

## Soret-driven double diffusive magneto-convection in couple stress liquid

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**Abstract.** The stability analysis of Soret driven double diffusive convection for electrically conducting couple stress liquid is investigated theoretically. The couple stress liquid is confined between two horizontal surfaces and a constant vertical magnetic field is applied across the surfaces. Linear stability analysis is used to investigate the effect of various parameters on the onset of convection. Effect of magnetic field on the onset of convection is presented by means of Chandrasekhar number. The problem is analyzed as a function of Chandrasekhar number ( $Q$ ), positive and negative Soret parameter ( $Sr$ ) and couple stress parameter ( $C$ ), mainly. The results show that the  $Q$ , both positive and negative  $Sr$  and  $C$  delay the onset of convection. The effect of other parameters is also discussed in paper and shown by graphs.

### 1 Introduction

Magneto-convection arises due the interaction of electrically conducting fluid flow and the applied magnetic field. The linear and non-linear magneto-convection in an electrically conducting fluid has been extensively studied by many authors viz, Busse [1] and Chandrasekhar [2]. Since molten metals and semiconductor melts are electrically conducting, a magnetic field may be applied to control the thermally induced convective flows in these fluids. It has many applications such as crystal growth, geothermal reservoir, metallurgical applications involving continuous casting and solidification of metal alloys and others. Li [3] and Pan and Li [4] show that gravity and magnetic fields represent different mechanisms of flow reduction and that they may be combined to further suppress the convection in a modulated gravity field. Ozoe and Maruo [5] investigated magnetic and gravitational natural convection of metal silicon and performed two-dimensional numerical computations to obtain the rate of heat transfer. Siddheshwar and Pranesh [6] examined the effects of time-periodic temperature/gravity modulation on the onset of magneto-convection in electrically conducting fluids with internal angular momentum by making a linear stability analysis. Recently Siddheshwar et al. [7] investigated a weak non-linear stability problem of magneto-convection in an electrically conducting Newtonian fluid, confined between two horizontal surfaces, under a constant vertical magnetic field, and subjected to an imposed time-periodic boundary temperature or gravity modulation.

Double-diffusive convection has attracted considerable interest in the last two decades. Such systems are often

termed thermohaline, referring to their most common occurrence in oceans and other large water bodies, with salt and heat playing the roles of the two components. Interesting dynamics ensues when the two components affect the density stratification in opposite senses, and convection may occur even when the total density gradient is gravitationally stable. If the gradient of two stratifying agencies such as heat and salt having different diffusivities are simultaneously present in a fluid layer many interesting convective phenomena can occur that are not possible in a single component fluid. An excellent review of the studies related to double diffusive convection has been reported by Turner ([8]-[10]), Huppert and Turner [11] and Platten and Legros [12].

Dougall [13] investigated the double diffusive convection caused by coupled molecular diffusion. Hurle and Jake-man [14] concluded from his experimental and theoretical work that in cross diffusion Dufour term is very small and may be neglected in comparison to Soret effect. Due to this fact one may leave the Dufour term while study the flow in liquids. Soret effect is a flux of salt caused by a spatial gradient of temperature. This phenomena has many applications such as in geophysics, oil reservoirs and ground water. Platten and Chavepeyer [15] was studied oscillatory motions in Benard cell due to Soret effect. According to their study, the linear theory oscillatory motions are observed in the two-component system with negative Soret coefficients and the order of magnitude of the period of oscillations is confirmed by experiments. Knobloch [16] investigated the thermal convection in a binary fluid driven by the Soret and Dufour effects. By his studied, he concluded that equations are identical to the thermosolutal problem except for a relation between the thermal and solute Rayleigh numbers. The onset of Rayleigh-Benard convection in liquid mixtures has been studied experimentally by Lee et al. [17]. They stated by his work that the

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overstable motions are possible when heated from below and only stationary motions exist when heated from above. Porta and Surko [18] has been performed experimentally the study of convective instability in a fluid mixture heated from above with negative Soret coefficient. According to their investigation, although the linear analysis predicts that the instability occurs at zero wave-number, a large wave-number pattern is observed. Bourich et al. [19] have given an excellent review of literature on the Soret effect convection either in fluid or porous media. Malashetty et al. [20] investigated double diffusive convection in a two-component couple stress liquid layer with Soret effect using both linear and non-linear stability analyses. Recently Srivastava et al. [21] investigated an electrically conducting two component Boussinesq fluid-saturated-porous medium in the presence of Soret coefficient.

