

# Applicability Problem in Optimum Reinforced Concrete Structures Design

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**Abstract.** Optimum reinforced concrete structures design is very complex problem, not only considering exactness of calculus but also because of questionable applicability of existing methods in practice. This paper presents the main theoretical mathematical and physical features of the problem formulation as well as the review and analysis of existing methods and solutions considering their exactness and applicability.

## 1 Introduction

Search for an effective and applicable method for optimum design of the reinforced concrete structures is not a new subject, but the great majority of solutions proposed in literature is aimed at finding minimal weight of a given structure, although the decision making process is usually, if not always, aimed at minimal price. Therefore, material and labour costs should also be important issues in design and construction of the reinforced concrete structures. Besides that, obtained solution should not be only mathematically acceptable, but also applicable in reality, i.e. at the building site.

Common approach in optimum design of the reinforced concrete structures is focused on optimizing the cross-sectional dimensions and total quantity of reinforcement but usually does not include more detailed analysis of the reinforcement pattern and possibility of its proper placing and fixing at the building site. However, in reality, reinforcement design includes detailed specification of many data beyond the determination of area of steel, such as the selection of bar diameters and the number of bars, as well as longitudinal distribution of bars, positioning of bars within critical sections, determination of curtailment points, specification of the size and spacing of stirrups, etc. As a consequence, proposed theoretical methods are usually adequate only for the preliminary analysis but not sufficiently applicable in engineering practice.

The aim of this paper is to present structural, mathematical and practical aspect of optimal reinforced structures structural design, as well as to provide concise and clear overview of the solutions proposed in literature and their basic features in order to enable other researchers to easily find adequate benchmark problems and to develop proper criteria for optimality assessment.

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## 2 Literature Overview

Development of optimization methods in the field of reinforced structures design was closely connected with available computation possibilities. Until the information technology was not developed enough to support very complex calculus models and procedures, problem of optimal reinforced structures design was usually solved by considering only basic variables such as cross-sectional dimensions and total amount of the reinforcement, while the problem of reinforcement bars placing within a concrete members remained almost untouched or was avoided by introducing too generalized assumptions [1]. Friel [2] derived an equation for determining optimal ratio of steel to total concrete area in a singly reinforced beam, while Chou [3] used Lagrange multiplier method for minimizing total cost of the T-shaped beam. Kirsch [4] presented iterative procedure in three levels of optimization for minimizing the cost of continuous girders with rectangular cross section, in which the total amount of the reinforcement is minimized at the first level, cross-sectional dimension are minimized at the second level, while the third level of optimization is minimizing the design moments. Lakshmanan and Parameswaran [5] derived a formula for direct determining of optimal span to cross-sectional depth ratio so the iterative trial and error procedure can be avoided, while Prakash et al. [6] based their cost-minimization method on Lagrangian and simplex methods. Kanagasundaram and Karihaloo [7,8] introduced the crushing strength of concrete as an additional variable along with cross-sectional dimensions and steel ratio to optimize the cost of simply supported and multi-span beams with rectangular and T-sections using sequential linear programming and convex programming. Chakrabarty [9,10] presented cost-optimization method for rectangular beams using the geometric programming and Newton–Raphson method, while Al-salloum and Siddiqi [11] proposed optimal design of singly reinforced rectangular beams by taking the derivatives of the augmented Lagrangian function with respect to the area of steel reinforcement. Coello et al. [12] proposed the cost optimal design of singly reinforced rectangular beam using Genetic Algorithms by considering cross-sectional dimensions and the reinforcement area as variables. More detailed overview of literature on cost-optimization of reinforced concrete structures up to 1998 can be found in [13].

Further development of the information technology enabled researchers to develop innovative methods of design by including the optimality aspect more thoroughly and to include more realistic requirements and optimality criteria, as well as the reinforcement detailing. One of the first papers that deal with reinforcement placing details was presented by Koumouis and Arsenis [14]. This method is based on multi-criterion optimization using Genetic Algorithms for finding a compromise between minimum weight, maximum uniformity and the minimum number of bars for a group of members. After that, researchers have started to introduce reinforcement detailing data as variables in optimization methods, usually by using one of two basic approaches. In the first one, reinforcement spacing demands are included into calculus as constraints [15-18], while the other one uses previously developed data base of possible reinforcement patterns [19-22]. Constraints in the first approach are based on maximum allowable number of reinforcement layers (usually one or two) and maximum allowable number of bars per layer (usually up to four or five). The second approach is in fact simplification of the first one because the data-base of allowable reinforcement patterns is developed by introducing the same limitations and demands proposed by a given code of practice. Only a few papers, such as [23] have no previously adopted arbitrary limitations.

It can be observed that the main problem in comparing efficiency and applicability of different approaches is the fact that they are based on different codes of practice, i.e. on different reinforcement placing rules and restrictions. Because of that, and as opposite of

the steel structures, there is no standard benchmark problems for testing a given method so the parametric sensitivity analysis is the only available tool for applicability assessment.

### 3 Optimization Problem

The first variable studied in optimum structural design is the cross-sectional shape for all members that the structure is composed of that will result in the most suitable stress distribution. In order to obtain mostly uniform forms within one building, the cross-sectional shapes are usually selected from a relatively limited list of possible or available sections. When the shape is determined and fixed, the second and possibly the most challenging task is the placement of reinforcing bars within concrete members, often called detailing. From the optimization point of view, this task generally belongs to the field of topology optimization, where the number of bars, their shape and space position are searched for. The type and form of a chosen parameterization of the shape will determine the computational complexity of a given problem.

