

# Parameters extraction for perovskite solar cells based on Lambert W-function

Junyu Ge, Manlin Luo, Wanli Pan, Na li and Wei Peng

School of Physics and Electronics, Hunan University, Changsha 410082, China

**Abstract.** The behaviors of the solar cells are decided by the device parameters. Thus, it is necessary to extract these parameters to achieve the optimal working condition. Because the five-parameter model of solar cells has the implicit equation of current-voltage relationship, it is difficult to obtain the parameters with conventional methods. In this work, an optimized method is presented to extract device parameters from the actual test data of photovoltaic cell. Based on Lambert W-function, explicit formulation of the model can be deduced. The proposed technique takes suitable method of selecting sample points, which are used to calculate the values of the model parameters. By comparing with the Quasi-Newton method, the results verify accuracy and reliability of this method.

## 1 Introduction

Compared with other energy resources, solar energy is sustainable and far more abundant than our projected energy needs as a species. Thus, it is believed as the largest source of electricity in the future<sup>[1]</sup>. Solar energy can be used in photothermal conversion, photochemical conversion and photovoltaic conversion. Using the photovoltaic effect, solar cells convert light into electric current, which is convenient for energy storage and transport. This advantage makes solar cell a promising electrical device. Perovskite solar cell has become one of the most widely used solar cells due to its high power conversion efficiency and low material costs<sup>[2]</sup>. A new category of perovskite solar cell has been put into use these years, with remarkable rapid progress made in energy conversion efficiency. In 2014, its energy conversion efficiency has increased to a confirmed data of 17.9%. The unique quality of perovskite solar cell contributes to the outstanding performance<sup>[3]</sup>.

In order to do the quantitative analysis of solar cells, it is significant to utilize a photovoltaic model which can represent the electrical characteristics of the solar cells accurately. Among different solar cell models used to describe the I-V relationship, single and double diode models are used practically. The single-diode model is now most widely used to describe solar cells' behavior. It is the simplest model with only five parameters and satisfied with most of the applications<sup>[4]</sup>. The five parameters are listed as follows: the series photocurrent ( $I_{ph}$ ), the resistance ( $R_s$ ), the shunt resistance ( $R_{sh}$ ), the ideality factor ( $n$ ) and the saturation current ( $I_0$ ).

This single-diode model is used to evaluate solar cells' performance, optimize device structure and fabrication process as well.

There are different methods to extract model parameters, such as genetic algorithm, differential evolution method and particle swarm optimization<sup>[5]</sup>. Generally, these algorithms require much time and computational cost to get high accuracy<sup>[6]</sup>. Herein, we present a method based on Lambert W-function to extract perovskite solar cells' parameters<sup>[7]</sup>. Due to the simple calculation process, this approach has high accuracy and calculation speed.

The key factor for precise result and efficient computation is choosing points. Points selected around short-circuit current ( $I_{sc}$ ), open-circuit voltage ( $V_{oc}$ ) and max power point have significant effects for the outcomes. In this paper, distinction between different point selection methods is studied. Moreover, the initial values of the parameters have no impact on the results<sup>[8]</sup>.

## 2 Methods

All the work is based on a single-diode model here. The single-diode model is depicted in Figure 1. It is well known that the output current of this model can be described as<sup>[9]</sup>:

$$I = I_{ph} - I_0 \left( \exp\left(\frac{V + IR_s}{nV_{th}}\right) - 1 \right) - \frac{V + IR_s}{R_{sh}}, \quad (1)$$

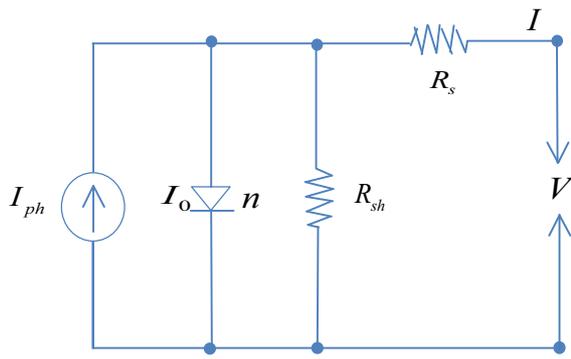


Figure 1. Single-diode model of solar cells.

The  $V_{th}$  is given by  $V_{th}(T) = \frac{kT}{q}$ , where  $q$  is the electron charge ( $1.60217646 \times 10^{-19}$  C),  $k$  is the Boltzmann constant ( $1.3806503 \times 10^{-23}$  J/K) and  $T$  is the Kelvin temperature of the p-n junction<sup>[10]</sup>.

