d\xi^* \tag{9}$$

$$b_{ijkl}^* = 3\int_{A^*}\left\{\alpha^4 \frac{\partial w_i^*}{\partial \xi^*}\frac{\partial w_j^*}{\partial \xi^*}\frac{\partial w_k^*}{\partial \xi^*}\frac{\partial w_l^*}{\partial \xi^*} + \alpha^2\frac{\partial w_i^*}{\partial \xi^*}\frac{\partial w_j^*}{\partial \xi^*}\frac{\partial w_k^*}{\partial \eta^*}\frac{\partial w_l^*}{\partial \eta^*} - 2sin\theta\alpha^3\frac{\partial w_i^*}{\partial \xi^*}\frac{\partial w_j^*}{\partial \xi^*}\frac{\partial w_k^*}{\partial \eta^*}\frac{\partial w_l^*}{\partial \xi^*} + \alpha^2\frac{\partial w_i^*}{\partial \eta^*}\frac{\partial w_j^*}{\partial \eta^*}\frac{\partial w_k^*}{\partial \xi^*}\frac{\partial w_l^*}{\partial \xi^*} + \frac{\partial w_i^*}{\partial \eta^*}\frac{\partial w_j^*}{\partial \eta^*}\frac{\partial w_k^*}{\partial \eta^*}\frac{\partial w_l^*}{\partial \eta^*} - 2sin\theta\alpha\frac{\partial w_i^*}{\partial \eta^*}\frac{\partial w_j^*}{\partial \eta^*}\frac{\partial w_k^*}{\partial \eta^*}\frac{\partial w_l^*}{\partial \xi^*} - 2sin\theta\alpha^3\frac{\partial w_i^*}{\partial \eta^*}\frac{\partial w_j^*}{\partial \xi^*}\frac{\partial w_k^*}{\partial \xi^*}\frac{\partial w_l^*}{\partial \xi^*} - 2sin\theta\alpha\frac{\partial w_i^*}{\partial \eta^*}\frac{\partial w_j^*}{\partial \xi^*}\frac{\partial w_k^*}{\partial \eta^*}\frac{\partial w_l^*}{\partial \xi\eta^*} + 4sin^2\theta\alpha^2\frac{\partial w_i^*}{\partial \eta^*}\frac{\partial w_j^*}{\partial \xi^*}\frac{\partial w_k^*}{\partial \eta^*}\frac{\partial w_l^*}{\partial \xi^*}\right\} d\eta^* d\xi^* \tag{10}$$

$$m_{ij}^* = \int_{A^*} w_i^* w_j^* \, d\eta^* d\xi^* \tag{11}$$

where:

$$w_i(\xi, \eta) = Hw_i^*\left(\frac{\xi}{a}, \frac{\eta}{b}\right) = Hw_i^*(\xi^*, \eta^*) \tag{12}$$

$$\alpha = \frac{b}{a} \quad ; \quad \xi^* = \frac{\xi}{a} \quad ; \eta^* = \frac{\eta}{b} \tag{13}$$

The dynamic behavior of the isotropic skew plate is governed by Hamilton's principle which leads to the following set of non linear algebraic equations:

$$2a_i k_{ir}^* + 3a_i a_j a_k b_{ijkr}^* - 2\omega^* a_i m_{ir}^* = 0, \quad r=1\ldots n \tag{14}$$

where:

$$\omega^* = \frac{a_i a_j k_{ij}^* + a_i a_j a_k a_l b_{ijkl}^*}{a_i a_j m_{ij}^*} \tag{15}$$

$$\omega^2 = \frac{D}{\rho b^4 \cos^4 \theta} \omega^{*2} \tag{16}$$

3 Numerical details

In the present work, calculations have been made using 9 plate functions obtained as product the 1st three clamped-clamped beam functions. and different values of θ (0°, 15°, 30°and 45°).

Two types of validations have been performed:

1) The linear results for various values of θ .
2) The non linear results obtained here and in previous works for various values of θ .

The results and comparisons are summarized in Tables 1, 2 and 3. A satisfactory agreement can be noticed in spite of the little difference observed in the results, which may be due to use of the Berger approximation enforcing the immovable boundary condition in an averaged manner [4] whereas there is no such approximation involved in the present work.

Table 1. Nondimentional frequency parameters $\omega b^2 \sqrt{\frac{\rho}{D}} \cos^2 \theta$ for a C-C-C-C skew plate

	$\omega b^2 \sqrt{\frac{\rho}{D}} \cos^2 \theta$ For values of skew angle, θ, deg, of			
	0	15	30	45
Leissa [1]	36.11	36.67	38.15	40.08
Present Work	35.99	36.52	37.92	39.72

Table 2. Ratio frequency parameters to the first nonlinear mode shape for a C-C-C-C square plate

W. Han and M. Petyt [3]		Present Work	
w^*_{max}	ω^*_{nl}/ω^*l	w^*_{max}	ω^*_{nl}/ω^*_l
0.20	1.010	0.20	1.010
0.40	1.041	0.40	1.042
0.60	1.089	0.60	1.090
0.80	1.152	0.80	1.154
1.00	1.226	1.00	1.227
1.20	1.308	1.20	1.310
1.40	1.397	1.40	1.396
1.50	1.443	1.50	1.442

Table 3. Ratio frequency parameters to the first nonlinear mode shape for a C-C-C-C skew plate

w^*_{max}	Skew Angles					
	G. Prathap and T. K. Varadan [4]			Present Work		
	15°	30°	45°	15°	30°	45°
0.2	1.009	1.008	1.007	1.010	1.010	1.010
0.4	1.037	1.033	1.029	1.041	1.040	1.039
0.6	1.082	1.073	1.065	1.089	1.086	1.089
0.8	1.142	1.126	1.112	1.152	1.147	1.145
1.0	1.214	1.191	1.171	1.224	1.220	1.214
1.2	1.297	1.266	1.239	1.306	1.298	1.2917
1.4	1.388	1.349	1.314	1.392	1.383	1.375
1.5	1.437	1.393	1.355	1.437	1.427	1.419

4 Conclusion

The theoretical model for thin elastic skew plate at large displacement has been developed and numerical results have been obtained for the nonlinear frequency amplitude dependence. Good agreement has been found between the nonlinear frequencies of the first mode from the present method and the published results discussed in reference [4].It will be noticed that the increase in the frequencies with increasing amplitude is more pronounced in the case of rectangular plates, and its less pronounced for skew plate with increasing skew angle θ.

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