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Natural Convection in a Rotating Nanofluid Layer

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Abstract. In this paper, we study the effect of rotation on the thermal instability in a horizontal layer of a Newtonian nanofluid. The nanofluid layer incorporates the effect of Brownian motion along with thermophoresis. The linear stability based on normal mode technique has been investigated. We observe that the value of Rayleigh number can be increased by a substantial amount on considering a bottom heavy suspension of nano particles. The effect of various parameters on Rayleigh number has been presented graphically.

1 Introduction:

Nanofluids are engineered colloidal suspensions of nanometer sized($1-100nm$) particles in ordinary heat transfer liquids. The common heat transfer fluids, known as base fluids, include water, ethylene glycol, engine oils, to name a few, while the nanoparticles used include metallic or metallic oxide particles (Cu, Cuo, Al_2O_3), carbon nanotubes, etc. The first scientist to use the term "Nanofluids"was Choi [1] in the year 1995 while working at the A.N.L.,USA. He was working on improved heat transfer mediums to be used in industries like power manufacturing, transportation, electronics, air conditioning etc..

Prior to the development of technology for manufacture of nano-meter sized particles, micro-meter sized particles were used in ordinary heat transfer fluids to enhance there thermal properties. The possibility of their usage was suggested by Maxwell[2] more than a century ago. But the use of these posed problems such as settling, producing drastic pressure drops, clogging channels, and premature wear on channels and components. These difficulties are overcome by the usage of nanoparticles. The smaller particles provide much larger relative surface area than micro-sized particles improving the heat transfer properties. Because of the superior properties of nanofluids like reduced pumping power due to enhanced heat transfer, minimal clogging, innovation of miniaturized systems leading to savings of energy and cost, over the base fluids, made Choi[3] to regard nanofluids as the next generation heat transfer fluids.

In the past one and a half decade, many researchers have shown interest in studying the enhanced heat transfer characteristics of nanofluids. These include Masuda et al.[4], Eastman et al.[5], Das et al.[6], Xie et al.[7-10], Wang et al.[11], Patel et al.[12]. They used nanoparticles of copper, silver, gold, copper-oxide, alumina, SiC, in base fluids such as water, ethylene-glycol, toulene, etc.. The nanoparticle concentration used by these ranged from 0.11 vol.% to 4.3 vol.%, and the thermal conductivity enhancement observed by them ranged from 10% to 40%. These were very promising data obtained by these workers. There were also studies conducted to account for the unusual behavior, with Eastman[13] coming out to claim that further studies are needed to account for the observed phenomenon. Buongiorno[14], conducted an extensive study to account for the unusual behavior of nanofluids focussing on Inertia, Brownian diffusion, thermophoresis, diffusophoresis, Magnus effects, fluid drainage and gravity settling, and proposed a model incorporating the effects of Brownian diffusion and the thermophoresis. With the help of these equations, studies were conducted by Tzou [15,16], Kim et al.[17-19] and more recently by Nield and Kuznetsov [20,21].

Kuznetsov and Nield [22] studied the onset of thermal instability in a porous medium saturated by a nanofluid, using Brinkman model and incorporating the effects of Brownian motion and thermophoresis of nanoparticles. They concluded that the critical thermal Rayleigh number can be reduced or increased by a substantial amount, depending on whether the basic nanoparticle distribution is top-heavy or bottom-heavy, by the presence of the nanoparticles. The corresponding Horton-Rogers-Lapwood Problem was investigated by Nield and Kuznetsov[20] for the Darcy Model. Agarwal et al.[23] studied thermal instability in a rotating porous layer saturated by a nanofluid for top heavy and bottom heavy suspension considering Darcy Model. Kuznetsov and Nield [24], and Nield and Kuznetsov [21] also studied the effect of local thermal non-equilibrium (LTNE) on the onset of convection in a nanofluid saturated porous medium and in a nanofluid layer. They found that in case of linear non-oscillatory instability, the effect of LTNE can be significant for some circumstances but remains insignificant for a typical dilute nanofluids. From the literature survey it is clear that the following is true about nanofluids:

(a) Nanoparticles influence the thermal conductivity of base fluids in a positive way.

(b) Nanoparticle concentration being present at the lower boundary may result differently from to their presence at the upper boundary.

(c) Rotation is known to show stabilizing effect in viscous fluids without nanoparticles. We need to investigate

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the same for nanofluids too, and also that whether all other parameters behave conventionally.
Thus the aim of the present study is to explore the above possibilities in rotating nanofluid layer. Assuming that the nanoparticles being suspended in the nanofluid using either surfactant or surface charge technology, preventing the agglomeration and deposition of these, in the present article, we study linear thermal instability in a rotating nanofluid layer, under the classical Rayleigh Bénard problem.