Non-Newtonian fluids has lot of importance in modern technology and industries. In last few decades the theory of polar fluids has received much attention and this is due to the fact that most of the Newtonian fluids cannot precisely describe the characteristics of the fluid flow with suspended particles. These fluids have many applications that occur in industry such as the extrusion of polymer fluids, solidification of liquid crystals, cooling of metallic plate in a bath, exotic lubricants and colloidal and suspension solutions. Due to polar effect, the couple stress liquid has distinct features in non-Newtonian fluids. Stokes [22] proposed the constitutive equations for couple stress fluids. In his studied, he suggested simplest theory for the micro fluids, which allows polar effects such as the presence of couple stress, body couples and non-symmetric tensors. Siddeshwar and Pranesh [23], Sharma and Sharma [24] and Malashetty and Basavarja [25] investigated Rayleigh-Benard convection in fluids with stress non-linearly proportional to velocity gradient.

To the best of author's knowledge no study is available on magneto-convection in double-diffusive couple stress liquid under Soret effect. In view of this, the same is studied in the paper.

## 2 Mathematical Formulations

We consider an electrically conducting two component liquid of depth  $d$ , confined between two infinite parallel horizontal planes at  $z = 0$  and  $z = d$ . Cartesian co-ordinates have been taken with the origin at the bottom of the layer, and the  $z$ -axis vertically upwards. The surfaces are extended infinitely in  $x$  and  $y$  directions and constant temperature gradient  $\Delta T$  and salinity gradient  $\Delta S$  are maintained across the layer. A constant magnetic field  $H_0 \hat{k}$  is applied externally in vertical upward direction (as shown in Fig. 1). Under the Boussinesq approximation, the non-dimensional governing equations for the study of magneto double-diffusive convection in an electrically conducting couple stress liquid are (following Siddheshwar et al. [7] and Srivastava et al. [21]),

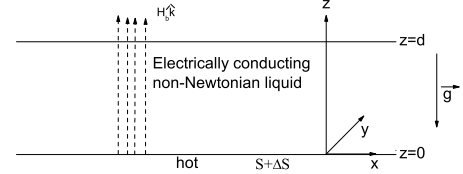


Fig. 1. Schematic of physical configuration.

$$\frac{1}{Pr} \frac{\partial (\nabla^2 \psi)}{\partial t} = -R \frac{\partial T}{\partial x} + R_S \frac{\partial S}{\partial x} + \nabla^4 \psi - C \nabla^6 \psi + \frac{1}{Pr} \frac{\partial (\psi, \nabla^2 \psi)}{\partial (x, z)} + QPm \frac{\partial}{\partial z} (\nabla^2 \phi) - QPm \frac{\partial (\phi, \nabla^2 \phi)}{\partial (x, z)} \quad (1)$$

$$\frac{\partial T}{\partial t} = -\frac{\partial \psi}{\partial x} + \nabla^2 T + \frac{\partial (\psi, T)}{\partial (x, z)} \quad (2)$$

$$\frac{\partial S}{\partial t} = -\frac{\partial \psi}{\partial x} + \frac{1}{Le} \nabla^2 S + S_r \frac{R}{R_S} \nabla^2 T + \frac{\partial (\psi, S)}{\partial (x, z)} \quad (3)$$

$$\frac{\partial \phi}{\partial t} = \frac{\partial \psi}{\partial z} + Pm \nabla^2 \phi + \frac{\partial (\psi, \phi)}{\partial (x, z)} \quad (4)$$

