

by the potential energy E of the spring, and the mass-spring system obtain the Newton's second law.

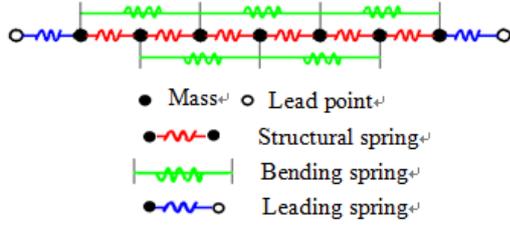


Figure 2. The model of cable

$$\ddot{\mathbf{X}} = \mathbf{M}^{-1} \left(-\frac{\partial E}{\partial \mathbf{X}} + \mathbf{F} \right) \quad (1)$$

The configuration of cable harness is defined by the position of mass points. Resolving this equation could get the stable configuration.

2.2 The Definition of Spring

2.2.1 Structural Spring

Structural spring connecting the two successive points will preserve the length of cable. The energy function for the structural spring is

$$E_i(\mathbf{x}) = \frac{k_s}{2} (|\mathbf{x}_i - \mathbf{x}_{i-1}| - l) (|\mathbf{x}_i - \mathbf{x}_{i-1}| - l) \quad (2)$$

Where k_s is the elastic modulus of the structural spring, and l is the natural length. The restoring forces acting on the i th particle as the first derivative of the energy is

$$\begin{aligned} f_{2i}(\mathbf{x}) &= -\frac{\partial E}{\partial \mathbf{x}} = -\left(\frac{\partial E_i}{\partial \mathbf{x}_i} + \frac{\partial E_{i+1}}{\partial \mathbf{x}_i} \right) \\ &= k_s \left(\frac{l}{|\mathbf{x}_i - \mathbf{x}_{i-1}|} - 1 \right) (\mathbf{x}_i - \mathbf{x}_{i-1}) \\ &\quad - k_s \left(\frac{l}{|\mathbf{x}_{i+1} - \mathbf{x}_i|} - 1 \right) (\mathbf{x}_{i+1} - \mathbf{x}_i) \end{aligned} \quad (3)$$

The damping spring is designed to show the internal damping of the cable during the configuration simulation. And the damping force is:

$$\begin{aligned} f_{di}(\mathbf{x}) &= -k_d \frac{(\mathbf{x}_i - \mathbf{x}_{i-1})(\mathbf{x}_i - \mathbf{x}_{i-1})^T}{|\mathbf{x}_i - \mathbf{x}_{i-1}|^2} (\mathbf{v}_i - \mathbf{v}_{i-1}) \\ &\quad + k_d \frac{(\mathbf{x}_{i+1} - \mathbf{x}_i)(\mathbf{x}_{i+1} - \mathbf{x}_i)^T}{|\mathbf{x}_{i+1} - \mathbf{x}_i|^2} (\mathbf{v}_{i+1} - \mathbf{v}_i) \end{aligned} \quad (4)$$

2.2.2 Bending Spring

Bending spring is a linear spring connecting the i th mass point with the $(i+2)$ th point used to describe the elastic bending behavior, as the dashed line shown in Figure 3.

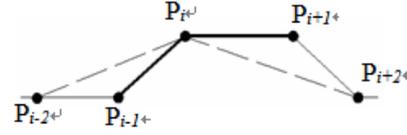


Figure 3. The bending spring

The energy of the bending spring in terms of the positions of the mass points is:

$$E_i(\mathbf{x}) = \frac{k_b}{2} (|\mathbf{x}_i - \mathbf{x}_{i-2}| - 2l) (|\mathbf{x}_i - \mathbf{x}_{i-2}| - 2l) \quad (5)$$

Where k_b is the elastic modulus of the bending spring. The restoring forces acting on the x_i particle as the first derivative of the energy is

$$\begin{aligned} f_{2i}(\mathbf{x}) &= -\frac{\partial E}{\partial \mathbf{x}} = -\left(\frac{\partial E_i}{\partial \mathbf{x}_i} + \frac{\partial E_{i+2}}{\partial \mathbf{x}_i} \right) \\ &= k_b \left(\frac{l}{|\mathbf{x}_i - \mathbf{x}_{i-2}|} - 1 \right) (\mathbf{x}_i - \mathbf{x}_{i-2}) \\ &\quad - k_b \left(\frac{l}{|\mathbf{x}_{i+2} - \mathbf{x}_i|} - 1 \right) (\mathbf{x}_{i+2} - \mathbf{x}_i) \end{aligned} \quad (6)$$

2.2.3 Leading Spring

In assembly product, the shape of cable harness is affected by the plug and bandage. A leading spring is assumed as the plug to constraint the position of mass. Leading spring is similar with the bending spring; however the stiffness is much greater. The restoring force is proportional in angle, as shown in Figure 4. Where the i th mass is a fixed point and the $(i+1)$ th mass is a movable point.

