

The Effect of the Rotor Static Eccentricity on the Electro-Mechanical Coupled Characteristics of the Motorized Spindle

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Abstract. High-speed motorized spindle is a multi-variable, non-linear and strong coupling system. The rotor static eccentricity is inevitable because of machining or assembling error. The rotor static eccentricities have an important effect on the electromechanical coupled characteristics of the motorized spindle. In this paper, the electromechanical coupled mathematical model of the motorized spindle was set up. The mathematical model includes mechanical and electrical equation. The mechanical and electrical equation is built up by the variational principle. Furthermore, the inductance parameters without the rotor static eccentricity and the inductance parameters with rotor static eccentricity have been calculated by the winding function method and the high speed motorized spindle was simulated. The result show that the rotor static eccentricity can delay the starting process of the motorized spindle, and at steady state, the rotor circuit currents are still large because of the rotor static eccentricity.

1 Introduction

The high speed motorized spindle is a multi-variable, non-linear and strong coupling system. The electromechanical coupled phenomenon of motorized spindle has an important influence to its performance. Currently, each components of the high-speed motorized spindle is designed individually. This method separate mechanical parameters from electromagnetic parameters and thermal parameters of spindle system. Consequently, the electro-mechanical coupled phenomenon of the high-speed motorized spindle is neglected. So it is necessarily to analyze the multi-physical processes and multidimensional coupling characteristics of the high-speed motorized spindle.

Generally, the multi-physical coupled phenomenon of the high-speed motorized spindle unit has attracted attention in the literature. The thermal model was coupled with the spindle dynamic model through bearing heat generation and thermal expansion of the whole system based on the bearing configuration. The thermo-mechanical model of the high-speed motorized spindle was set up in [1]. It was shown that bearing orientation had a significant effect on spindle stiffness. A complete bearing load-deflection analysis including thermal expansion was derived and was coupled with an analysis of spindle dynamic response in [2]. Several methods setting up the electro-mechanical coupling modeling were given in [3]. Founded on the calculus of variations [4], the electro-mechanical coupling dynamical model of the motorized spindle was developed in [5], which provided the theoretical basis for further research on the electromechanical coupled dynamical performance of high speed motorized spindle system. A modified

mechanical-electric coupling model of the high speed grinding motorized spindle was created in [6], and a new strategy of suppressing high-order harmonic mechanical-electric coupling vibration by optimizing inverter operating parameters was proposed. An integrated model was presented in [7] to study the electro-thermo-mechanical coupled dynamic characteristics of motorized spindles. And the coupling relationship among the electrical, thermal, and mechanical behaviors of the motorized spindle was explained. In order to predict and control the thermal properties of motorized spindles under work condition and their effects on the dynamic characteristics effectively, a bearing thermo-mechanical dynamic model which takes preload methods and thermal responses into account is presented in [8], and then friction loss and support stiffness of the bearing are analyzed.

The present studies have got great achievements on thermo-mechanical coupled and electro-mechanical coupled properties of the motorized spindle. However, the rotor static eccentricities also have an important effect on the electromechanical coupled characteristics of the motorized spindle. In this paper, the electromechanical coupled mathematical model of the motorized spindle was set up and the high speed motorized spindle was simulated. The effect of the rotor static eccentricity on the electromechanical coupled characteristics of the motorized spindle is revealed.

2 The electromechanical coupled mathematical model of high speed motorized spindle

2.1 The Structure Characteristics of the Motorized Spindle

The motor of the high-speed motorized spindle is directly attached inside the spindle housing and mounted on the rotating shaft by interference fit. The spindle of the machine tool is directly driven by the motor. Discarding the traditional belt drive and gear drive, the high-speed motorized spindle is a new structure of the spindle and it makes the high-speed machine tool achieve to zero-transmission. Eliminating the intermediate links between the electric motor and the spindle, which greatly simplifies the structure of the main transmission system of the machine tools and the power of the motor is passed to the tool of the machine tools directly. Figure 1 is a typical structure of the high-speed motorized spindle.