where the non-dimensional parameters, Prandtl number  $Pr = \frac{\nu}{\kappa_T}$ , thermal Rayleigh number  $R = \frac{\beta_T g \Delta T d^3}{\nu \kappa_T}$ , solutal Rayleigh number  $R_S = \frac{\beta_S g \Delta T d^3}{\nu \kappa_T}$ , Chandrasekhar number  $Q = \frac{\mu_m H_0^2 d^2}{\rho_0 \nu \nu_m}$ , magnetic Prandtl number  $Pm = \frac{\nu_m}{\kappa_T}$ , couple stress parameter  $C = \frac{\mu_c}{\mu d^2}$ , Soert parameter  $S_r = \frac{D \beta_S}{\kappa_T \beta_T}$  and Lewis number  $Le = \frac{\kappa_T}{\kappa_S}$ . In these parameters,  $\beta_T$  and  $\beta_S$  are coefficient of thermal and solutal expansions respectively,  $\kappa_T$  and  $\kappa_S$  are the thermal and solutal diffusivity respectively,  $D$  is cross diffusion due to  $T$  component,  $\nu$  and  $\nu_m$  are kinematic and magnetic viscosity respectively and  $\mu_m$  is magnetic permeability. Now linearizing the above set of equations and combining Eqs. (1) and (4), we get

$$\left[ \left( \frac{\partial}{\partial t} - Pm \nabla^2 \right) \left( \frac{1}{Pr} \frac{\partial}{\partial t} \nabla^2 - \nabla^4 (1 - C \nabla^2) \right) - QPm \nabla^2 \frac{\partial^2}{\partial z^2} \right] \psi + R \left( \frac{\partial}{\partial t} - Pm \nabla^2 \right) \frac{\partial T}{\partial x} - R_S \left( \frac{\partial}{\partial t} - Pm \nabla^2 \right) \frac{\partial S}{\partial x} = 0 \quad (5)$$

$$\left( \frac{\partial}{\partial t} - Pm \nabla^2 \right) T + \frac{\partial \psi}{\partial x} = 0 \quad (6)$$

$$\left( \frac{\partial}{\partial t} - \frac{1}{Le} \nabla^2 \right) S + \frac{\partial \psi}{\partial x} - S_r \frac{R}{R_S} \nabla^2 T = 0 \quad (7)$$

The Soret parameter  $S_r$  may take either positive or negative value. A positive Soret parameter indicates that

the solute diffuse towards the cooler plates, while negative Soret parameter indicates the reverse. Eqs. (5) – (7) are solved for stress-free, isothermal, isohaline boundary conditions, as given below

$$\psi = T = S = 0 \quad \text{at} \quad z = 0, 1. \quad (8)$$

### 3 Stability Analysis

In this section, we predict the threshold of both marginal and oscillatory convection, using linear theory. To make this study, assume the solutions to be periodic waves of the form

$$\begin{pmatrix} \psi \\ T \\ S \end{pmatrix} = e^{\sigma t} \begin{pmatrix} \Psi \sin \pi \alpha x \\ \Theta \cos \pi \alpha x \\ \Phi \cos \pi \alpha x \end{pmatrix} \sin \pi z \quad (9)$$

where  $\Psi$ ,  $\Theta$  and  $\Phi$  are the amplitudes of stream function, temperature and concentration field respectively. Substituting (9) in the Eqs. (5)-(7), we obtain a matrix equation

$$\begin{bmatrix} \left[ \left( \frac{\sigma}{Pr} + a^2 \eta \right) (\sigma + Pma^2) + QPm\pi^2 \right] a^2 R (\sigma + Pma^2) \pi \alpha & & \\ & \pi \alpha & (\sigma + a^2) \\ & \pi \alpha & Sra^2 \frac{R}{R_s} \end{bmatrix} \begin{pmatrix} \Psi \\ \Theta \\ \Phi \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad (10)$$