2 Governing Equations:

We consider a nanofluid layer, confined between two free-free horizontal boundaries at z=0 and z=d, heated from below and cooled from above. The boundaries are perfect conductors of heat and nanoparticle concentration. The nanofluid layer is extended infinitely in x and y-directions, and z-axis is taken vertically upward with the origin at the lower boundary. The fluid layer is rotating uniformly about z-axis with uniform angular velocity $\mathbf{\Omega}$. The Coriolis effect has been taken into account by including the Coriolis force term in the momentum equation, whereas, the centrifugal force term has been considered to be absorbed into the pressure term. In addition, the local thermal equilibrium between the fluid and solid has been considered, thus the heat flow has been described using one equation model. T_h and T_c are the temperatures at the lower and upper walls respectively such that $T_h > T_c$. Employing the Oberbeck-Boussinesq approximation, the governing equations to study the thermal instability in a nanofluid layer are [14-16,20-21]:

$$\nabla \cdot \mathbf{v} = 0 \tag{1}$$

$$\begin{aligned}\rho_f\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = &-\nabla p + \mu\nabla^2\mathbf{v} \\ &+[\phi\rho_p + (1-\phi)\rho_f(1-\beta(T_f - T_c))]g \\ &+\frac{2}{\delta}(\mathbf{v}\times\mathbf{\Omega})\end{aligned} \tag{2}$$

$$\begin{aligned}(\rho c)_f\left[\frac{\partial T_f}{\partial t} + \mathbf{v}\cdot\nabla T_f\right] &= k_f\nabla^2 T_f \\ &+(\rho c)_p\left[D_B\nabla\phi\cdot\nabla T_f + D_T\frac{\nabla T_f\cdot\nabla T_f}{T_f}\right]\end{aligned} \tag{3}$$

$$\frac{\partial \phi}{\partial t} + \mathbf{v}\cdot\nabla\phi = D_B\nabla^2\phi + \frac{D_T}{T_c}\nabla^2 T_f \tag{4}$$

where $\mathbf{v} = (u, v, w)$ is the fluid velocity. In these equations, ρ is the fluid density, $(\rho c)_f$, $(\rho c)_p$, the effective heat capacities of the fluid and particle phases respectively, and k_f the effective thermal conductivity of fluid phase. D_B and D_T denote the Brownian diffusion coefficient and thermophoretic diffusion respectively, p is pressure, g is the acceleration due to gravity, μ denotes viscosity of fluid. It is assumed that the Brownian motion and the thermophoresis processes remain coherent.

Assuming the temperature(T) and volumetric fraction(ϕ) of the nanoparticles to be constant at the stress-free boundaries, we may assume the boundary conditions on T and ϕ to be:

$$\mathbf{v} = 0,\ \ T = T_h,\ \phi = \phi_1\ at\ z = 0, \tag{5}$$

$$\mathbf{v} = 0,\ \ T = T_c,\ \phi = \phi_0\ at\ z = d, \tag{6}$$

where ϕ_1 is greater than ϕ_0. To non-dimensionalize the variables we take

$$\begin{aligned}(x^*, y^*, z^*) &= (x, y, z)/d, \\ (u^*, v^*, w^*) &= (u, v, w)d/\alpha_f, \\ t^* = t\alpha_f/d^2, \alpha_f &= \frac{k_f}{(\rho c)_f}, \\ p^* = pd^2/\mu\alpha_f,\ \phi^* &= \frac{\phi-\phi_0}{\phi_1-\phi_0},\ T^* = \frac{T-T_c}{T_h-T_c}.\end{aligned}$$

Equations (1)-(6), then take the form (after dropping the asterisk):

$$\nabla\cdot\mathbf{v} = 0, \tag{7}$$

$$\begin{aligned}\frac{1}{Pr}\left(\frac{\partial\mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p + \nabla^2\mathbf{v} - Rm\hat{e}_z \\ +RaT\hat{e}_z - Rn\phi\hat{e}_z + \sqrt{Ta}(\mathbf{v}\times\hat{k}),\end{aligned} \tag{8}$$

$$\begin{aligned}\frac{\partial T_f}{\partial t} + \mathbf{v}\cdot\nabla T_f = \nabla^2 T_f + \frac{N_B}{Le}\nabla\phi\cdot\nabla T_f \\ +\frac{N_A N_B}{Le}\nabla T_f\cdot\nabla T_f,\end{aligned} \tag{9}$$