The Eq.(1) is an implicit non-linear transcendental function which has no direct general analytical solution. Lambert W-function is used to obtain the explicit formulation of the model, and this function can be described as:

$$W(x) \exp[W(x)] = x \quad (2)$$

"W(x)" represents the usual short-hand notation for the principal branch of Lambert W-function.

In our work,  $R_{sh} \gg 0$ ,  $\frac{V + I R_s}{R_{sh}} \approx 0$ . In this way, the

Eq.(1) can be written as:

$$I = I_{ph} - I_0 \left[ \exp\left(\frac{V + R_s I}{nV_{th}}\right) - 1 \right] \quad (3)$$

Then,

$$\exp\left(\frac{R_s I}{nV_{th}} + \frac{V}{nV_{th}}\right) = -\frac{I}{I_0} + \frac{I_{ph} + I_0}{I_0} \quad (4)$$

Independent variable  $I$  is replaced by  $x$ , then the solution of Eq. (4) can be expressed as<sup>[11]</sup>:

$$\exp(ax + b) = cx + d \quad (5)$$

Variables are defined as follows:

$$\begin{cases} \frac{R_s}{nV_{th}} = a \\ \frac{V}{nV_{th}} = b \\ \frac{1}{-I_0} = c \\ \frac{I_{ph} - I_0}{I_0} = d \end{cases} \quad (6)$$

Multiply  $-\frac{a}{c} \exp(-ax - \frac{ab}{c})$  at both sides of the equation, then  $x$  can be calculated by:

$$x = -\frac{d}{c} - \frac{1}{a} W\left[-\frac{a}{c} \exp\left(\frac{bc - ab}{c}\right)\right] \quad (7)$$

Substitute  $a, b, c, d$  with Eq.(6):

$$I = \frac{R_{sh}(I_{ph} + I_0) - V}{R_s + R_{sh}} - \frac{nV_{th}}{R_s} W(X), \quad (8)$$

Here,

$$X = \frac{R_s R_{sh} I_0}{nV_{th}(R_s + R_{sh})} \exp\left(\frac{R_{sh}(R_s I_{ph} + R_s I_0 + V)}{nV_{th}(R_s + R_{sh})}\right).$$

Integral-equation method is also available for the formulation simplification. But differential-equation method is utilized here for the simpler computation. The differential  $dV/dI$  can be expressed as:

$$\frac{dV}{dI} = -R_s + \frac{nV_{th} R_{sh}}{V + (R_s + R_{sh})I - \frac{nV_{th}}{R_{sh}}(I_0 - I_{ph})} \quad (9)$$

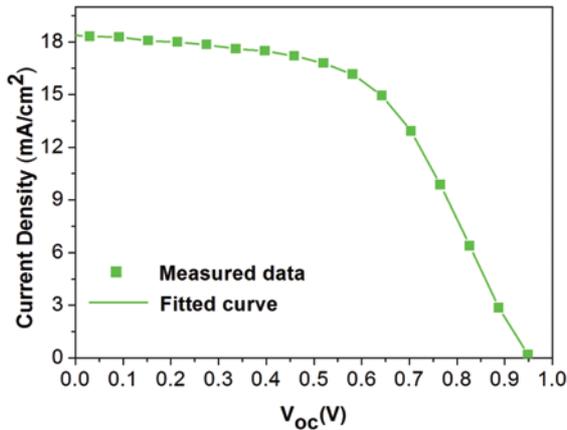
This equation can be greatly simplified with algebraic method, and then the differential  $dV/dI$  can be given as below:

$$\frac{dV}{dI} = -C_1 - \frac{C_2}{-V - C_3 I - C_4} \quad (10)$$

It is not an implicit equation anymore, and the main advantage is speed. The number of unknown parameters is reduced from five ( $R_s, R_{sh}, n, I_{ph}, I_0$ ) to four ( $C_1, C_2, C_3, C_4$ ), which are defined by the following expressions:

$$\begin{cases} C_1 = R_s, \\ C_2 = nV_{th}R_{sh}, \\ C_3 = R_s + R_{sh}, \\ C_4 = nV_{th} + R_{sh}(I_0 + I_{ph}) \end{cases} \quad (11)$$

Thus, the value of the five parameters can be easily obtained by solving the Eq. (11).



