

Mathematical Modelling Stress Distribution in a Metallic Plate with an Asymmetrical Notch

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Abstract. The paper is focused on analyzing σ_x , σ_y , τ_{xy} , arising in a steel plate with stress raisers when alternate bend fatigue. The research based on the finite-element method is carried out by software ANSYS. Stress distribution is studied in two planes.

1. Introduction

A sample stressed in a cantilevered manner was used for the purpose of research into high- and low-cycle fatigue of various structural steel classes [1-4]. Stress values around the notch-shaped stress raiser were calculated according to elasticity theory, methods of which are characterized by certain inaccuracy.

Numerous works have been addressed to the behavior of materials with various stress raisers. However, investigation into the behavior of materials with asymmetrical stress concentrators is a complex issue, which is to be solved according to the theory of strength of materials. A lot of software has been developed recently to solve mechanics problems of strength of materials, making the matters at issue more simplified.

2. Problem statement and analysis of results

This work is focused on mathematical modelling stress distribution in a metallic plate with an asymmetrical notch. For this purpose stress distribution in a steel plate with asymmetrical semi-cylindrical notch is analyzed. An assumption is made that a plate body is subject to alternate bending. The data were analyzed by software CAE ANSYS on the base of finite-element method [5 - 8].

A model of the sample was given in a two-dimensional view to simplify the calculation (Fig.1). The problem was solved on assumption of plate elasticity, with one its end fastened, while the other end is subject to the vertically acting force of 1000 N.

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As the defects are accumulated most intensively around the stress raisers the values σ_x , σ_y , τ_{xy} were analyzed at distances ± 10 mm from the stress raiser in the X-direction and in the minimal section of the sample in the Y-direction (Fig.1).

The results of calculation by the software ANSYS are given below, the finite element scheme is assigned according to notch shape and dimensions.

The values σ_x on the sample surface (Fig.2a) vary parabolically, moreover, the minimal value - 1175 MPa is registered in the center of the stress raiser, rising to - 200MPa symmetrically with respect to the distance from the centre. The values σ_x in the center are - 390MPa at a distance of 1.5 mm (Figure 2b) from the surface. Therefore, the values increase on the whole parabola, it is not symmetrical though: for $x=-10$ mm $\sigma_x=-225$ MPa, whereas at a distance of +10 mm $\sigma_x=-100$ MPa. At a distance of 2.4 mm from the surface (Fig.2c) the values σ_x have an opposite sign. At the same time σ_x in the stress raiser center equals 200MPa, σ_x is -125MPa for $x=-10$ mm, and - 10MPa for $x=+10$ mm, respectively. At a distance 3.1 mm (Figure 2d) the values σ_x for $x=+10$ mm and $x=-10$ mm are quite the same. However, the increase up to $\sigma_x=610$ MPa is registered in the stress raiser center. The stresses on the sample surface in the minimal section of the stress raiser (Fig.2a and 2e) have a quite similar modulus, but their nature becomes different: tension stresses transform into compression ones.

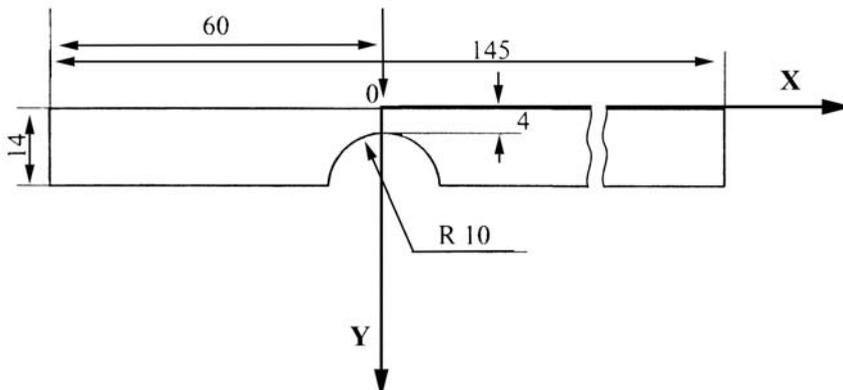


Fig.1. Plate geometry in Cartesian coordinate system

Focusing on the varying values σ_y , an emphasis is to be laid on proximity of the plot given in Fig.2 a, since the values σ_y are zero on the surface due to the dispersion of values when calculating CAE ANSYS.

Analyzing the distribution σ_y at a distance of 1.5 mm from the surface (Fig.3b) a parabolic distribution is detected with the maximum in the center ($\sigma_y=38$ MPa) and minimums at the extremities of intervals under consideration ($\sigma_y=-10$ MPa).

