Imperfect Preventive Maintenance Model Study Based On Reliability Limitation

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Abstract. Effective maintenance is crucial for equipment performance in industry. Imperfect maintenance conform to actual failure process. Taking the dynamic preventive maintenance cost into account, the preventive maintenance model was constructed by using age reduction factor. The model regards the minimization of repair cost rate as final target. It use allowed smallest reliability as the replacement condition. Equipment life was assumed to follow two parameters Weibull distribution since it was one of the most commonly adopted distributions to fit cumulative failure problems. Eventually the example verifies the rationality and benefits of the model.

Introduction

Equipment is the major asset of the modern enterprise. Preventive maintenance is an ideal repair method in eliminating fault. The traditional repair assume that the equipment will return to perfect state or minimal repair state after repair, which is not consistent with the actual situation. Most of the equipment will restore to a state between them, which is called imperfect maintenance[1]. For example, the proper adjustment of the engine is imperfect maintenance, because the engine will not return to a new state but a state improved a lot after the adjustment.

Pham and Wang divided imperfect maintenance into eight categories: (p,q) rule, $(p(t),q(t))$ rule, improvement factor method, the virtual age method, impact model method, (α,β) rule, composite (p,q) rules and other[2]. Malik proposed the concept of improvement factor[3]. It can measure age improvement degree and failure rate after repair. Chan and Shaw gives two types of failure rate recovery mode that depends on the length of service and maintenance effect[4]: fixed reduced type and ratio of reduced type.

The traditional preventive maintenance method calculates the maintenance cycle according to the average life, reliability or single repair cost. It doesn't consider the balance between reliability and repair cost. By introducing the reliability constraints, the paper establishes a better preventive maintenance model considering both cost and reliability.

Imperfect preventive maintenance model

The following are several basic assumptions of the model:

(1) Equipment conduct (N-1)[th] imperfect maintenance then it will be replaced in the Nth in the infinite time domain.

(2) Equipment will conduct minimum repair in the periodic preventive maintenance interval h if failure occur. The minimum repair can only make equipment return to a normal operation state, it doesn't change failure rate.

(3) The effect of imperfect preventive maintenance can be achieved by age reduction factor. Age reduction factor is influenced by equipment age and preventive maintenance cost rate. It is a dynamic value.

(4) Single preventive maintenance cost shows a linear relationship with repair numbers. It is expressed as $c_{pmi} = c_f + ic_v$ .Equipment is worn with increase of device age, each maintenance time becomes longer, which is $t_{pmi} = (1/k) \cdot i \cdot h$, $1/k$ is time adjustment parameter.

(5) Single minimal repair cost c_{mr} and single repair time t_{mr} are constant. Production loss cost per unit time c_s is constant.

Figure.1 is the periodic imperfect preventive maintenance strategy model diagram.

Figure 1. imperfect preventive maintenance model

Equipment operation is a degradation process. Imperfect preventive maintenance can improve the life of

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equipment. Age reduction factor η_i is used to indicate the recovery degree after the preventive maintenance [5].

$$\eta_i = \left(a \cdot \frac{c_{pmi}}{c_{pr}} \right)^{b \cdot i}, i = 1, \ldots, N-1 \qquad (1)$$

a is repair cost adjustment factor, and $0 \le a \le c_{pr}/c_{pmi}$. c_{pmi} is single preventive maintenance cost. c_{pr} is one replacement cost, and $c_{pmi} \le c_{pr}$. b is adjustment coefficient of repair times, and $0 < b < 1$.

We can use age reduction factor to obtain the effective age. The bigger the age reduction factor is, the smaller effective age is. This paper uses the age proportion reduced model, only the maintenance of the last age can get the percentage recovery. The effective age before and after the maintenance is expressed as formula (2).

$$T_1^- = h, T_1^+ = (1-\eta_1)h \qquad (2)$$

Similarly, the effective age before and after the i th maintenance is shown in formula (3).

$$T_i^- = (i - \sum_{j=1}^{i-1} \eta_j)h, T_i^+ = (i - \sum_{j=1}^{i} \eta_j)h \qquad (3)$$

After implementation of $(N-1)^{th}$ prevention maintenance, replacement will be implemented in the Nth maintenance cycle. The preventive replacement timeline is shown in formula (4).

$$T_N^- = (N - \sum_{j=1}^{N-1} \eta_j)h \qquad (4)$$

The Weibull distribution form can describe the failure regularity of mechanical equipment, it is widely used in practical production. The failure rate expression of the distribution is shown in formula (5). β is the shape parameter. α is a scale parameter.

$$\lambda(t) = \frac{\beta}{\alpha} \left(\frac{t}{\alpha} \right)^{\beta-1} \qquad (5)$$

Equipment failure numbers w_i in the ith imperfect maintenance cycle can be obtained according to the failure rate.