$$\frac{\partial\phi}{\partial t} + \mathbf{v}\cdot\nabla\phi = \frac{1}{Le}\nabla^2\phi + \frac{N_A}{Le}\nabla^2 T_f, \tag{10}$$

$$\mathbf{v} = 0,\ \ T = 1,\ \phi = 1\ at\ z = 0, \tag{11}$$

$$\mathbf{v} = 0,\ \ T = 0,\ \phi = 0\ at\ z = 1. \tag{12}$$

Here $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$,
$Ta = (\frac{2\mathbf{\Omega}K}{\nu\delta})^2$, is the Taylor′s number,
$Pr = \frac{\mu}{\rho_f\, k_T}$, is the Prandtl number,
$Le = \frac{\alpha_f}{D_B}$, is the Lewis number,
$Ra = \frac{\rho g\beta d^3(T_h - T_c)}{\mu\alpha_f}$,
is the Thermal Rayleigh number
$Rm = \frac{[\rho_p\phi_0 + \rho(1-\phi_0)]gd^3}{\mu\alpha_f}$,
is the basic density Rayleigh number,
$Rn = \frac{(\rho_p - \rho)(\phi_1 - \phi_0)gd^3}{\mu\alpha_f}$,
is the concentration Rayleigh number,
$N_B = \frac{(\rho c)_p(\phi_1 - \phi_0)}{(\rho c)_f}$,
is the modified particle density increment, and

$$N_A = \frac{D_T(T_h - T_c)}{D_B T_c(\phi_1 - \phi_0)}, \text{isthemodifieddiffusivityratio.}$$

3 Basic Solution

At the basic state the nanofluid is assumed to be at rest, therefore the quantities at the basic state will vary only in $z-$direction, and are given by

$$\mathbf{v} = 0,\ T = T_b(z),\ \phi = \phi_b(z)\ p = p_b(z). \tag{13}$$

Substituting eq.(13) in eqs. (9) and (10), we get

$$\frac{d^2T_b}{dz^2} + \frac{N_B}{Le}\frac{d\phi_b}{dz}\frac{dT_b}{dz} + \frac{N_A N_B}{Le}(\frac{dT_b}{dz})^2 = 0, \qquad (14)$$

$$\frac{d^2\phi_b}{dz^2} + N_A\frac{d^2T_b}{dz^2} = 0. \qquad (15)$$

employing an order of magnitude analysis[22], we have:

$$\frac{d^2T_b}{dz^2} = 0, \quad \frac{d^2\phi_b}{dz^2} = 0 \qquad (16)$$

The boundary conditions for solving (16) can be obtained from eqs. (11) and (12) as:

$$T_b = 1,\ \phi_b = 1,\ \text{at z} = 0, \qquad (17)$$

$$T_b = 0,\ \phi_b = 0,\ \text{at z} = 1. \qquad (18)$$

The remaining solution $p_b(z)$ at the basic state can easily be obtained by substituting T_b in eq.(16), and then integrating eq.(8) for p_b.
Solving eq(16), subject to conditions (17) and (18), we obtain:

$$T_b = 1 - z, \qquad (19)$$

$$\phi_b = 1 - z. \qquad (20)$$

4 Stability Analysis

Superimposing perturbations on the basic state as listed below:

$$\mathbf{v} = \mathbf{v}',\ p = p_b + p',\ T = T_b + T', \phi = \phi_b + \phi'. \qquad (21)$$

We consider the situation corresponding to two dimensional rolls for the ease of calculations, and take all physical quantities to be independent of y. The reduced dimensionless governing equations after eliminating the pressure term and introduction of the stream function, ψ, come out as

$$\frac{1}{Pr}\frac{\partial}{\partial t}(\nabla_1^2\psi) = \nabla_1^4\psi - Ra\frac{\partial T_f}{\partial x} + Rn\frac{\partial \phi}{\partial x} + \frac{\partial(\psi, \nabla_1^2\psi)}{\partial(x,z)} + Ta\frac{\partial^2}{\partial z^2}\frac{\partial\psi}{\partial x} \qquad (22)$$

$$\frac{\partial T_f}{\partial t} + \frac{\partial\psi}{\partial x} = \nabla_1^2 T_f + \frac{\partial(\psi, T_f)}{\partial(x,z)} \qquad (23)$$

$$\frac{\partial\phi}{\partial t} + \frac{\partial\psi}{\partial x} = \frac{1}{Le}\nabla_1^2\phi + \frac{N_A}{Le}\nabla_1^2 T_f + \frac{\partial(\psi,\phi)}{\partial(x,z)} \qquad (24)$$

where $\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$.