**Figure 2.** I - V characteristics of perovskite solar cells.

The curve fitting is performed with practical measured I-V data, this I-V curve of a perovskite solar cell was plotted in Figure 2. Moreover, this set of data is fitted with eleven-degree polynomial function in MATLAB. Four point were chose from the I-V curve to get the values of  $R_s$ ,  $R_{sh}$ ,  $n$ ,  $I_{ph}$  and  $I_0$  according to Eq.(11). Calculating the coefficients of algebraic equation has simplified the process of obtaining higher quality results. The exact expressions of  $I_{ph}$  and  $I_0$  can be obtained by combining Eqs.(12) and (13)<sup>[11]</sup>:

$$I_{ph} = \frac{C_4 - C_2 / (C_3 - C_1)}{C_3 - C_1} \quad (12)$$

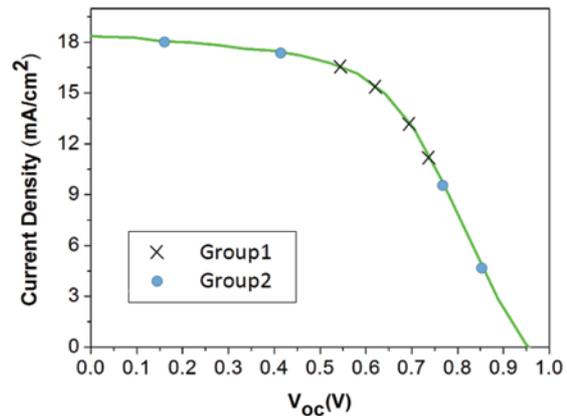
$$I_0 = \frac{(I_{ph} - I)R_{sh} - (V - IR_s)}{R_{sh}(\exp((V + IR_s) / nV_{th}) - 1)} \quad (13)$$

### 3 Result and discussion

A method suitable for obtaining parameters has been developed. In the extraction process, it is found that the calculation results were closely related to the four sample points, these points are chose to solve the equations.

Furthermore, tests were carried out for comparison. First, points were only selected from the left side or right side of the max power point respectively. We failed to get reasonable results.

Then, two groups of points were chose from the I-V curve for parameters extraction, the positions of these points are shown in Figure 3. Two points from each side of the max power point were selected. But the distances between any two adjacent points of the Group1 are shorter than that of the Group2. The calculation results were shown in Table 1.



**Figure 3.** The selected points in the I-V curve for parameters extraction.

In order to estimate the accuracy of our method, the parameters were also extracted by Quasi-Newton method as Table1 shown. When comparing the extracted results of three groups, considerable error can be found in the results of Group2. The value of  $R_s$  in Group2 is about 20  $\Omega$  lower than others, and the difference of  $I_0$  is more than an order of magnitude. By comparison, disparities between Group1 and Quasi-Newton methods are unremarkable.

**Table 1.** Results of parameters extraction by Lambert W-function.

	$R_s(\Omega)$	$R_{sh}(\Omega)$	n	$I_{ph}(A)$	$I_0(A)$
Group 1	93.08366	3622.54335	2.23511	0.00169	$9.26446 \times 10^{-8}$
Group 2	73.35790	3308.57911	2.18622	0.00170	$1.07354 \times 10^{-7}$
Quasi-Newton methods	91.00027	3623.95822	2.23999	0.00169	$9.30074 \times 10^{-8}$

According to the slight variation between Group1 and Quasi-Newton methods, the results which solved by the four points of Group1 are regarded as the true parameters in the perovskite solar cells. This conclusion was proved to be reliable after repeating this process. The accuracy of result is largely dependent on the position of selected points. In order to get accurate presentation, the calculation points should be chose on both sides of the max power point with an appropriate distance between each others. When the starting point and the ending point are chose nearer the short-circuit current and the open-circuit voltage respectively, the values of parameters exhibit higher quality, and show a better fit to the consequence of Quasi-Newton method.

## 4 Conclusion

In this paper, an efficient method has been proposed to extract all the parameters of a solar cell's mathematical model. Based on a single I-V curve under the one-diode photovoltaic model, parameters can be easily calculated. With the help of Lambert W-function, the explicit analytic expression for I-V relationship can be obtained. This expression is directly used to determine the values of the parameters through the selecting points form fitted curve. It is shown that our method can be easily used to analyze perovskite solar cells. The result accuracy of this method is as high as that of the numerical methods. But the computational process is simpler, and the computational speed is faster.

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