

# Numerical simulation for Human Quiet Standing System

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**Abstract.** Human quiet standing system was investigated with a 2-joint (ankle and hip) sagittal model of body movement with perturbation noise. In the case of the noise perturbations, the influence on the stability of time delay is discussed numerically in this paper. Analytical analyses on the mean of the station response reveal that the presence of time delay leads to periodic vibration in parameter space, and the mean value increases as the reflexes delay increasing. But the mean square of the stationary response varies with not only the noise but also the delay. We find the stochastic factor has no affection on the stability of human, but it will make the amplify oscillations diffusion, which can be seen from the shape change of the figure on the stationary joint probability density and the stationary probability density. All these conclusions are shown by the numerical simulation. In conclusion, the influence of reflexes delay and perturbation on the stability and quality of stance has been studied by successfully combined with the biomechanical concepts and tools..

## 1 Introduction

A large numbers of publication work [1-4] at the equilibrium stability of upright standing were based on the simple biomechanical model of the human body as single inverted pendulum. The multi-joint properties of the body were rarely taken into account in the previous studies. Furthermore, the delay was not taken into account in the feedback loop [5, 6]. However, the existence of delays essentially complicates postural and requires the strict adjustment of the feedback gains to maintain upright [7]. Also, the human nervous system operates in a very noisy environment and hence the noise effects on human postural. The noise perturbations have been considered into the single-link biomechanical model of the human body, but not in the double-link biomechanical model. In the present study, we take into account both delay in the feedback loop and stochastic perturbations in the double-joint body configuration. In 2009, a spatial double inverted pendulum moving in a three dimensional space was considered to be a model of human standing postural control by Xinjilefu [8]. Postural control was possible by on-line minimization of the system Lagrangian have been showed. The influence of reflexes delay and perturbations on the stability and quality of stance has been not studied by numerical method.

As one can see in [9], how the delay effect on the human posture during stance in the double pendulum with perturbation have been discussed. Is it consistent with [9]? Motivated by the above findings, the purpose of this study is to show the reflex delay and noise in the double pendulum system in numerical simulation.

This Letter is organized as follows. In Section 2, the effects of time delay and additive noise on dynamics of

system (1) are discussed in detail by numerical Simulation. Finally, conclusions are drawn.

## 2 Numerical simulation

The equations of human quiet standing have been built in [9].

$$\begin{cases} (\frac{1}{3}m_1 + m_2)l_1^2\ddot{\theta}_1 + \frac{1}{2}m_2l_1l_2\ddot{\theta}_2 \cos(\theta_1 - \theta_2) + \frac{1}{2}m_2l_1l_2\dot{\theta}_2^2 \sin(\theta_1 - \theta_2) \\ -\mu_2(\dot{\theta}_2 - \dot{\theta}_1) - (\frac{1}{2}m_1 + m_2)gl \sin \theta_1 + \alpha_1\theta_1 + \mu_1\dot{\theta}_1 - \alpha_2(\theta_2 - \theta_1) \\ = \beta f(\theta_1(t-\tau)) + \gamma \eta(t), \\ \frac{1}{2}m_2l_1l_2\ddot{\theta}_1 \cos(\theta_1 - \theta_2) + \frac{1}{3}m_2l_2^2\ddot{\theta}_2 - \frac{1}{2}m_2l_1l_2\dot{\theta}_1^2 \sin(\theta_1 - \theta_2) \\ - \frac{1}{2}m_2gl_2 \sin \theta_2 + \alpha_2(\theta_2 - \theta_1) + \mu_2(\dot{\theta}_2 - \dot{\theta}_1) = 0. \end{cases} \quad (1)$$