$$w_i = \int_{T_{i-1}^-}^{T_i^-} \lambda(t) dt$$
$$= \left(\frac{h}{\alpha} \right)^\beta \left[\left(i - \sum_{j=1}^{i-1} \eta_i \right)^\beta - \left(i - 1 - \sum_{j=1}^{i-1} \eta_i \right)^\beta \right] \qquad (6)$$

Total costs C_{total} include total preventive repair costs C_{pm}, total minimum repair costs C_{mr}, production loss costs C_p and replacement costs C_{pr}. It is expressed as formula (7).

$$C_{total} = C_{pm} + C_{mr} + C_p + C_{pr}$$
$$= \sum_{i=1}^{N-1} c_{pmi} + \sum_{i=1}^{N} c_{mr} w_i + c_s \left(\sum_{i=1}^{N-1} t_{pmi} + \sum_{i=1}^{N} t_{mr} w_i \right) + C_{pr}$$
$$= \sum_{i=1}^{N-1} (c_f + ic_v) + \sum_{i=1}^{N} c_{mr} w_i + c_s \left(\sum_{i=1}^{N-1} \frac{i}{k} \cdot h + \sum_{i=1}^{N} t_{mr} w_i \right) + C_{pr}$$
$$= (c_{mr} + c_s t_{mr}) \sum_{i=1}^{N} w_i + \left[(N-1)c_f + \frac{N(N-1)c_v}{2} \right] + c_s \frac{N(N-1)h}{2k} + C_{pr}. \qquad (7)$$

Equipment total running time T before replacement includes preventive maintenance interval h and preventive maintenance time. The repair cost rate $C(h, N)$ is expressed as formula (8). It is a Concave function. This paper omits the proof process.

$$C(h,N) = \frac{C_{total}}{T}$$
$$= \frac{(c_{mr} + c_s t_{mr}) \sum_{i=1}^{N} \left(\frac{h}{\alpha} \right)^\beta \left[\left(i+1 - \sum_{j=1}^{i} \eta_{j-1} \right)^\beta - \left(i - \sum_{j=1}^{i} \eta_{j-1} \right)^\beta \right] + \left[(N-1)c_f + \frac{N(N-1)c_v}{2} \right] + c_s \frac{N(N-1)h}{2k} + C_{pr}}{Nh + \frac{N(N-1)h}{2k}}. \qquad (8)$$

This paper adopts periodic imperfect preventive strategies under failure rate limits. It is time to conduct preventive replacement when the equipment reliability is equal to the predetermined limit level R_{min}. Equipment failure process obeys Weibull distribution. T_N^- is replacement time. Then h can be obtained through the following methods.

$$R(T_N^-) = 1 - F(T_N^-) = \exp\left[-\left(T_N^- / \alpha \right)^\beta \right] = R_{min} \qquad (9)$$

$$h = \frac{\alpha(-\ln R_{min})^{1/\beta}}{N - \sum_{j=1}^{N-1} \eta_j} = \frac{\alpha(-\ln R_{min})^{1/\beta}}{N - \sum_{j=1}^{N-1} \left(a \cdot \frac{c_{pmi}}{c_{pr}} \right)^{b \cdot i}}. \qquad (10)$$

Set $N = 1$, get corresponding h, then get $C(h, N)$; set $N = N+1$; If $C(h, N) > C(h, N+1)$, record $C(h, N+1)$ as the minimum new cost.

Verification Example

Equipment fault time obey the Weibull distribution. The parameter is shown in Table 1.

Table 1. model parameter

parameter	value	parameter	value
α	221	c_f	5000
β	3	c_v	100
b	0.002	k	500
c_{pr}	4000000	R_{min}	0.6
c_s	5000	c_{mr}	8000
t_{mr}	0.4		

Age reduction factor curve is shown in Figure.2. Age reduction factor decreases with increasing device age gradually. The age reduction factor of first imperfect preventive maintenance is $\eta_1 = 0.9868$ .while the 50th age reduction factor is $\eta_{50} = 0.5493$, the equipment only can restore to the state of 54.93%.

Using iterative algorithm I obtain the maintenance cost of different repair numbers. The relationship between repair cost rate and the maintenance numbers is shown in Figure.3. Repair cost rate decrease at first with the increase of N but it starts to increase in the end. This is consistent with the actual situation. It gets the minimum value when $N = 13, h = 89.8622$ .The maintenance interval h which changes with repair numbers is shown in Figure.4.

Figure 2. age reduction factor changes with N

Figure 3. repair cost rate changes with N

Figure 4. h changes with N

Conclusion

Equipment can't become as good as new after maintenance. The article construct imperfect preventive maintenance model based on periodic prevention. Age reduction factor is used to describe the effective age after maintenance. By adding reliability constraint condition, such policies can ensure that reliability of the equipment is kept above a certain predetermined level and thus the risk of sudden functional failure is low. The method mentioned in the article has certain reference significance on the costs optimization of enterprise. The imperfect preventive maintenance model can quantitively determine the maintenance interval and provide suggestion for the operation of the company maintenance policy.

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