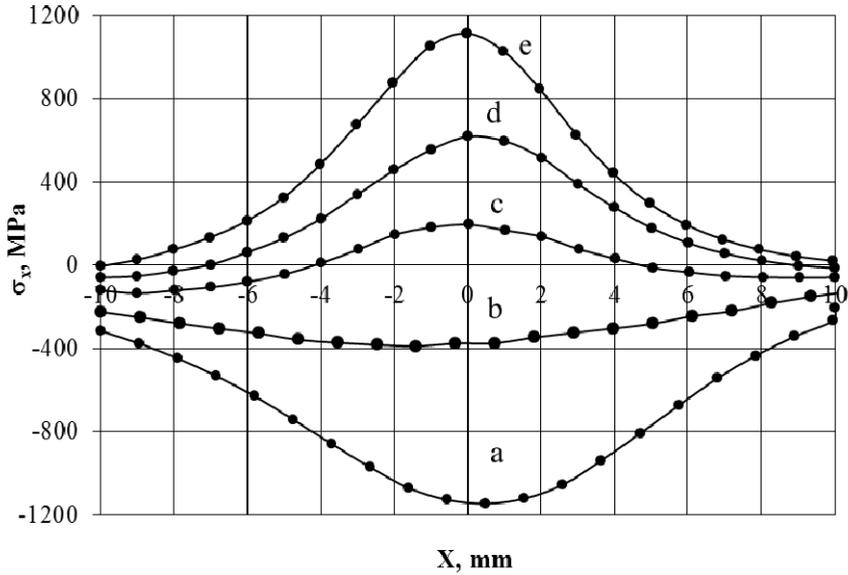


Fig.2. Stress distribution σ_x in the plate. Sections at the distances from the surface: a) on the surface, b) 1.5 mm, c) 2.4 mm, d) 3.1 mm, e) 4 mm.

The distribution σ_y is nearly the same at a distance of 2.4 mm from the surface (Fig.3c), the maximum increases up to 60 MPa and the minimum decreases to -20 MPa. It is similar for the distance of 3.1 mm: -73 MPa and 28 MPa, respectively (Fig.3d).

Strong changes are detected at a distance of 4 mm (Fig.3e). Two maximums are apparent around the stress raiser at distances of $x=-3$ mm and $x=3$ mm, equaling 80 MPa and 42 MPa, respectively. In the center of a stress raiser there is a minimum with the value 21 MPa. A probable cause of it is the surficial position of the curve to be analyzed in this area.

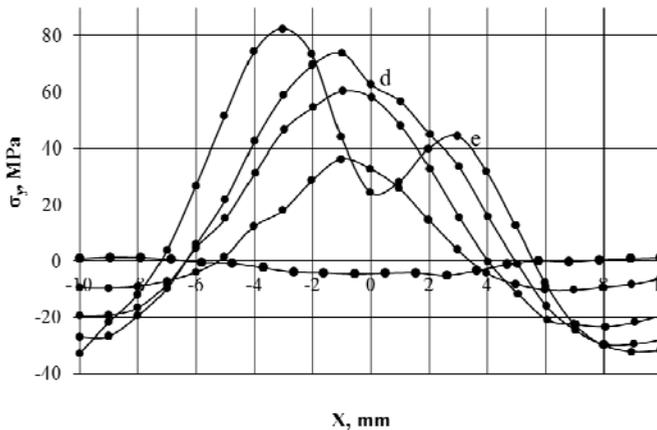


Fig.3. Stress distribution σ_y in the plate. Sections at the distances from the surface: a) on the surface, b) 1.5 mm, c) 2.4 mm, d) 3.1 mm, e) 4 mm.

The distribution of tangential stresses τ_{xy} seems to be more interesting (Fig.4). The values τ_{xy} approximate to zero on the surface at the distances $x=-10$ mm, in the center and for $x=10$ mm (Fig.4a). The minimum is detected at a distance $x=-3$ mm ($\tau_{xy}=-20$ MPa), and the

maximum 18MPa is located symmetrically with the center at a distance $x=4$ mm. At a distance of 1.5 mm from the surface (Figure 4 b) the distribution τ_{xy} differs in numerical values only. In the stress raiser center the values are in the range of zero, being 3MPa. However, at a distance of 2.4 mm deep into the surface (Fig.4c) the plot is displaced $\tau_{xy}(X)$ because the value in the concentrator center $\tau_{xy}=50$ MPa, and the τ_{xy} plot – X- axis cross point is displaced too. Positions and values of the maximums and minimums are not displaced significantly (Fig.4c).