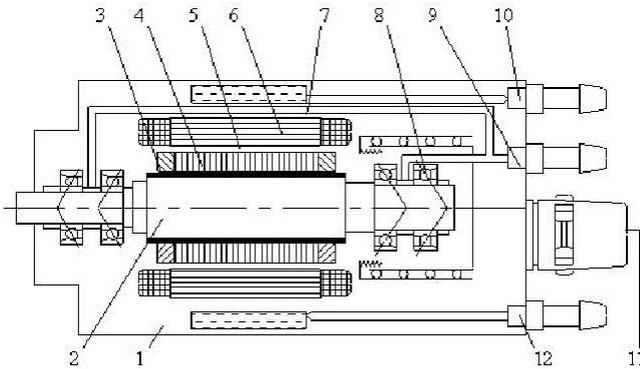


Figure 1. A typical structure of the high-speed motorized spindle

- 1. spindle housing 2. spindle 3. Interference connection sleeve
- 4. rotor 5. air gap 6. stator 7. coolant jacket
- 8. bearing 9. lubricating oil entrance 10. coolant outlet 11. power interface 12. coolant entrance

2.2 The Physical Model of the High-Speed Motorized Spindle

The motor of the high-speed motorized spindle is directly mounted on the rotating shaft by interference fit. Figure 2 is its physical model.

Theoretically, the high-speed motorized spindle can be regarded as a high speed motor in. According to the structure characteristics of motorized spindle, the motorized spindle can be simplified as a high speed motor. When the three-phase alternating current inputted into the stator winding of the high speed electric spindle, the stator winding produces a rotating magnetic field. The rotating magnetic field passes through the air gap and chains with the rotor winding. Simultaneously, the rotating magnetic field cuts the rotor winding and the three-phase induction current emerges in the rotor winding. The rotor current is affected by the magnetic force in the magnetic field, which makes the rotor whirling.

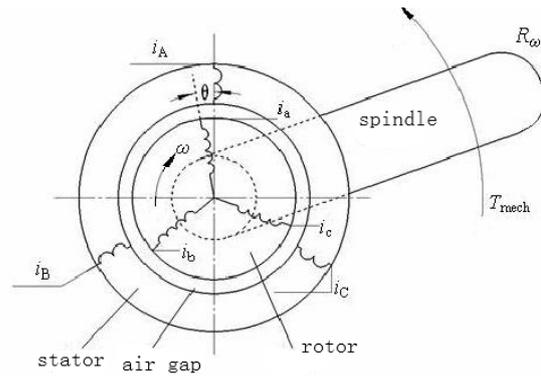


Figure 2. The physical model of the high-speed motorized spindle

2.3 The Electromechanical Coupled Mathematical Model of High-Speed Motorized Spindle

Grounded the physical model, using the variational principle the electromechanical coupled mathematical model of high speed motorized spindle can be set up [9].

The kinetic energy of the system is

$$T = W_m + W_{mech} = \frac{1}{2} \sum_j \sum_m L_{jm} i_j i_m + \frac{1}{2} J \theta^2 \quad (1)$$

where j, m stand for A, B, C, a, b, c and W_m is the kinetic energy of the electromagnetic system and W_{mech} is the kinetic energy of mechanical system and L_{jm} is the inductance parameters of electromagnetic system and J is moment of inertia.

The potential energy of the system is

$$V = 0 \quad (2)$$

Lagrange function is

$$L = T - V = T \quad (3)$$

The dissipation function of the system is

$$F_R = \frac{1}{2} R_s \sum_{k=1}^3 i^2 + \frac{1}{2} R_r \sum_{k=4}^6 i^2 + \frac{1}{2} R_n n^2 \quad (4)$$

where R_s is the stator resistance and R_r is the rotor resistance and R_n is the electromagnetic loss impedance and n is speed of the rotor.