The equations (22) -(24) are solved subject to idealized stress-free, isothermal, iso-nano concentration boundary conditions so that temperature and nano concentration perturbations vanish at the boundaries, that is

$$\psi = \frac{\partial^2\psi}{\partial z^2} = T = \phi = 0 \text{ at } z = 0, 1 \qquad (25)$$

The choice of these boundary conditions, though not very liable physically, eases the difficulty of mathematical calculations not ignoring the physical effects totally[25,26]. This type of boundary conditions are encountered in some places like in the case of geothermal regions where the fluid layer cannot be isolated from the surroundings avoiding fluid inclusion to its full extent.

The critical Rayleigh numbers for stationary and oscillatory onset of convection and the frequency of oscillations, ω, are obtained as

$$Ra^{st} = \frac{1}{\alpha_c^2}(\delta^6 + Ta\pi^2) + Rn(Le - N_A) \qquad (26)$$

$$Ra^{osc} = Ra^{st} + \omega^2 \left[\frac{RnN_ALe^2}{\delta^4 + \omega_c^2 Le^2} + \frac{Ta\pi^2(1 - 1/Pr)}{\alpha_c^2 Pr(\delta^4 + \omega_c^2/Pr^2)}\right], \qquad (27)$$

$$\omega^2 = \frac{-X_2 + \sqrt{X_2^2 - 4X_1X_3}}{2X_1} \qquad (28)$$

where $\delta^2 = \pi^2 + \alpha_c^2$ and $\alpha_c = \frac{\pi}{\sqrt{2}}$, is the critical wave number.

$X_1 = \frac{T_2 Le^2}{Pr^2}$,

$X_2 = \frac{T_1}{Pr^2} + T_2\delta^4(Le^2 + \frac{1}{Pr^2}) + T_3 Le^2$,

$X_3 = (T_1 + T_2\delta^4 + T_3)\delta^4$.

$T_1 = RnLe\delta^2(1 - Le + N_A)$,

$T_2 = \frac{\delta^4}{\alpha_c^2}(1 + \frac{1}{Pr})$,

$T_3 = \frac{Ta\pi^2\delta^2}{\alpha_c^2}(1 - \frac{1}{Pr})$.

These expressions can be obtained from [20] by dropping the terms pertaining to porous media.
It is quite obvious from eq.(28) that oscillatory convection is possible only when

$$X_2^2 - 4X_1X_3 > 0. \qquad (29)$$

5 Results and Discussion:

Analytical expressions have been obtained for the Rayleigh numbers pertaining to stationary and oscillatory convections. The expression for stationary Rayleigh number is

$$Ra^{st} = \frac{1}{\alpha_c^2}(\delta^6 + Ta\pi^2) + Rn(Le - N_A) \qquad (30)$$

For ordinary fluids, $Le = 0 = N_A$ and non-rotating case $Ta = 0$, we obtain

$$Ra^{st} = \frac{1}{\alpha_c^2}(\delta^6) \qquad (31)$$

which is a classical result for all fluids. Thus it is interesting to observe in this case, that to the value of Rayleigh number for ordinary fluids, we have added a positive term in the form of $Rn(Le - N_A)$. We can say that this is a positive term as the experimentally determined values of Rn

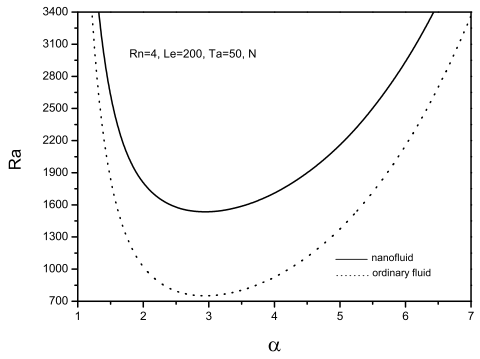

Fig. 1. Comparison of Values of Thermal Rayleigh number for Nanofluid and Ordinary fluids.

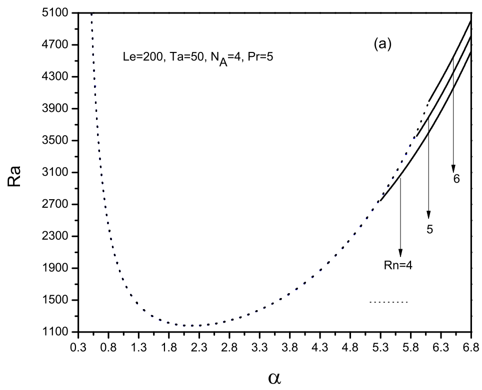

Fig. 2. Linear stability curves showing oscillatory vs stationary convection for different values of $(a)Rn$.