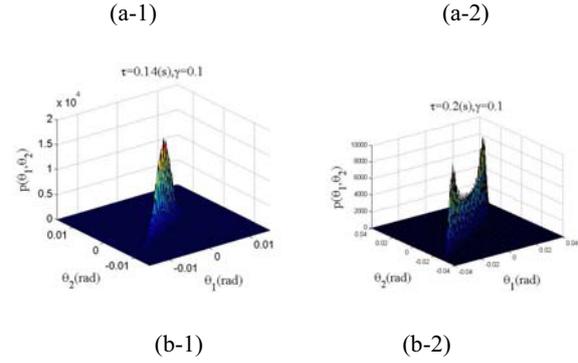
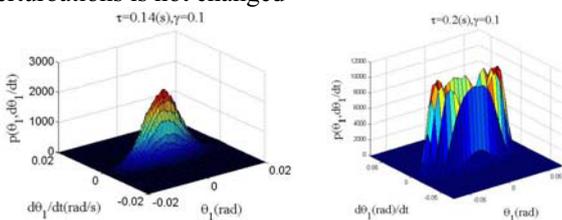
In [9], in the case of the combined effects of the noise, the influence on the stability of the reflexes delay will be firstly discussed. The stationary response of Eq. (1) will be calculated to study the influence. Because of the system is infinite dimensional stochastic system, it is difficult to calculate the accurate stationary response. Here, the approximate stationary response of stochastic oscillatory near critical delay of deterministic system is derived employing multi-scale method to find the relation with the delay and noise.

The influence of the reflexes delay and perturbation on the stability is found. To verify adequately the obtained results of the influences on the human during standing and explain the mechanism of human maintain upright stance, the stationary probability density function figures and time history plane portrait are analyzed as following. All the parameter values are the same as [9] taken.

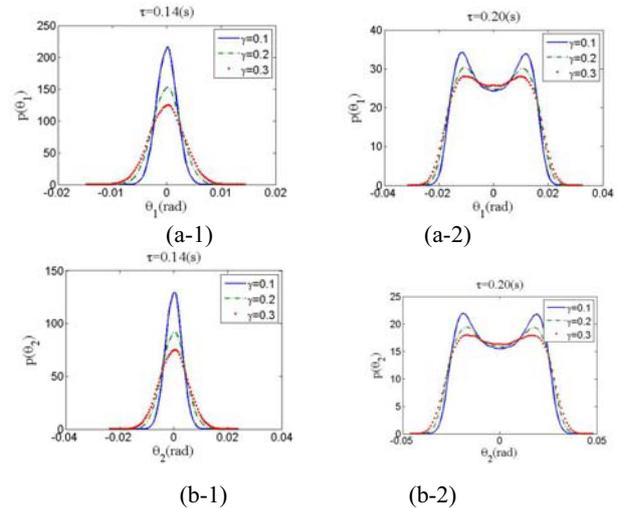
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Fig. 1 and Fig. 2 show the shape of the joint stationary probability density and the stationary probability density varies with the different reflex delay. For the human reflex delay  $\tau = 0.14 < \tau_c = 0.184$ , the shape of the stationary joint probability density  $p(\theta_1, \dot{\theta}_1)$  and  $p(\theta_1, \theta_2)$  have a single-peak bell shaped in Fig. 1(1). In Fig. 2(1), the function  $p(\theta_1)$  have only a single maximum at the point (0,0) too. It shows the probability when human at upright position is maximum. The human during standing are always controlled to the upright equilibrium position even if we are perturbed by the noise, as is shown in Fig. 5(1) and Fig. 6(1). The random trajectories of human movement perturbations noise are concentrated in the vicinity of point (0,0). The equilibrium point (0,0) is stable, which is natural to name this type of stochastic attractor by stochastic equilibrium point.