The voltage of each loop is regarded as the generalized force. Substituting equation (1), (2), (3), (4) into the Lagrange equation, can get the voltage equation of each loop.

$$\mathbf{U} = \mathbf{L} \frac{d\mathbf{I}}{dt} + \mathbf{I} \frac{d\mathbf{L}}{dt} + \mathbf{I}\mathbf{R} \quad (5)$$

As usual \mathbf{U} is the voltage matrix, \mathbf{L} is the matrix of inductance, \mathbf{I} is the current matrix, \mathbf{R} is the resistance matrix. The external load W_{mech} is regarded as the

generalized force. The mechanical equation of the system is

$$T_e = J \frac{dn}{dt} + R_n n + T_{mech} \quad (6)$$

The electromechanical coupled mathematical model of high speed motorized spindle includes the voltage equation of each loop and the mechanical equation of the system. Simultaneous equations of voltage equation and mechanical movement equation are the mathematical model of the system. Therefore, the electro-mechanical coupled mathematical model of the high speed motorized spindle is

$$\begin{cases} \mathbf{U} = \mathbf{L} \frac{d\mathbf{I}}{dt} + \mathbf{I} \frac{d\mathbf{L}}{dt} + \mathbf{I}\mathbf{R} \\ T_e = J \frac{dn}{dt} + R_n n + T_{mech} \\ \frac{d\theta}{dt} = n_p n \end{cases} \quad (7)$$

where n_p is pole number of the motor and

$$T_e = \frac{1}{2} n_p \mathbf{I}^T \mathbf{L} \mathbf{I} \quad (8)$$

In order to further study on the spindle system, the unknown parameters in the mathematical model must be calculated. The most important parameters are the inductance parameters of each loop. Because of machining or assembling error, the rotor static eccentricity is inevitable. The static eccentricity of the rotor leads to emerge the dislocation between the stator and the rotor. The dislocation can cause distortion of the inductance parameters. So, the inductance parameters of each loop with static eccentricity must be calculated.

3 The inductance parameters calculation

3.1 The effective air-gap length

If the rotor exists static eccentricity, the air-gap length must be fixed. The effective air-gap length in the case of eccentricity can be described as, [10]

$$a(\theta) = a_0 - a_0 \rho \cos \theta \quad (9)$$

where θ is the angular position of winding and a_0 is the air-gap length in the case of no eccentricity and ρ is degree of eccentricity.

3.2 The Winding Function

The winding function is the function of air gap magnetic wave inputting one ampere of current into the winding. As show in [11], the winding function of winding x is given by

$$P(\theta) = \frac{2}{a_0 \sqrt{1-a^2(\theta)}} \sum_{i=0}^{\infty} \left(\frac{1 - \sqrt{1-a^2(\theta)}}{a(\theta)} \right)^i \cos i\theta \quad (10)$$

where i is the ordinal number of the current harmonics.

3.3 The Expression for Inductance Parameters

The inductance between any two windings “x” and “y” in any electrical machine is given by the following expression, [12]

$$L_{xy} = \mu_0 l r \int_{-\pi}^{\pi} P_x(\theta) P_y(\theta) a^{-1}(\theta) d\theta \quad (11)$$

where $P_x(\theta)$ is winding function of the winding “x” and $P_y(\theta)$ is winding function of the winding “y” and μ_0 is permeability of vacuum and l is the length of the stack, r is the radius of the stator.

The element of the matrix of inductances in expression (7) and (8) can be calculated by the expression (11). So all inductance parameters can be calculated. The basic parameters of the specific high speed motorized spindle studied in this paper are show in Table I .

Table I Original parameters of the high speed motorized spindle

Parameter	Parameter value	Parameter	Parameter value	Parameter	Parameter value
Rated power (kw)	20	Rated current (A)	46	Rotor resistance (Ω)	2.00
Rated speed (kr/min)	15	Rated torque (Nm)	12.7	Pole number	4
Rated voltage (V)	350	Stator resistance (Ω)	0.50	Rotor slot	28

The inductance parameters matrix of the stator without the rotor static eccentricity is

3.4 Inductance Parameters of the Stator

According the relative position of each stator coil, the inductance parameters of the stator can be calculated by expression (10).

$$\mathbf{L}_{ss} = \begin{bmatrix} 0.2614 & -0.1362 & -0.1362 \\ -0.1362 & 0.2614 & -0.1362 \\ -0.1362 & -0.1362 & 0.2614 \end{bmatrix}$$

The inductance parameters matrix of the stator with 20% rotor static eccentricity is

$$\mathbf{L}_{ss} = \begin{bmatrix} 0.2645 & -0.1358 & -0.1358 \\ -0.1189 & 0.2645 & -0.1358 \\ -0.1358 & -0.1358 & 0.2645 \end{bmatrix}$$

3.5 Inductance Parameters of the Rotor

The inductance parameters matrix of the rotor \mathbf{L}_{rr} is 29×29 . Figure3 shows the inductance parameter between 1st rotor loop and other rotor loop without the rotor static eccentricity. In Figure7 n is the serial number of the rotor loop. Figure4 shows the inductance parameter between 1st rotor loop and other rotor loop with 20% rotor static eccentricity.