For non-trivial solution of  $\Psi$ ,  $\Theta$  and  $\Phi$ , we follow the same processor applied by Srivastava et al. [21], we get

$$R = (\sigma + a^2) \frac{P1}{P2} \quad (11)$$

where  $a^2 = \pi^2 \alpha^2 + \pi^2$ ,

$$P1 = \left[ \left\{ \left( \frac{\sigma}{Pr} + a^2 \eta \right) (\sigma + Pma^2) + QPm\pi^2 \right\} a^2 (\sigma Le + a^2) + R_s \pi^2 a^2 Le (\sigma + Pma^2) \right],$$

where  $\eta = 1 + Ca^2$  and

$$P2 = \pi^2 a^2 [(\sigma + Pma^2)(\sigma Le + a^2) + SrLea^2(\sigma + Pma^2)]$$

For the direct bifurcation (i.e., steady onset), we have  $\sigma = 0$  at the margin of stability. Then Rayleigh number at which marginally stable steady mode exists, becomes

$$R^{st} = \frac{(a^4 \eta + Q\pi^2) a^2 + R_s \pi^2 a^2 Le}{\pi^2 a^2 (1 + SrLe)} \quad (12)$$

The minimum value of Rayleigh number  $R^{st}$  at the critical wave number  $\alpha = \alpha_c$ , where  $\alpha_c^2 = \chi$  satisfies the equation

$$\eta \chi^4 + 2\eta \chi^3 + \left( \eta - \left( \frac{1}{\xi} + Q \right) + \frac{R_s Le (\eta - 1)}{\pi^2} \right) \chi^2 - 2 \left( \frac{1}{\xi} + Q \right) \chi - \left( \frac{1}{\xi} + Q \right) = 0 \quad (13)$$

In the absence of magnetic field the Eq. (12) reduces to

$$R^{st} = \frac{a^6 \eta}{\pi^2 a^2 (1 + SrLe)} + \frac{R_s Le}{(1 + SrLe)} \quad (14)$$

which gives the result of Malashetty et al. [20]. Further when  $R_s = 0$  and absence of Soret effect, Eq. (14) reduces to

$$R^{st} = \frac{a^6 \eta}{\pi^2 a^2} \quad (15)$$

which is same the result of Siddheshwar and Pranesh [23]. If  $C = 0$  that is when couple stress is absent, we get the classical results of Chandrasekhar [2], that is critical Rayleigh number  $R_c^{st} = 657.5$  and wave-number is  $\alpha_c^2 = 0.5$ .

The growth rate  $\sigma$  is in general a complex quantity such that  $\sigma = \sigma_r + i\sigma_i$ . The system with  $\sigma_r \leq 0$  is always stable, while for  $\sigma_r \geq 0$  it will become unstable. For neutral stability state  $\sigma_r = 0$ . Therefore, we now set  $\sigma = \sigma_i$  in Eq.(11) and clear the complex quantities from the denominator, to obtain

$$R = \Lambda_1 + i\sigma_i \Lambda_2 \quad (16)$$

The expression for  $\Lambda_1$  and  $\Lambda_2$  is not presented for brevity.

Since  $R_T$  is a physical quantity, it must be real. Hence, from Eq. (16) it follows that either  $\sigma_i = 0$  (steady onset) or  $\Lambda_2 = 0$  ( $\sigma_i \neq 0$ , oscillatory onset). For oscillatory onset  $\Lambda_2 = 0$  ( $\sigma_i \neq 0$ ), gives a dispersion relation of the form (on dropping the subscript  $i$ )

$$C_0 (\omega^2)^2 + C_1 (\omega^2) + C_2 = 0 \quad (17)$$

where the constants  $C_i$ 's are as under

$$C_0 = a^2 (1 + Le) \frac{Le}{Pr} - Lea^2 \left\{ \frac{(1 + SrLe)}{Pr} - \eta Le \right\}$$

$$C_1 = a^6 (1 - Pm^2 Le) \left\{ \frac{(1 + SrLe)}{Pr} - \eta Le \right\} + a^6 (1 + Le) \eta (1 + SrLe) + a^6 Pm^2 (1 + Le) \frac{Le}{Pr} + R_s Le \pi^2 a^2 \{1 + (Sr - 1) Le\} + Le \{PmLe + (1 + SrLe)\} QPm\pi^2 a^2 - QPm\pi^2 a^2 (1 + Le) Le.$$

$$C_2 = a^{10} Pm^2 \left\{ \frac{(1 + SrLe)}{Pr} - \eta Le \right\} + a^{10} Pm^2 (1 + Le) \eta (1 + SrLe) + QPm\pi^2 a^6 (1 + Le) (1 + SrLe) + R_s Le \pi^2 a^2 \{1 + (Sr - 1) Le\} a^4 Pm^2 - QPm\pi^2 a^6 \{PmLe + (1 + SrLe)\}$$