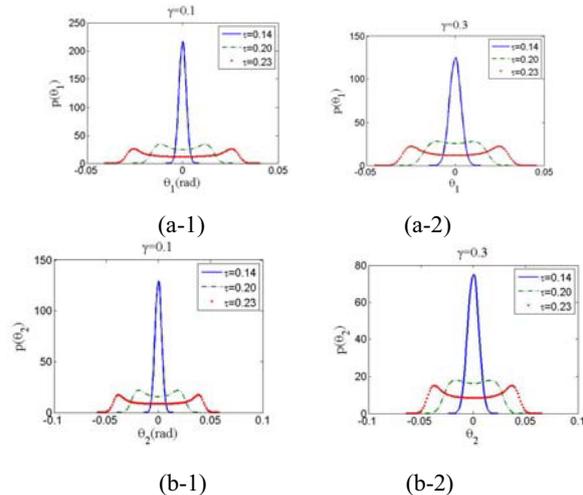
We can see that in this regime the additive noise plays the dominant role in amplifying the oscillations. It reflects mechanism of human maintain upright stance. However, if human reflexes delay  $\tau = 0.20$  is beyond the critical delay, the shape of the joint stationary probability density  $p(\theta_1, \dot{\theta}_1)$  and  $p(\theta_1, \theta_2)$  become a crater-like form but not single-peak bell shaped form. Its center locates at the equilibrium point (0,0) in Fig.4(b). In Fig. 2(2), the function  $p(\theta_1)$  possesses a maximum at the points of deterministic cycle and a single minimum at the point(0,0). It shows that the probability is small when human during standing is close to upright position. Furthermore, the probability density is smaller as the human stand is more close to upright position. At this moment, as above section stated human will start to periodically vibration. The random trajectories of human quiet standing system (1) are concentrated in the vicinity of deterministic cycle, as is shown in Fig.4(2) and Fig.5(2). The equilibrium point (0,0) is unstable and a stochastic limit cycle appeared, as is shown in Fig.6. This phenomenon varies with reflexes delay is be defined as phenomenological bifurcation in random vibration theory. Fig. 2 and Fig. 3 shows the variation of the parameter  $\gamma$  of noise perturbation doesn't change the location of the extremal points of the related probability density function. In other words, noise perturbation has no influence on the stability of human. An increase of  $\gamma$  just only results into a growth of the dispersion of the stochastic human quiet standing system around the deterministic attractors (i.e. around the stable point (0,0) for  $\tau < \tau_c$  and around the stable limit cycle for  $\tau > \tau_c$ ). Note that the deterministic bifurcation value  $\tau_c = 0.184$  under additive Gaussian noise perturbations is not changed



**Fig.1.** The stationary joint probability density of human standing system (1) for  $\gamma = 0.1$ , (a-1), (b-1)  $\tau = 0.14$ , (a-2), (b-2)  $\tau = 0.20$ , the other parameters are set as Tab.1, (a): The stationary joint probability density  $p(\theta_1, \dot{\theta}_1)$  the angular displacement and velocity in the ankle; (b): the stationary joint probability density  $p(\theta_1, \theta_2)$  of the angular displacement in the ankle and in the hip



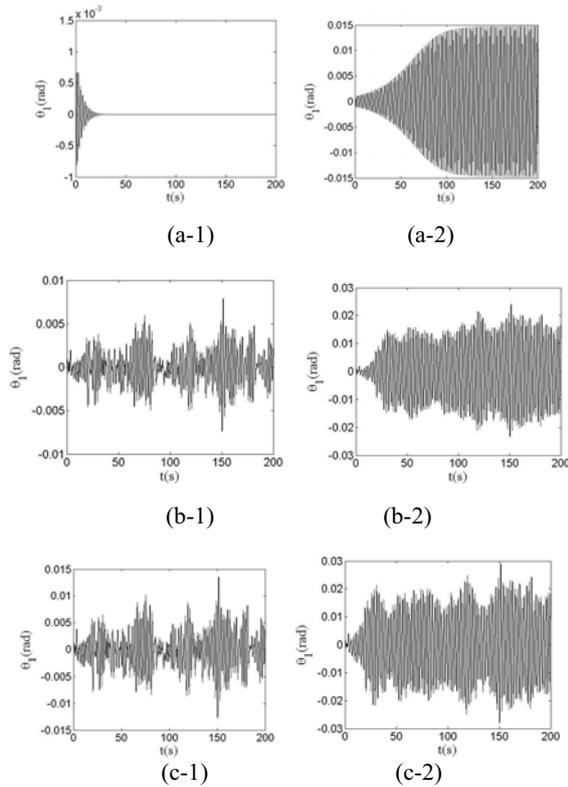
**Fig. 2.** (a): the stationary probability density  $p(\theta_1)$  of the angular displacement in the ankle; (b): the stationary probability density  $p(\theta_2)$  of the angular displacement in the hip for  $\gamma = 0.1$  ( solid),  $\gamma = 0.2$  (dashed),  $\gamma = 0.3$  (dotted), (a-1),(b-1):  $\tau = 0.14$ , (a-2), (b-2):  $\tau = 0.20$ .