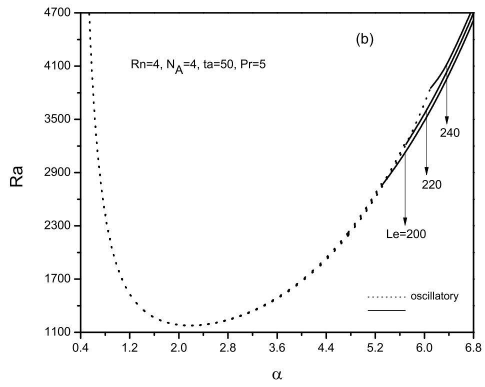

Fig. 3. Linear stability curves showing oscillatory vs stationary convection for different values of $(b)Le$.

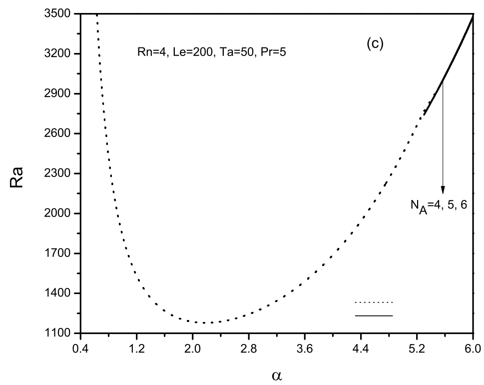

Fig. 4. Linear stability curves showing oscillatory vs stationary convection for different values of $(c)N_A$.

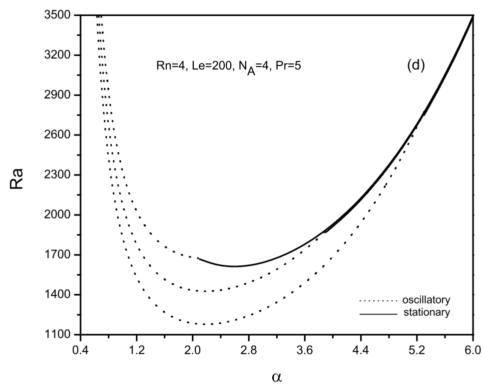

Fig. 5. Linear stability curves showing oscillatory vs stationary convection for different values of $(d)Ta$.

are in the range $1 - 10$, for N_A are $1 - 10$, while of Le are large enough, of the order $10 - 10^6$. Thus the value of Ra_{cr} will be higher in the case of nanofluids than ordinary fluids implying a delay in the onset of convection in this case. Thus to say, more heat is required by nanofluids for convection to start in. This behavior may be attributed to the property of high thermal conductivity of nanofluids which delays the occurrence of density differences across the fluid layer brought about by heating, thus delaying the onset of convection. This implies that the heat transferred by nanofluids will be more than ordinary fluids, making them ideal heat transfer mediums. This fact is also well documented by fig.1.

In figures $2(a)-(b)$ and $3(c)-(d)$, we present the linear stability curves showing oscillatory and stationary modes of convection. Ra^{st} and Ra^{osc} are being plotted against wave number α for $Rn = 4, Le = 200, N_A = 4, Ta = 50$ and $Pr = 5$. From the figures 2 and 3, we observe that initially when α is small, the onset of convection occurs as oscillatory convection. Then at intermediate value of α, the critical value for onset of convection is achieved through oscillatory convection. Finally when α is large, mode of convection changes to stationary convection. Therefore, it can be said that Exchange of Stabilities occurs [25].

In all these curves it is to be noted that the convection sets in oscillatory mode and slowly switches over to the stationary mode. There seems to be negligible effect of the parameters concentration Rayleigh number Rn, Lewis number Le and modified diffusivity ratio N_A, on the overstable regime while on the damped oscillations, these seem to be independent of the effect of modified diffusivity ratio N_A and Taylor number Ta as well. Rn and Le have stabilizing effect on the damped oscillations. An increase in their value increases the critical Thermal Rayleigh number thus stabilizing the system by delaying the onset of convection for demand of more thermal energy.

6 Conclusions:

We considered linear thermal instability analysis in a horizontal rotating layer of a nanofluid, heated from below and cooled from above, incorporating the effect of Brownian motion along with thermophoresis. Further bottom heavy suspension of nano particles has been considered. The effect of various parameters on the onset of thermal instability has been found. We draw the following conclusions:
1. More amount of heat is required by nanofluids than ordinary fluids for convection to start.
2. "Exchange of Stabilities "takes place in this case.
3. Rn, Le and Ta have stabilizing effects on the system.

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