Now Eq. (16) with  $\Lambda_2 = 0$ , gives

$$R^{osc} = \left[ a^2 D1 + \frac{QPma^2 \pi^2}{\omega^2 + Pm^2 a^4} D2 + R_s Le \pi^2 a^2 D3 \right] D4^{-1} \quad (18)$$

$$D1 = (a^4 - \omega^2 Le) \left( a^4 \eta (1 + SrLe) + \frac{\omega^2 Le}{Pr} \right) - a^4 (1 + Le) \omega^2 \left( \frac{1 + SrLe}{Pr} - \eta Le \right)$$

$$D2 = (a^4 - \omega^2 Le) \left( Pma^4 (1 + SrLe) - \omega^2 Le \right) + a^4 (1 + Le) \left( PmLe + (1 + SrLe) \omega^2 \right)$$

$$D3 = (a^4 (1 + SrLe + \omega^2 Le))$$

$$D4 = \left[ \pi^2 a^2 \left\{ \omega^2 Le^2 + a^4 (1 + SrLe)^2 \right\} \right]$$

Also for the oscillatory convection to occur,  $\omega^2$  must be positive.

### 4 Results and Discussion

Normal mode analysis is used to investigate the linear stability for the onset of magneto-convection in a couple stress, binary liquid with Soret effect. The stability of basic flow are governed by non-dimensional parameters: couple stress parameter ( $C$ ), Chandrasekhar number ( $Q$ ), Soret coefficient ( $Sr$ ), Lewis number ( $Le$ ), Prandtl number ( $Pr$ ), Solutal Rayleigh number ( $R_S$ ), and magnetic Prandtl number ( $Pm$ ). The entire stability analysis is divided into (i) Stationary convection and (ii) Oscillatory convection.

The neutral stability curves in  $R-\alpha$  plane, for different values of parameters are shown in Figs. 2-7. In the figures, we present the variation of  $R$  with respect to  $\alpha$  for fixed values of the parameters  $C = 1.0, Q = 25.0, Sr = -0.003, Le = 1.0, R_S = 10.0, Pm = 0.001$  and  $Pr = 2.0$ , with variation in one of the parameter. From these figures, it is clear that the neutral curves are connected in a topological sense. This connectedness allows the linear stability criteria to be expressed in terms of the critical Rayleigh number,  $R_c$ , below which the system is stable and unstable above.

The characteristic curves for different value of Chandrasekhar number  $Q$  for stationary and oscillatory modes have been presented in Fig. 2(a) and 2(b) respectively. We see that the effect of increasing  $Q$  is to increase the critical value of Rayleigh number for both stationary and oscillatory modes, implying that  $Q$  has a stabilizing effect. Similar effect is shown by couple stress parameter  $C$ , solutal Rayleigh number  $R_S$  and Lewis number  $Le$  in Fig. 3, 5 and 6 respectively. In Fig. 4(a)-4(b), we investigate the effect of positive and negative value of Soret parameter on the neutral stability for fixed values of other parameters. It can be observed that an increment in  $Sr$  decreases the minimum of Rayleigh number for stationary and oscillatory state for both positive and negative values thus destabilizing the system.

Variation of  $R$  with respect to wave number  $\alpha$ , for different value of  $Pm$ , has been depicted in Fig. 7. From the figure, we notice that for stationary convection there is no effect of  $Pm$ , whereas for oscillatory convection value of critical Rayleigh number decreases with increasing value of  $Pm$  thus destabilizing the system. Variation of  $R$  with wave number  $\alpha$  for different value of  $Pr$  is also shown by Table 1. It is clear that for oscillatory convection there is slight increase the value of  $R$  on increasing the value of  $Pr$  indicating that it stabilizes the system.