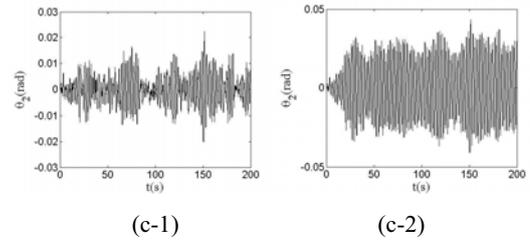
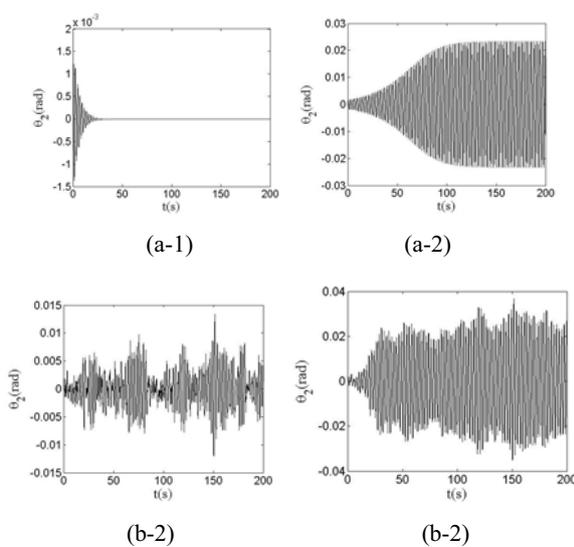
**Fig. 3.** (a): the stationary probability density  $p(\theta_1)$  of the

angular displacement in the ankle; (b): the stationary probability density  $p(\theta_2)$  of the angular displacement in the hip for  $\tau = 0.14$  (solid),  $\tau = 0.20$  (dashed),  $\tau = 0.23$  (dotted).

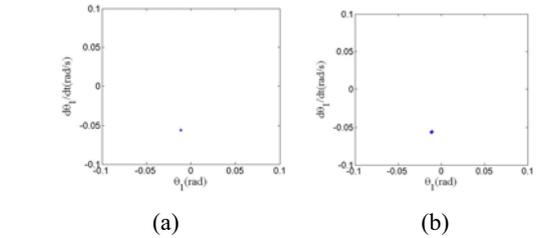
(a-1) (b-1):  $\gamma = 0.10$ , (a-2) (b-2):  $\gamma = 0.30$ .



**Fig. 4.** Time history plane portrait in the ankle for (a-1):  $\tau = 0.14$ ,  $\gamma = 0$ ; (a-2):  $\tau = 0.20$ ,  $\gamma = 0$ ; (b-1):  $\tau = 0.14$ ,  $\gamma = 0.1$ ; (b-2):  $\tau = 0.20$ ,  $\gamma = 0.1$ ; (c-1):  $\tau = 0.14$ ,  $\gamma = 0.3$ ; (c-2):  $\tau = 0.20$ ,  $\gamma = 0.3$ .



**Fig. 5.** Time history plane portrait in the ankle for (a-1):  $\tau = 0.14$ ,  $\gamma = 0$ ; (a-2):  $\tau = 0.20$ ,  $\gamma = 0$ ; (b-1):  $\tau = 0.14$ ,  $\gamma = 0.1$ ; (b-2):  $\tau = 0.20$ ,  $\gamma = 0.1$ ; (c-1):  $\tau = 0.14$ ,  $\gamma = 0.3$ ; (c-2):  $\tau = 0.20$ ,  $\gamma = 0.3$ .



**Fig. 6.** Poincaré mapping section when (a):  $\tau = 0.20$ ,  $\gamma = 0$ ; (b):  $\tau = 0.20$ ,  $\gamma = 0.1$ .

### 3 Conclusions

In this paper, in the case of the combined effects of the noise, the influence on the stability of time delay is discussed by numerical simulation. The results show human reflexes delay still has a great influence on stability in human quiet standing. The human will be still stable when reflexes delay is small in the case under perturbation noise. While reflex delay exceeds the critical time delay, human will start to periodic vibration. And the first of stationary response will increase as human reflexes delay increasing. This influence of delay on the human movement is same as [9]. By analysis the shape change of the figure on the stationary probability density, we can also find the stochastic factor has no affection on the stability of human, but it makes the amplify oscillations diffusion, as is shown in Fig.6. All these results are particularly important in the clinical realm.

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