The second highly pronounced maximum 50MPa is registered for $x=-1$ mm at a distance 3.1 mm from the surface (Fig.3d). The maximum, which was seen previously in Fig.4c, is displaced to $x=7$ mm ($\tau_{xy}=78$ MPa). At a distance of 2 mm from the center is the minimum ($\tau_{xy}=18$ MPa) registered.

At a distance 4 mm from the surface (Fig.3e) specifics of tangential stress distribution is similar to that registered when considering σ_x and σ_y . In this case negative area of the plot is almost mirrored relative to the axis X (Fig.3c), but the maximum on the plot is 150 MPa, corresponding to $X=-2$ mm, whereas the minimum is -95MPa for $x=+2$ mm.

This paper provides consideration of stress distribution over the minimal section of the sample (Fig.4), since there the maximal stresses are registered.

The σ_x versus Y curve is a linear one (Fig.5a) and expressed by the interpolation equation $\sigma_x=592Y-1188$, where Y is given in millimeters, and σ_x in MPa. The σ_y and σ_{xy} versus Y dependences (Fig.5b and 5c) are of the same type with maximums in the range 2.5-3 mm. For instance, the maximal value σ_y equals 60MPa and τ_{xy} is 49MPa, respectively.

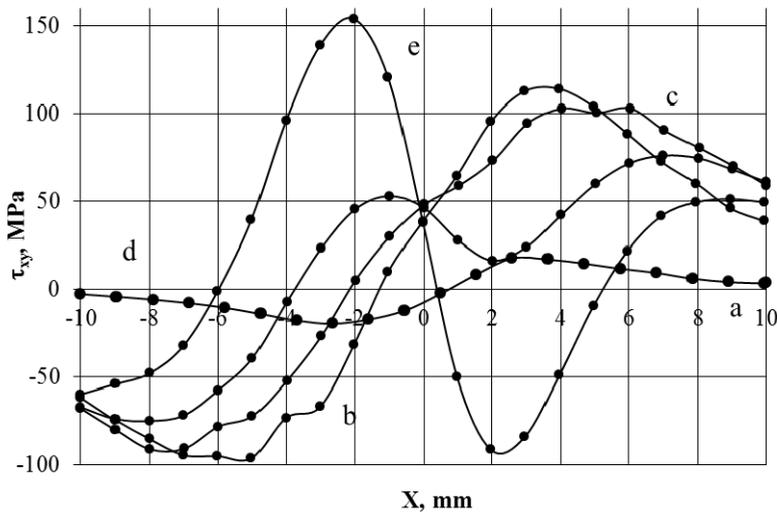


Fig.4. Stress distribution τ_{xy} in the plot. Sections at distances from the surface: a) on the surface, b) 1.5 mm, c) 2.4 mm, d) 3.1 mm, e) 4 mm.

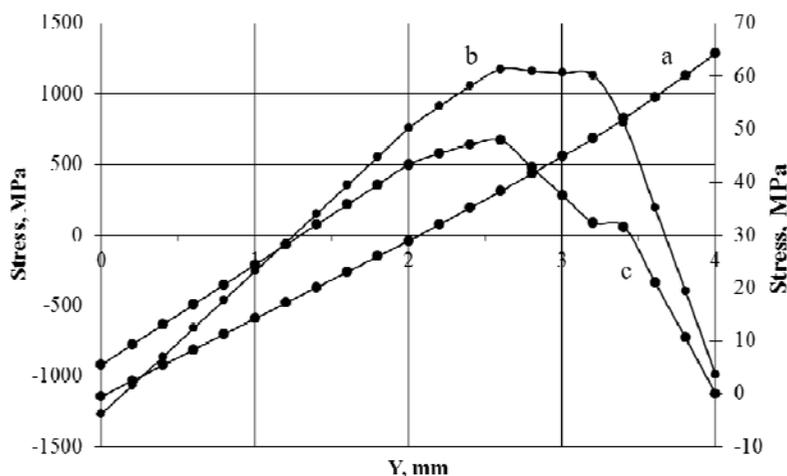


Fig.5. Stress distribution σ_x (a), σ_y (b), and τ_{xy} (c) in the plate. The curves b and c are in the right ordinate of the plot.

3. Conclusions

To sum up, a computer model of stress distribution in a sample with asymmetrical notch makes it obvious that notch displacement is the cause of inflection points and even double peaks on stress distribution curves. Therefore, various rupture points are possible when cyclic stressing.

4. Acknowledgement

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