If the optimal design of a given reinforced structure is cost-oriented, i.e. if the aim is to minimize the total price of a structure, than the objective function can be defined as:

$$F_{(x)} = V_c P_c + W_s P_s + A_f P_f \quad (1)$$

where  $V_c$  is the volume of concrete,  $W_s$  is weight of steel,  $A_f$  is total area of formwork and  $P_c$ ,  $P_s$  and  $P_f$  are unit price of concrete per  $m^3$ , of steel per kg and of formwork per  $m^2$ , respectively. Prices of the materials include material, fabrication and labor.

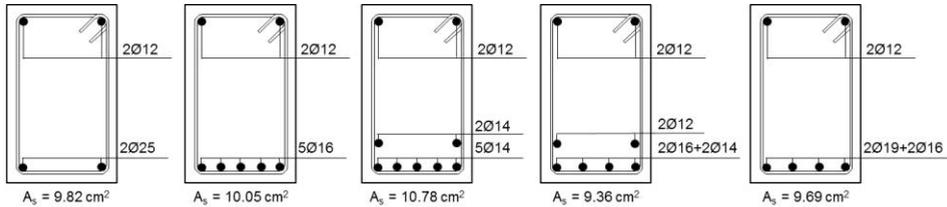
Since the structural geometry (spans and supports positions), material properties and prices and loads (except self-weight) are usually predefined in the designing process, variables in this problem are the cross-sectional dimensions, i.e. width  $b$  and depth  $h$ . Basically, there is no need to include the total steel area in critical cross sections in the variables because it can be calculated according to a given code of practice. However, this aspect of design should not be totally neglected since the complete solution of the problem should also include details about bars diameters and placing scheme, which can be quite a demanding task, especially considering the great variety of possible patterns and possibility of combining bars with different diameters or forming bundles made of same or different bars. It should be noted here that although codes of practice can vary more or less between different countries, they all generally come down to the same set of requirements because what is obligated in one country usually is accepted as a rule of thumb in another and vice versa.

### 4 Applicability Problem

The main problem in all approaches mentioned in the second section is detailing, i.e. forming the reinforcement pattern with sufficient amount of data and details that would be applicable for placing and fixing in situ. Having in mind that reinforcement bars come in more than ten different available diameters, this task is not as easy as it is usually considered. Consequently, there are numerous possibilities for achieving any given required total area of steel. The level of complexity gets even higher if we consider the possibility of using different bars diameters in the same cross section and the fact that reinforcement can also be grouped in bundles consisting of two, three or four same or different bars.

This variety of feasible solutions faces a designer with several further problems. The first and the most obvious one is how to take all possible reinforcement patterns into account. Complexity of this problem can be illustrated by a relatively simple example shown on Figure 1. For a given rectangular cross-section  $b/h = 25/35$  cm calculated

minimal area of steel is  $9.30 \text{ cm}^2$ . All presented patterns are theoretically acceptable, but the computer would always opt for the third one ( $9.36 \text{ cm}^2$ ), although an experienced designer would dismiss it as too complicated. Note that solutions with bundles and combinations of different bundles or of bundles and single bars are not presented, because in that case there would be more than 20 feasible solutions.



**Fig. 1.** Feasible reinforcement patterns for  $A_r=9.30 \text{ cm}^2$ .

## 5 Possible Solutions

One possible solution that can be found in the literature is to use previously developed database consisting of all possible patterns for all available bar diameters placed in cross-sections with different widths. However, proposed databases are usually extremely generalized, simplified and used only to show the basic principle and possibilities of a given methods. The only one that might be considered sufficiently detailed is presented in papers by Govindaraj and Ramasamy [19,20], but it should be noted that their database covers only very limited cross-sectional width span of 10 cm and provides almost 10.000 possible reinforcement patterns. This means that for greater span of allowable widths, for example 20–50 cm, database would consist of hundreds of thousands possible patterns and development of such database would be meticulous and demanding task. Besides that, even if such database would exist, the computer would always opt for the solution that would demand the minimal amount of steel, regardless of it applicability at the building site. But in reality, when faced with a variety of possible and theoretically acceptable solutions, a designer is supposed to find a compromise between the exactness and the applicability at the building site. Because of that, it is recommendable to use some additional criteria for estimating the applicability of a given reinforcement pattern, such as the one proposed in [24].

The second possible solution, and the more applicable one, is to divide the reinforcement problem into two separate and generally independent steps. The first one would be to find only the optimum amount of steel and its centrum, and it would be a part of a chosen optimization method, usually some metaheuristics. After the optimal solution is obtained, the second step would be to find feasible reinforcement pattern that would provide calculated amount of steel and that also could be placed within optimum cross-sectional width. This approach has several benefits. First of all, it does not affect the exactness of calculus because, due to a great number of possible reinforcement patterns, any given steel area can be sufficiently obtained, without compromising exactness of the calculus. Besides that, it would decrease number of variables and restrictions in the optimization process and thus would make it faster and more efficient.

## 6 Conclusion

Theoretical methods for optimum reinforced concrete structures design that can be found in the literature usually consider this problem only as the mathematical one, regardless of applicability of obtained solutions in practise. Purpose of this paper is an attempt to abridge a gap between theory and practise in this field by emphasizing importance of assessing obtained solutions from the practical point of view. The most critical points and problems are emphasized, as well as the possible solutions that would lead to more applicable solutions.

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