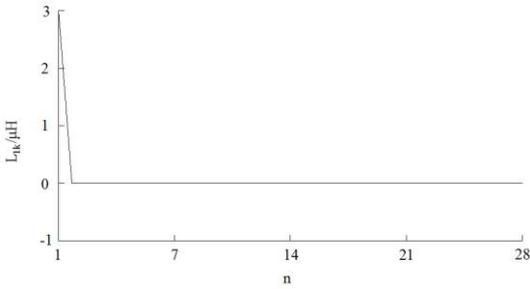


Figure 3. The inductance between rotor loop and other rotor loop without the rotor static eccentricity

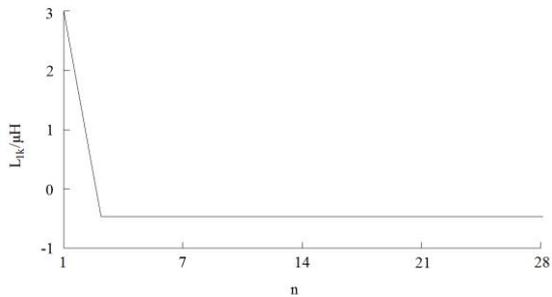


Figure 4. The inductance between rotor loop and other rotor loop with 20% rotor static eccentricity.

3.6 Inductance Parameters between the Stator and the Rotor

Figure5 shows the inductance parameter between the phase A of the stator and each of rotor loops without the rotor static eccentricity. Figure6 shows the inductance

parameter between phase A of the stator and each of rotor loops with 20% rotor static eccentricity.

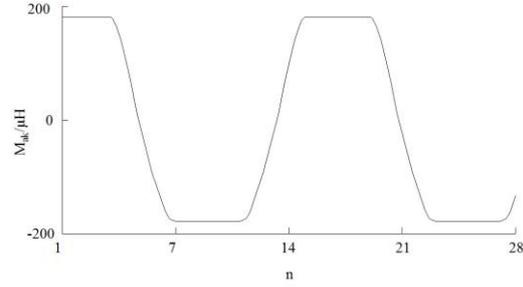


Figure 5. The inductance between the phase A of the stator and each of rotor loop without the rotor static eccentricity

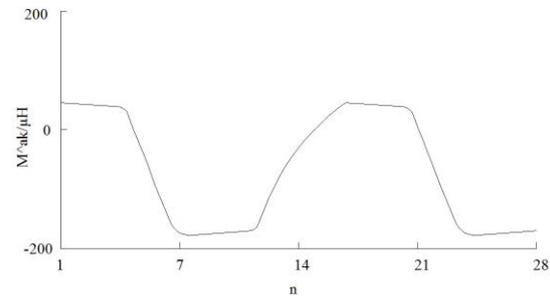


Figure 6. The inductance between phase A of the stator and each of rotor loop with 20% rotor static eccentricity

4 Simulations and verification

According to the expression (7), the state equation of the high speed motorized spindle can be summed as

$$\frac{d}{dt} \begin{bmatrix} \mathbf{I} \\ n \\ \theta \end{bmatrix} = \begin{bmatrix} -\mathbf{L}(\frac{d\mathbf{L}}{dt} + \mathbf{R}) & 0 & 0 \\ \frac{1}{2J} \mathbf{I}^T \frac{\partial \mathbf{L}}{\partial \theta} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{I} \\ n \\ \theta \end{bmatrix} + \begin{bmatrix} \mathbf{L}^{-1} & 0 \\ 0 & -\frac{1}{J} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ T_{mech} \end{bmatrix} \quad (12)$$

Using the basic parameters in Table.1 and the inductance parameters calculated previously, the high speed motorized spindle was simulated. The motorized spindle is started without load and the load of 10Nm is applied to the motorized spindle after completing the starting process (When the motorized spindle operates for 20s, the load of 10Nm is applied to the motorized spindle).