### 5 Conclusions

In the present article, linear stability analysis is performed, to study the magneto double diffusive convection in couple stress liquid in the presences of Soret coefficient. Linear stability analysis is performed using normal mode technique. The following conclusions are drawn:

1. Effect of increasing  $Q, C, R_S, Le$  is found to increase the onset of stationary, oscillatory convection, thus it delay the onset of convection.
2. On increasing the value of  $Sr$ , value of Rayleigh number corresponding to stationary and oscillatory convection decrease, thus it advances the onset of convection.
3. On increasing the value of  $Pm$ , there is no effect on stationary convection however Rayleigh number decreases with increasing value of  $Pm$ .

4. On increasing the value of  $Pr$ , Rayleigh number slightly increases with increasing value of  $Pr$ , thus stabilize the system.

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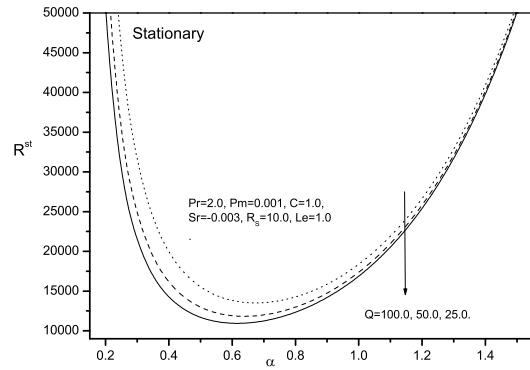


Fig. 2(a). Variation of  $R^{st}$  with  $\alpha$  for different Chandrasekhar number.

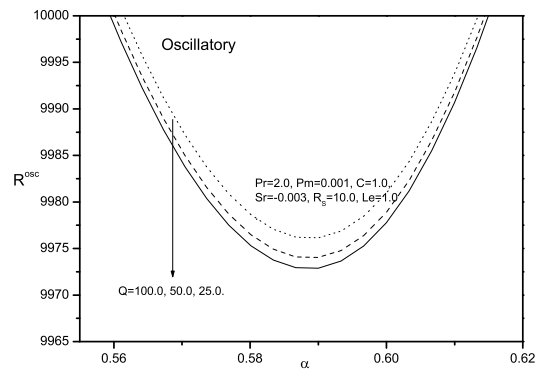


Fig. 2(b). Variation of  $R^{osc}$  with  $\alpha$  for different Chandrasekhar number.

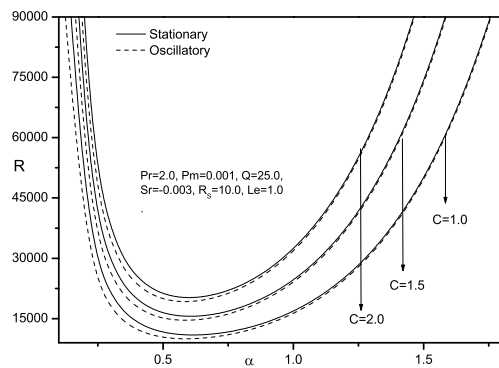


Fig. 3 Variation of  $R$  with  $\alpha$  for different couple stress parameter.

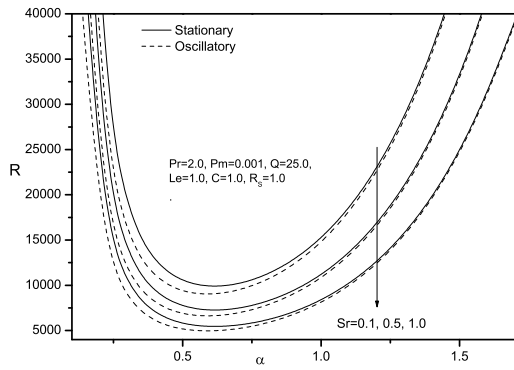


Fig. 4(a). Variation of  $R$  with  $\alpha$  for different value of positive Soret number.