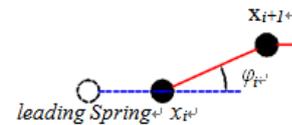


Figure 4. The leading spring

The energy of the leaning spring is given by following formula, where k_l is the elastic modulus of the leading spring:

$$E_i(\mathbf{x}) = \frac{k_l}{2} \varphi_i^2 \quad (7)$$

In order to simplify the expressions we set $\mathbf{a}_i = (\mathbf{x}_{i+1} - \mathbf{x}_i)$, and \mathbf{b} is the unit vector of the direction of leading spring, therefore the angle can be expressed as:

$$\varphi_i = \frac{k_l}{2} \left[\tan^{-1} \left(\frac{|\mathbf{a}_i \times \mathbf{b}|}{\mathbf{a}_i \cdot \mathbf{b}} \right) \right]^2 \quad (8)$$

Leading spring only works on the points beside the fix points. The restoring forces acting on the i th particle which is after the fixed points is:

$$f_{3i}(\mathbf{x}) = -\frac{\partial E_{i-1}}{\partial \mathbf{x}_i} = -k_l \varphi_{i-1} \frac{\partial \varphi_{i-1}}{\partial \mathbf{a}_{i-1}} \quad (9)$$

$$= k_l \varphi_{i-1} \frac{\mathbf{a}_{i-1} \times (\mathbf{a}_{i-1} \times \mathbf{b})}{\mathbf{a}_{i-1}^2 |\mathbf{a}_{i-1} \times \mathbf{b}|}$$

Meanwhile, the restoring forces acting on the i th particle which is before the fixed points is:

$$f_{3i}(\mathbf{x}) = -\frac{\partial E_{i+1}}{\partial \mathbf{x}_i} = k_l \varphi_{i+1} \frac{\mathbf{a}_{i+1} \times (\mathbf{a}_{i+1} \times \mathbf{b})}{\mathbf{a}_{i+1}^2 |\mathbf{a}_{i+1} \times \mathbf{b}|} \quad (10)$$

3 Computation of the Equation

We adopted the *Newmark-β* method which is used to resolve structural dynamics in blast wave or seismic load [8] to compute the differential equation. *Newmark-β* method is an implicit method that is possible to improve the stability.

According to the *Newmark-β* method, the position and the velocity of i th point mass on time t can be obtain by the following formulas:

$$\begin{cases} \mathbf{x}_t = \mathbf{x}_{t+\Delta t} + \Delta t \cdot \dot{\mathbf{x}}_{t+\Delta t} + \frac{1}{4} \Delta t^2 (\ddot{\mathbf{x}}_{t+\Delta t} + \ddot{\mathbf{x}}) \\ \dot{\mathbf{x}}_t = \dot{\mathbf{x}}_{t+\Delta t} + \frac{1}{2} \Delta t^2 (\ddot{\mathbf{x}}_{t+\Delta t} + \ddot{\mathbf{x}}) \end{cases} \quad (11)$$

Based on the equation from 1-10, the motion equation of cable could be deduced as a second-order differential equation set:

$$\begin{cases} f_{ix}(\ddot{x}_i, \ddot{y}_i, \ddot{z}_i) = 0 \\ f_{iy}(\ddot{x}_i, \ddot{y}_i, \ddot{z}_i) = 0 \\ f_{iz}(\ddot{x}_i, \ddot{y}_i, \ddot{z}_i) = 0 \end{cases} \quad (12)$$

The integration methods of an ordinary differential equation include explicit methods and implicit methods. Equation 12 is solved with Newton iteration. And acceleration in three coordinate axes is gotten. Then put acceleration in equation 11 to get the numerical solution of position and velocity. The shape of the cable harness is obtained by repeating the above calculations until the velocity becomes smaller than some specified threshold. Therefore the shape of cable could be simulated.

4. Experimental Verification

In order to simulate the configuration of cable based on the above model, a simulation platform is edited based on Open Cascade (OCC) software. In the platform, the calculation result of above model is connected with a B-spline curve. And then with the `::onPipe()` command, the configuration of cable is realized by lofting a circle shape

section on that curve. The Simulation result is given in Figure5 with the condition is that the distance between the two ends is 70cm, angle between the directions of two ends is 60 degree, and the height difference is 30cm. And the other parameters are shown in Table1.

Table1: The Value of Parameters

Parameters	Value
length(cm)	100
segments	20
k_s/m	$4.75 \cdot 10^3$
k_b/m	$5 \cdot 10^2$
k_d/m	$2.5 \cdot 10^3$
k_l/m	$3 \cdot 10^3$



Figure 5. Simulation of cable

To verify the correctness of the mass-spring model and calculation algorithm, an experimental bench is constructed based on 3D laser scan technology, as shown in Figure 6. The bench includes an aluminum base, clamp mechanism, articulated arm laser scanner and a pc. Clamp mechanism includes straight rod, cross universal joint and rod holder. The position and angle of cable's two-end could be gotten through the graduation in the holder. We use Geomagic Qualify, the reverse engineering software, to process the scanning data, and then the 3D geometry configuration could be gained.

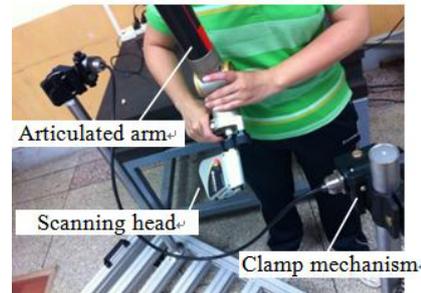


Figure 6. Configuration of cable

5. Conclusion

In this paper, the simulation of cable harness is complete based on an improved mass-spring model. *Newmark-β* method is used to get spatial deformable of cable geometry configuration. Moreover, the actual geometry configuration of cable is achieved with 3D scanning technology. Through comparison, the accuracy of the model has been verified. In the future, we mainly focus on the design of the parameters of the different kinds of spring.

6. Acknowledgement

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