Figure7 shows the stator phase “A” current, rotor current, spindle speed and electromagnetic torque for no-load condition without the rotor static eccentricity. Figure8 shows the stator phase “A” current, rotor current, spindle speed and electromagnetic torque for no-load condition with 20% rotor static eccentricity.

As show in Figure7, it took 10s to complete the starting process and the rotor circuit current is very low

after the motorized spindle entering the rated condition without load. After the load of 10Nm is applied to the motorized spindle, the motorized spindle quickly enter a new dynamic equilibrium. As show in Figure8, it took 15s to complete the starting process and the rotor circuit current is still large after the motorized spindle entering the rated condition without load. After the load of 10Nm is applied to the motorized spindle, the motorized spindle enter a new dynamic equilibrium slowly.

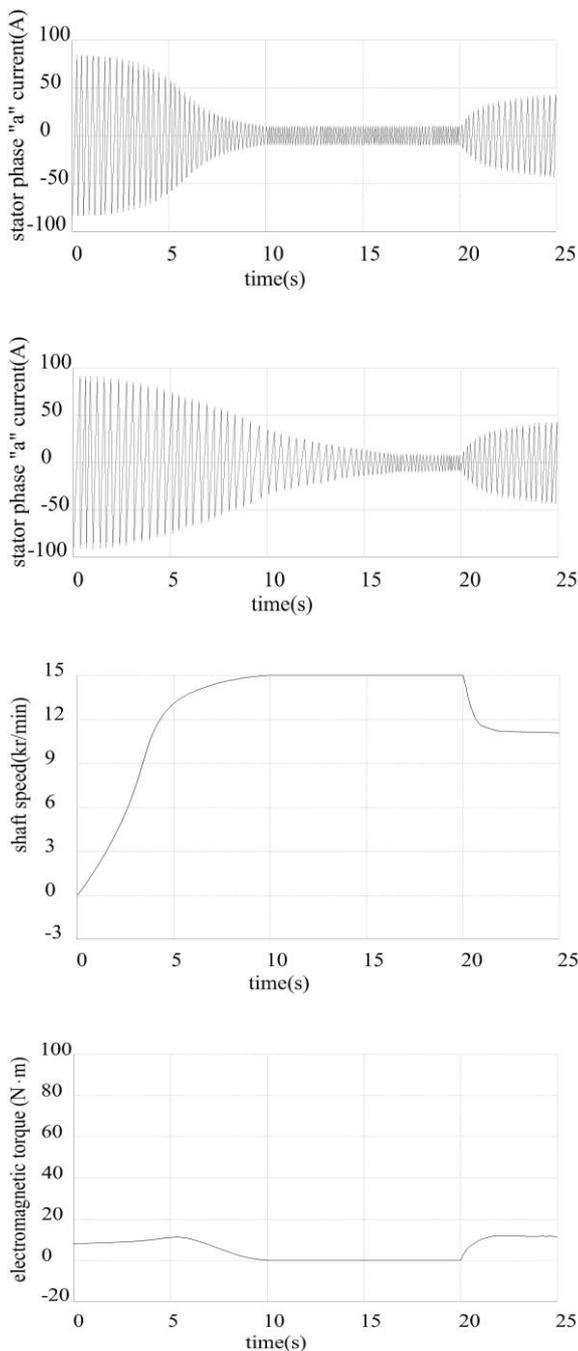


Figure7. The stator phase “A” current, rotor current, spindle speed and electromagnetic torque without the rotor static eccentricity