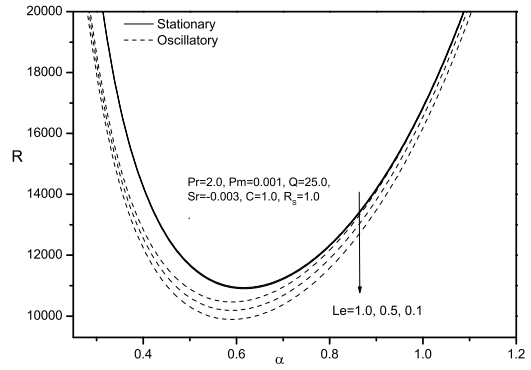


Fig. 6. Variation of  $R$  with  $\alpha$  for different value of Lewis number.

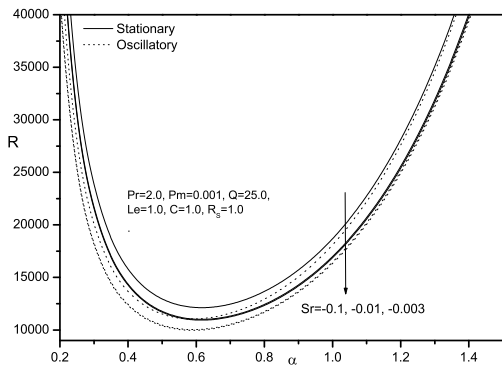


Fig. 4(b). Variation of  $R$  with  $\alpha$  for different value of negative Soret number.

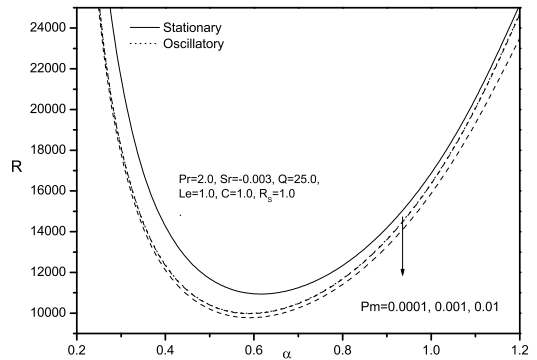


Fig. 7. Variation of  $R$  with  $\alpha$  for different value of magnetic Prandtl number.

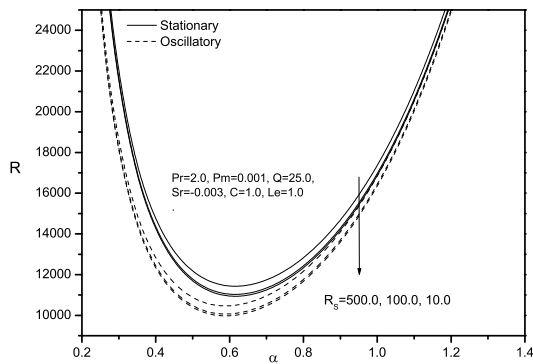


Fig. 5. Variation of  $R$  with  $\alpha$  for different value of solutal Rayleigh number.

**Table 1.** Variation of  $R^{osc}$  with  $\alpha$  for different value of  $Pr$ .

$\alpha$	$R$		
	$Pr=2.0$	$Pr=6.0$	$Pr=10.0$
0.1	110219.75	110219.75	110222.46
0.2	30902.7	30902.7	30903.47
0.3	16509.16	16509.16	16509.58
0.4	11853	11853	11853.3
0.5	10170.54	10170.54	10170.79
0.6	9836.86	9836.86	9837.1
0.7	10348.64	10348.64	10348.88
0.8	11559.47	11559.47	11559.72
0.9	13474.95	13474.95	13475.23
1	16186.75	16186.75	16187.06
1.1	19850.25	19850.25	19850.6
1.2	24678.65	24678.65	24679.06
1.3	30944.1	30944.1	30944.58
1.4	38982.4	38982.4	38982.97
1.5	49199.7	49199.7	49200.39
1.6	62080.43	62080.43	62081.26
1.7	78195.78	78195.78	78196.82
1.8	98212.09	98212.09	98213.41
1.9	122896.94	122896.94	122898.67
2	153116.02	153116.02	153118.45

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