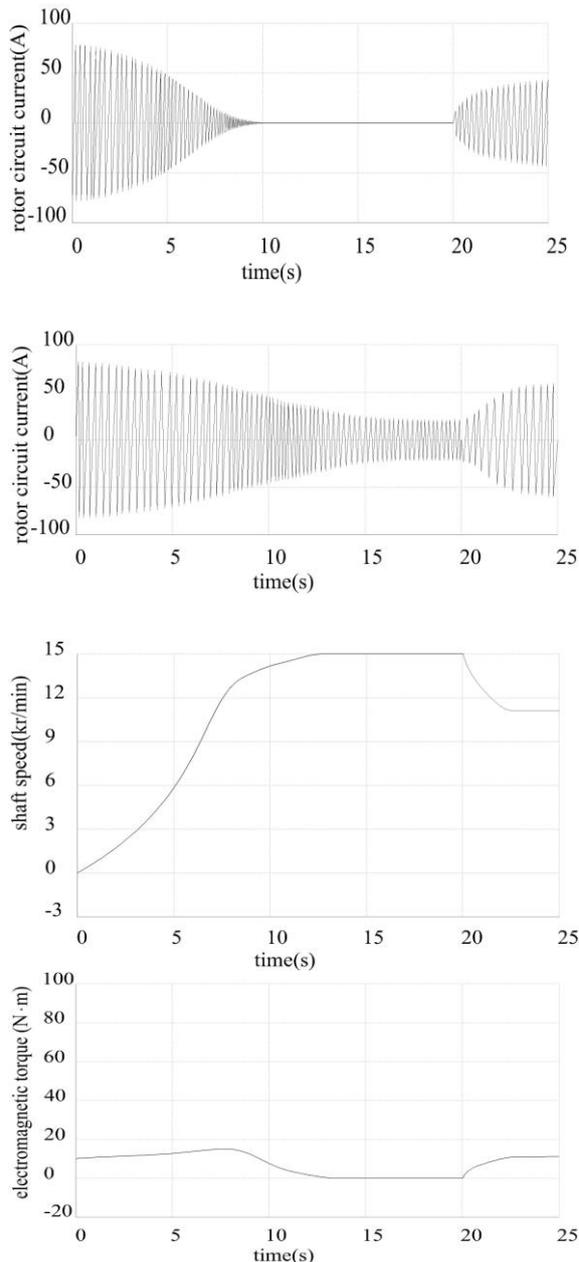


Figure 8. The stator phase “A” current, rotor current , spindle speed and electromagnetic torque with 20% rotor static eccentricity

Compared Figure7 with Figure8, it is clear that the starting time under 20% rotor static eccentricity is larger than under healthy rotor. Because of the rotor static eccentricity, the magnetomotive force in air gap is delayed. The process magnetic energy transforming into mechanical energy is delayed, so the start time is extended.

It is clear that at steady state, the rotor circuit currents in healthy motorized spindle are very low and can be neglected. However, the rotor circuit currents under 20% rotor static eccentricities are also large. At steady state, the spindle speed very close to the synchronous speed. The cutting action between the rotor and rotating magnetic field is very weak so the rotor circuit currents in healthy motorized spindle are very low. Under 20% rotor static eccentricity, although the rotor speed is very close

to synchronous speed, the cutting action between the rotor and rotating magnetic field is also strong because of the rotor static eccentricity. The induced electromotive force in the rotor is still high so the rotor circuit currents under 20% rotor static eccentricities are also large.

In addition, after the load is applied to the motorized spindle, it takes more time to enter a new dynamic equilibrium in Figure 8. This delay in achieving the new dynamic equilibrium can be attributed to the generation of backward magnetomotive force due to the static eccentricity.

5 Conclusions

This paper has set the electromechanical coupled mathematical model of high speed motorized spindle by variational principle. The mathematical model can quantitatively explain the mechanical-electrical energy conversion, particularly the mutuality between inputting voltage, electric current, outputting torque and rotate speed. The inductance parameters without the rotor static eccentricity and the inductance parameters with 20% rotor static eccentricity have been calculated. Furthermore, the high-speed motorized spindle was simulated and the following have been proved conclusively.

- 1) The rotor static eccentricity can delay the starting process of the high speed motorized spindle.
- 2) At steady state, the rotor circuit currents are still large because of the rotor static eccentricity.
- 3) After the load is applied to the motorized spindle, it takes more time to enter a new dynamic equilibrium because of the rotor static eccentricity.

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